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Resolving the pulsations of subdwarf B stars: PG 0154+182, HS 1824+5745 and HS 2151+0857

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ABSTRACT

We continue our programme of extended single-site observations of pulsating subdwarf B (sdB) stars and present the results of extensive time-series photometry to resolve the pulsation spectra for use in asteroseismological analyses. PG 0154+182, HS 1824+5745 and HS2151+0857 were observed at the MDM Observatory during 2004 and 2005. Our observations are sufficient to resolve the pulsations of all three target stars. We extend the number of known frequencies for PG 0154+182 from one to six, confirm that HS 1824+5745 is a monoperiodic pulsator and extend the number of known frequencies to five for HS 2151+0857. We perform standard tests to search for multiplet structure, measure amplitude variations as pertains to stochastic excitation and examine the mode density to constrain the mode degree ℓ.

Key words: stars: individual: PG 0154+182 – stars: individual: HS 1824+5745 – stars: individual: HS 2151+0857 – stars: oscillations - subdwarfs – stars: variables: other.

1 INTRODUCTION

Subdwarf B (sdB) stars are thought to be stars with masses about 0.5 $M_\odot$ with thin ($<10^{-2}$ $M_\odot$) hydrogen shells and temperatures from 22 000 to 40 000 K (Heber et al. 1984; Saffer et al. 1994), making them exceedingly blue. Pulsating sdB stars come in two varieties: short period (90–600 s) named EC 14026−2647 stars after that prototype, officially V361 Hya stars or sdBV stars, with amplitudes typically near 1 per cent and long period (45 min to 2 h) named PG 1716+426 stars after that prototype or LPsdBV stars, with amplitudes typically <0.1 per cent. For more on these stars, see Kilkenny (2001) and Green et al. (2003). For this work, our interest is the EC 14026 (sdBV) class of pulsators as their periods are short, so many pulsation cycles can be observed during each run from a single site.

Pulsating sdB stars potentially allow the opportunity to discern their interior structure using asteroseismology, obtaining estimates of total mass, luminosity, shell mass, radiative levitation, gravitational diffusion and helium fusion cross-sections. However, to apply the tools of asteroseismology, the pulsation frequencies (periods) must first be resolved. Variable star discovery surveys seldom resolve or detect the complete set of pulsations. Multisite campaigns, because of the complexity of organization, have only observed a few sdB pulsators.

Our programme is to resolve poorly studied sdB pulsators, typically from single-site data. This method has proven useful for the sdBV stars Feige 48 and KPD 2109+4401 (Reed et al. 2004; Zhou et al. 2006). For these observations, we obtained most of our data at a modest sized telescope (1.3 m) combined with higher signal-to-noise ratio (S/N) observations on a 2.4-m telescope. This combination allows us to effectively resolve the pulsation spectrum of sdBV stars and detect low-amplitude pulsations that were below the detection limit of the discovery data. Our high S/N data from larger telescopes can detect pulsation amplitudes as low as 0.5 millimodulation amplitudes (mma; equivalent to 0.05 per cent), insuring that we do not miss low-amplitude modes. Our typical temporal resolution is better than 1.5 μHz.

This paper reports the results of our follow-up observations on the sdBV stars PG 0154+182 (hereafter PG 0154), HS 1824+5745 (hereafter HS 1824) and HS 2151+0857 (hereafter HS 2151). PG 0154 (B = 15.3) was reported as a pulsator by Koen et al. (2004, hereafter K04) who detected a single frequency at 6090 μHz. Their observations consisted of three <3 h observing runs which were obtained under less than ideal conditions and were certainly insufficient to resolve the pulsations. From optical spectroscopy and Two Micron All-Sky Survey (2MASS) infrared data, they also determined that PG 0154 has a main-sequence G5–K5 companion. HS 1824 (B = 15.6) and HS 2151 (B = 16.5) were determined to be variable by Østensen et al. (2001, hereafter Ø01) as part of a four-star discovery paper. (Observations of the other two variables will be reported elsewhere.) They detected a single frequency in HS 1824 and four in HS 2151. Their observations consisted of three ≈1-h runs for HS 1824 and two ≈1 h and one 2.5-h runs for HS 2151. As such, the pulsations were likely unresolved and ideal for

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follow-up observations. Both HS stars are apparently single with $T_{\rm eff} = 33\,100$ and 34 500 K and $\log g = 6.0$ and 6.1 for HS 1824 and HS 2151, respectively.

2 OBSERVATIONS

Data were obtained at MDM Observatory's 2.4- and 1.3-m telescopes using an Apogee Alta U47+ CCD camera. This camera is connected via USB2.0 for high-speed readout: our binned (2 × 2) images had an average dead time of 1 s. The observations used a red cut-off filter (BG38) so the effective bandpass covers that of B and V filters and is essentially that of a blue-sensitive photomultiplier tube. Such a set-up allows us to maximize light throughput while maintaining compatibility with observations obtained with photomultiplier photometers. Tables 1 through 3 provide the details of our observations including date, start time, integration time and run length.

Table 1. Observations of PG 0154+182 during 2004 on the MDM 1.3-m telescope.

Run	UT start (h:min:s)	Date (UT)	Length (h)	Integration (s)
mdr277	02:50:30	October 06	3.7	20
mdr279	02:50:30	October 07	3.8	20
mdr281	02:50:30	October 08	3.7	20
mdr283	02:50:30	October 09	4.2	20
mdr286	02:50:30	October 10	1.2	20
mdr288	02:50:30	October 11	4.1	20
mdr291	02:50:30	October 12	0.4	20
mdr293	02:50:30	October 13	6.6	20
mdr296	02:50:30	October 14	0.8	20

Table 2. Observations of HS 1824+5745 during 2005.

Run	UT start (h:min:s)	Date (UT)	Length (h)	Int. (s)	Telescope (m)
mdm052505	04:35:00	May 25	6.9	20	1.3
mdm052605	04:12:00	May 26	7.4	20	1.3
mdm052705	08:32:00	May 27	3.1	20	1.3
mdm053005	04:13:30	May 30	7.4	20	1.3
mdm053105	04:43:00	May 31	6.9	20	1.3
mdm060105	04:13:30	June 01	7.4	20	1.3
mdm060305	03:37:00	June 03	7.9	20	1.3
mdm060405	03:48:30	June 04	7.7	20	1.3
mdm060505	03:48:30	June 05	7.8	20	1.3
mdm060605	03:45:00	June 06	7.8	20	1.3
mdm061305	04:17:11	June 14	6.6	20	1.3
mdm061505	03:50:00	June 15	7.6	20	1.3
mdm061605	03:33:00	June 16	8.1	20	1.3
hs18_061705	03:36:00	June 17	3.0	20	1.3
hs18_061905	03:33:00	June 19	1.9	30	1.3
hs18_062005	03:26:00	June 20	4.4	25	1.3
hs18_062205	05:40:00	June 22	5.9	25	1.3
hs18_070605	04:17:00	July 06	1.2	7	2.4
hs18_070705	04:14:00	July 07	1.5	7	2.4
hs18_070805	03:21:00	July 08	5.4	7	2.4
hs18_070905	03:18:00	July 09	2.2	7	2.4
hs18_071005	03 15 30	July 10	2.1	7	2.4
hs18_071105	03:20:50	July 11	4.1	7	2.4
hs18_071205	03:23:30	July 12	3.1	7	2.4

Table 3. Observations of HS 2151+0857 during 2005.

Run	UT start (h:min:s)	Date (UT)	Length (h)	Int. (s)	Telescope (m)
hs21_061805	09:10:00	June 18	2.1	25	1.3
hs21_061905	07:40:00	June 19	3.9	26	1.3
hs21_062005	08:06:00	June 20	3.5	28	1.3
hs21_070605	06 01 00	July 06	5.8	10	2.4
hs21_070705	05:55:50	July 07	5.9	10	2.4
hs21_070805	08:55:00	July 08	2.9	10	2.4
hs21_070905	05:43:40	July 09	6.1	10	2.4
hs21_071005	05:27:00	July 10	6.4	10	2.4
hs21_071205	06:33:41	July 12	5.2	10	2.4

PG 0154 was observed as a secondary target during our fall 2004 KPD 2109 data run (reported in Zhou et al. 2006). We obtained short data runs (<6.6 h) over nine nights. HS 1824 was the subject of a concerted effort, with more than 125 h of data obtained during three different observing runs, two at the 1.3 m and the last at the 2.4-m telescopes. After the first observing run on HS 1824, we began observing HS 2151 as a secondary target. More than 42 h of data were obtained of HS 2151 during three nights on the 1.3 m and six on the 2.4-m telescopes. Though only the observations of HS 1824 conform to our normal programme (long time-series observations covering several weeks), we feel that we have successfully resolved all of the pulsations.

Standard procedures of image reduction, including bias subtraction, dark current and flat field correction, were followed using IRAF[1] packages. Intensities were extracted using IRAF aperture photometry with extinction and cloud corrections using the normalized intensities of several field stars. As sdB stars are substantially hotter than typical field stars, differential lightcurves are not flat due to differential atmospheric and colour extinctions. A low-order polynomial was fit to remove these trends from the data on a night-by-night basis. Finally, the lightcurves are normalized by their average flux and centred around zero so the reported differential intensities are $\Delta I = (I/\langle I \rangle) - 1$. Amplitudes are thus given as mma with an amplitude of 10 mma corresponding to 1.0 per cent or 9.2 millimag. Sample lightcurves are shown in Fig. 1.

3 PULSATION ANALYSIS

PG 0154. The temporal spectrum and window function for PG 0154 are plotted in Fig. 2. The top panel shows a Fourier transform (FT; also known as temporal or pulsation spectrum) of the original data, with the two lower panels showing the effects of prewhitening by four and six frequencies, respectively. The inset is the window function, which is a single, noise-free sine wave (of arbitrary amplitude) sampled at the same times as the data. The central peak of the window is the input frequency with other peaks indicating the aliasing pattern of the data. In order to determine the significance of the two low-amplitude peaks, we convolved a slightly smoothed window function which included the four highest amplitude peaks with the data noise as determined using the Breger et al. (1994, hereafter B94) criterion, which is four times the average value of the FT (the dashed line in Fig. 2). Using such a conservative estimate of the noise, we can say with confidence that

[1] IRAF is distributed by the National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.

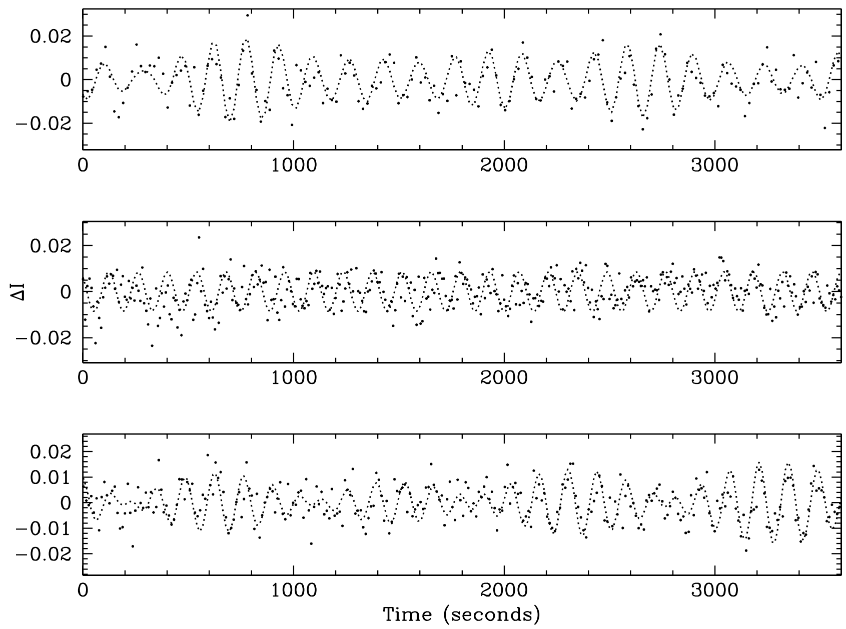

Figure 1. Sample 1-h lightcurves of the three targets. From top to bottom are PG 0154 (obtained on the 1.3 m), HS 1824 (obtained on the 2.4 m) and HS 2151 (obtained on the 2.4 m). The dotted line is our least-squares solution of the entire data sets.

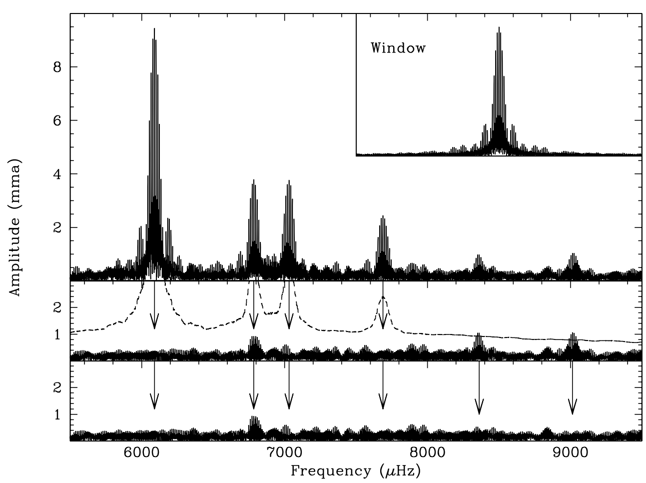

Figure 2. Temporal spectra (FT) for PG 0154 showing the original data (top), and pre-whitening by four and six frequencies (middle and bottom panels, respectively). Arrows indicate pre-whitened frequencies while the dashed line in the middle panel shows the 4σ detection limit. Inset is the data window plotted at the same horizontal scale.

Table 4. Periods, frequencies and amplitudes for PG 0154. Formal least-squares errors are in parentheses.

ID	Period (s)	Frequency (μHz)	Amplitude (mma)
$f1$	110.9305 (18)	9014.65 (15)	1.1 (2)
$f2$	119.5827 (22)	8362.09 (15)	1.1 (2)
$f3$	130.0694 (10)	7688.20 (06)	2.5 (2)
$f4$	142.2167 (08)	7031.52 (04)	3.9 (2)
$f5$	147.3860 (09)	6784.90 (04)	3.9 (2)
$f6$	164.2108 (03)	6089.73 (01)	9.5 (2)

both low-amplitude peaks are real and significantly above the noise. Frequencies, amplitudes and phases were determined by simultaneously fitting a non-linear least-squares solution to the data. Our solution for the frequencies and amplitudes is provided in Table 4. No other frequencies were detected up to the Nyquist frequency near 25 000 μHz.

We readily confirm the frequency detected by K04 and detect five additional pulsation frequencies. As the closest two frequencies are separated by nearly 300 μHz, the pulsations in PG 0154 can readily be resolved in about 1 h of observations, a very fortunate circumstance since our data runs were short in duration. We also examined individual observing runs of sufficient length and readily detected all six pulsation frequencies. Though there was some amplitude variability (to be discussed in Section 4.3) which is responsible for the peak in the residuals at 6784 μHz, the frequencies were stable over the course of our observing runs.

HS 1824. The majority of our 2005 campaign was devoted to confirming that HS 1824 has just a single pulsation frequency. The formal least-squares solution for the frequency, using all of the data is $f = 7190.413\,397 \pm 0.000\,005$ μHz which is on the low end of Ø01, but within their errors. Using the B94 criterion provides a limit of undetected pulsations at 0.48 mma outside of the window pattern caused by the single peak. Additional peaks were searched for up to the Nyquist frequency near 50 000 μHz, but none were detected. However, just because HS 1824 is monoperiodic does not make it uninteresting. It has both amplitude and phase variations that will be discussed in Section 4. The average amplitude of the pulsation was 2.4 mma over the course of our observations, but ranged from 2 to 6 mma. Such amplitude variation points out a weakness in the pre-whitening process. Pre-whitening algorithms typically remove a constant amplitude sinusoidal function from the data at the frequency, amplitude and phase determined via non-linear least-squares fitting (or several for multiple pulsations). Fig. 3 shows how this can affect the pre-whitening process. A 30-μHz region centred on the pulsation frequency is shown for HS 1824. The top panel shows the original FT (the inset shows a 14 000-μHz span) and the middle panel shows the FT pre-whitened by one frequency (solid line) overplotted on the window function (dashed line) that

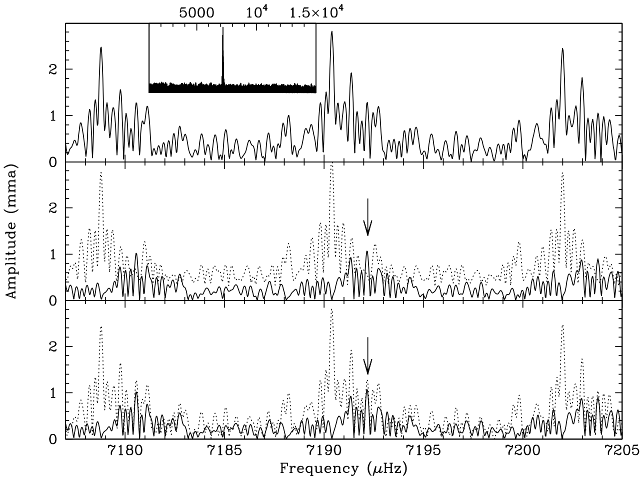

Figure 3. Temporal spectra (FT) for HS 1824 showing the pitfalls of pre-whitening. The top panel shows the original FT (the inset spans a larger region) and the middle panel shows the original FT pre-whitened by one frequency (solid line) using a window function fit from the entire data set (dashed line). The bottom panel shows the same FT as the middle panel, but the window function incorporates the amplitude and phase variations determined from individual nights of data.

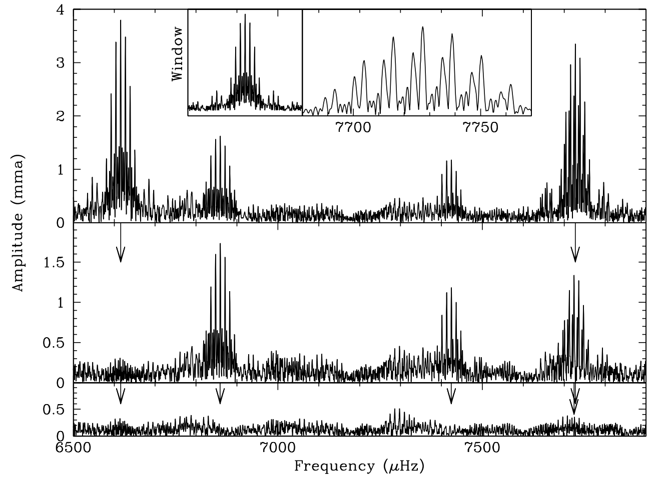

Figure 4. Temporal spectra (FT) for HS 2151 showing the original data (top), and pre-whitening by two (middle) and five frequencies (bottom). Arrows indicate pre-whitened frequencies. Inset shows the data window and an expanded view of the close pair of frequencies.

matches the entire data set. The arrow points to a peak that is still above the window function, which could be (mistakenly) deemed intrinsic to the star. The bottom panel shows the same pre-whitened FT as the middle panel, except that the window function is created using the least-squares information from individual runs. In this case, there are no peaks left above the window and we correctly deduce the monoperiodic nature of the pulsations.

HS 2151. The majority of our data for HS 2151 were obtained during our observing run in 2005 July, with just three relatively short runs obtained during the late-June run. As such, we examined these data as three separatesets: the June data, the July data and the combined data. The June data alone are not sufficient to resolve the close doublet near 7725 μHz, and the combined data have a slightly more complex window function than just the July data. The B94 noise criterion for the combined and July runs are within 0.02 mma of each other (0.51 and 0.53 mma, respectively), and so the best results were obtained using the July data only. The temporal spectrum of HS 2151 is shown in Fig. 4. The top panel shows the original FT of the July data with the next two panels showing the pre-whitening sequence of two and five frequencies, respectively. The insets show the window function and an enlarged section of the original FT to show the resolved doublet. Arrows indicate the pre-whitened frequencies and our least-squares solution is provided in Table 5. No further frequencies were detected up to the Nyquist frequency near 39 000 μHz.

Table 5. Periods, frequencies and amplitudes for HS 2151. Formal least-squares errors are in parentheses.

ID	Period (s)	Frequency (μHz)	Amplitude (mma)
$f1$	129.4133 (07)	7727.18 (4)	3.01 (13)
$f2$	129.4671 (13)	7723.97 (8)	1.51 (13)
$f3$	134.6929 (17)	7424.30 (9)	1.20 (12)
$f4$	145.7850 (13)	6859.41 (6)	1.74 (12)
$f5$	151.1561 (07)	6615.68 (3)	3.84 (12)

4 DISCUSSION

4.1 Comparison with the discovery data

The goal of these observations was to obtain better data than in the discovery papers and to resolve the pulsation frequencies for asteroseismic analysis. For the sake of comparison with the discovery data, we calculate the temporal resolution as $1/\Delta T$ with ΔT being the temporal extent of the observations. For the discovery data, we can determine temporal resolution from information provided in their papers, and estimate 'by eye' the discovery data detection limit as twice the top of the noise level in their FTs outside of the pulsations and windows.

For PG 0154, we were able to confirm the frequency from K04 as well as detect five new frequencies. Our observations have a temporal resolution of 1.4 μHz which is more than four times better than the discovery data and a detection limit of 0.76 mma which is more than six times better than in K04.

We certainly obtained more and better data than Ø01 for HS 1824, but it ended up only confirming the singe frequency they detected. Our temporal resolution is 0.25 μHz, which is 23 times better than theirs and our detection limit is 0.48 mma which is approximately four times better than Ø01.

For HS 2151, we confirm the four previously detected frequencies of Ø01 and detect an additional, closely spaced frequency. This detection was possible because of our 11 times better temporal resolution of 0.5 μHz and our detection limit was also approximately three times better than Ø01 at 0.53 μHz.

Compared to the discovery data of these three stars, we certainly obtained our goal of vastly superior temporal resolution and improved detection limits. However, it is interesting to note that both PG 0154 and HS 1824 showed a single peak in very limited discovery data, and while PG 0154 paid dividends with only relatively few observations, no new pulsations were detected in HS 1824 after a concerted effort.

4.2 Multiplet constraints on pulsation modes

For our goal of applying asteroseismological tools to these stars, one of the most helpful features would be observational constraints on the pulsation *modes* themselves, rather than just recording the pulsation frequencies. Pulsation modes can be described by three quantum numbers, n (or k), ℓ and m, and mathematically described by spherical harmonics. As rotation can break the m degeneracy by separating each degree ℓ into a multiplet of $2\ell+1$ components, multiplet structure is a very useful tool for observationally constraining pulsation degree (see Winget et al. 1991). Such multiplets should nearly equally be spaced in frequency, but such structure is seldom observed in sdBV stars. However, when multiplets are observed, they constrain not only the pulsation degree (ℓ) but also the rotation period and inclination (O'Toole, Heber & Benjamin 2004; Reed et al. 2004).

For PG 0154 there is a common value that emerges with an average frequency spacing of ≈670 μHz that includes *all* of the observed frequencies. The spacings are provided in Table 6 and range from 652 to 695 μHz. This is less than a 5 per cent variation and roughly of the order expected from theory. *If* these are rotationally split multiplets, $f1$ through $f4$ would necessarily belong to a single degree of $\ell \geqslant 2$, missing at least one component (just one for $\ell = 2$) and $f5$ and $f6$ would be two parts of an $\ell \geqslant 1$ multiplet. As such, all of the frequencies in PG 0154 can be explained using one $\ell = 1$ and one $\ell = 2$ mode–each missing one of their components. However, rotational splitting of 670 μHz means a rotation period of 1492 s, or about 25 min! At a canonical radius of $0.15\,R_\odot$, this would imply a rotation velocity of $\approx 400\,\mathrm{km\,s^{-1}}$–an easily measurable quantity. Spectroscopic constraints already exist and from fig. 7 of K04, it is obvious that PG 0154 is a normal, slowly rotating sdB star. Though it could be possible to conceal such a high velocity with a low inclination, another argument against this scenario is the orbital separation. Using the canonical sdB mass of $0.5\,M_\odot$ and a range of masses appropriate for the main-sequence companion, the orbital distance between the two mass centres is $<0.95\,R_\odot$. The two stars could not exist within such close proximity to each other without significant effects; at a minimum, the sdB star would have significant ellipsoidal variations like those observed for the close binary KPD 1930+2752 (Billères et al. 2000) and for most companion parameters, the two stars would not fit within the orbital separation.

Since the spacing is so large, it is conceivable that they could be successive overtones of the radial index, n. However, because $f4$ and $f5$ do not share this spacing, they would necessarily belong to differing degrees, ℓ, and theory predicts that differing degrees have different overtone spacing. However, a glance at fig. 4 of Charpinet et al. (2002) reminds us that theory expects these to be low-overtone pulsations, which do not obey the asymptotic relationship of even frequency spacing. The figure indicates that successive overtones should *differ* in spacing by at least 500 μHz, ruling out this possibility.

Table 6. Frequency spacings between pulsation frequencies for PG 0154.

Frequency pair	Difference (μHz)
$f1$–$f2$	652.6
$f2$–$f3$	673.9
$f3$–$f4$	656.7
$f5$–$f6$	695.4

Table 7. A 'Kawaler scheme' model fit to the observed frequencies using $f_o = 6349.9$ (6103.2), $\delta = 669.6$ and $\Delta = 247.3$ μHz.

Star	i	j	Model	Difference
$f1$	4	0 (1)	9028.1	−13.5
$f2$	3	0 (1)	8358.6	3.5
$f3$	2	0 (1)	7689.0	−0.8
$f4$	1	0 (1)	7019.4	12.1
$f5$	1	−1 (0)	6772.5	12.4
$f6$	0	−1 (0)	6103.4	−13.1

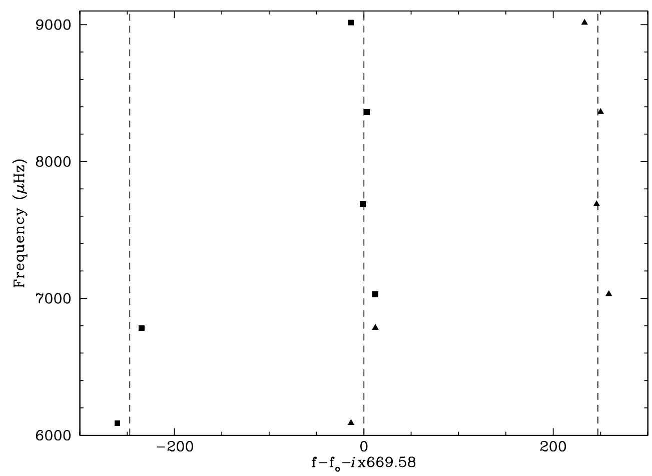

Figure 5. Echelle diagram for PG 0154 using 'Kawaler scheme' model fits. Squares correspond to the model with $f_o = 6349.9$ μHz and triangles for the model with $f_o = 6103.2$ μHz.

Another possibility is the 'Kawaler scheme' which has recently been introduced (Kawaler & Vučković 2006; Vučković et al., in preparation). Though it is not compatible with low-overtone (n) pulsation theory invoked for sdBV stars, it has been noticed that for several stars, an improved frequency fit can be obtained using an asymptotic-like formula

$$f(i, j) = f_o + i \times \delta + j \times \Delta,$$

where i has integer values, j is limited to values of −1, 0 and 1, and up till now δ has been a small spacing and Δ has been a large spacing. However, for PG 0154, the large spacing occurs too frequently to be restricted to just three values (−1, 0 and +1). For PG 0154, there are not enough frequencies to make a unique fit and so Table 7 gives the solutions for two possible values of f_o, and Fig. 5 shows the Echelle diagram of the fits using a different symbol for each fit. (Fractional multiples of the δ spacings, i.e. $\delta/2$, are also equally good fits, but the largest spacing comes directly from the data.) For both models, $\delta = 669.6$ μHz and $\Delta = 247.3$ μHz using $f_o = 6349.9$ μHz for the squares and $f_o = 6103.2$ μHz for the triangles. Part of the Kawaler scheme is to choose 'the (δ, Δ) pair for which we have at least two modes with the same value of i but different j' and we have satisfied this condition, but without coming to a unique conclusion. Still, we can say that the Kawaler scheme fits the data with a rms error of only 0.16 per cent and remains an option while a rotational explanation for the frequency splitting is clearly ruled out. A Kawaler scheme fit was also obtained switching the values for δ and Δ, but the differences were substantially larger.

For HS 2151, where we have five frequencies to work with, no two frequency spacings are similar. As such, we are left with the

following possibilities for the five pulsation frequencies detected. (i) Rotation is sufficiently slow that all m values are degenerate; (ii) at most one pair shares the same n and ℓ values with differing m, which seems very unlikely; (iii) our line of sight is along the pulsation axis, with $\sin i \approx 0$, making only $m = 0$ modes observable because of geometric cancellation (Pesnell 1985; Reed, Brondel & Kawaler 2005) or (iv) internal rotation is such that rotationally induced multiplets are widely spaced and uneven (Kawaler & Hostler 2005).

4.3 Constraints on pulsation degree via mode density

An open question involving sdBV stars is the mode degree ℓ of the pulsations. In resolved sdBV stars, we sometimes observe many more pulsation modes than $\ell = 0$, 1 and 2 can provide. Higher ℓ modes may be needed, but if so they must have a larger intrinsic amplitude because of the large degree of geometric cancellation (Charpinet et al. 2005; Reed et al. 2005). A general guideline would be one n order per ℓ degree per 1000 μHz. As such, the temporal spectrum can accommodate three frequencies per 1000 μHz without the necessity of invoking high-ℓ values.

For PG 0154, even ignoring the frequency spacing that interrelates several frequencies, it is still easy to accommodate all the frequencies without the need for $\ell \geqslant 3$ modes. The pulsation spectrum for HS 2151 is slightly more dense and we do not detect any related frequencies. However, it is still possible to fit the frequencies within the range observed without invoking $\ell \geqslant 3$ modes if the 6616 and 6859 μHz frequencies are related to those near 7725 μHz. As such, we can deduce that the pulsations in all three pulsators in this paper *can* be accommodated using $\ell = 0, 1$ and 2 modes. However, this by no means implies that they are.

4.4 Amplitude variability

Another feature we can examine is the amount of amplitude variability in the pulsations. If pulsating sdB stars are observed over an extended time period, it is common to detect amplitude variability in many, if not all, of the pulsation frequencies. Such variability can occasionally be ascribed to beating between pulsations too closely spaced to be resolved in any subsets of the data, but often appear in clearly resolved pulsation spectra, where mode beating cannot be the cause.

Fig. 6 shows the amplitudes of the pulsations of PG 0154 for individual nights. The scale is selected (for all of the amplitude and phase plots) such that a 10 per cent error bar occupies ≈10 per cent of the panel. All of the amplitudes are essentially constant to within the errors except for $f5$, which triples in amplitude over the course of the run. We initially suspected that it was caused by an unresolved doublet of frequencies, which could still be the case. However, several other frequencies show the same general trend (though on smaller scales) and the amplitude variations do not appear sinusoidal, which would be expected with mode beating. As such, we doubt our initial interpretation and rather think these variations are intrinsic. However, the only way to be sure is to obtain more observations. Still, the net result is that five of six frequencies are stable both in amplitude and phase (not shown as all of the phases are constant to within the errors).

Of significantly more interest are the amplitudes and phases of the single frequency present in HS 1824. These are shown in Fig. 7 and indicate that they are quite variable. Between observing days 41 and 45 (JD = 245 3531.5 and 245 3536.5), the amplitude rises from 0.9 to 4.2 mma, while the phases change by ≈40 per cent

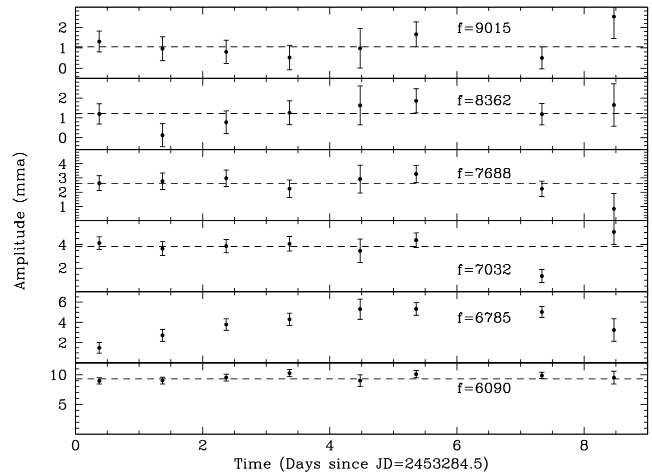

Figure 6. Pulsation amplitudes for frequencies detected in PG 0154. Error bars are the formal least-squares errors.

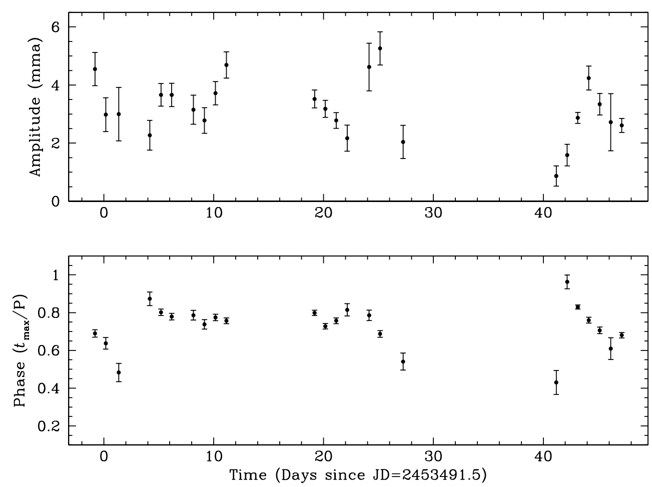

Figure 7. Pulsation amplitudes and phases for frequencies detected in HS 1824. Error bars are the formal least-squares errors.

between days 42 and 46. Clearly, HS 1824 is both amplitude and phase variable, possibly an indicator of stochastic pulsations (discussed in the next section). Note that phases are chosen such that the phase is the time of first maximum since JD = 245 3279.5 for PG 0154, JD = 245 3491.5 for HS 1824 and HS 2151 divided by the period.

Fig. 8 shows the amplitudes and phases of the three well-separated frequencies of HS 2151. The doublet near 7725 μHz is not resolvable on a nightly basis. In this case, two of the three frequencies ($f4$ and $f5$) have amplitude variations marginally larger than the errors which appear to follow the same trend. One possible explanation would be if they are two parts of a multiplet sharing an increase in driving power, though such a hypothesis cannot be tested using these data. For all three frequencies, the phases vary over the duration of our observing campaign. The highest amplitude frequency ($f5$) is generally the most stable, varying smoothly by 15 per cent over the first three nights and then remaining essentially constant. The other two frequencies ($f3$ and $f4$) both show an offset between the June and July data runs with $f4$ essentially constant during the individual month's data. This points towards a problem with timing rather than intrinsic changes of the star. We investigated this possibility

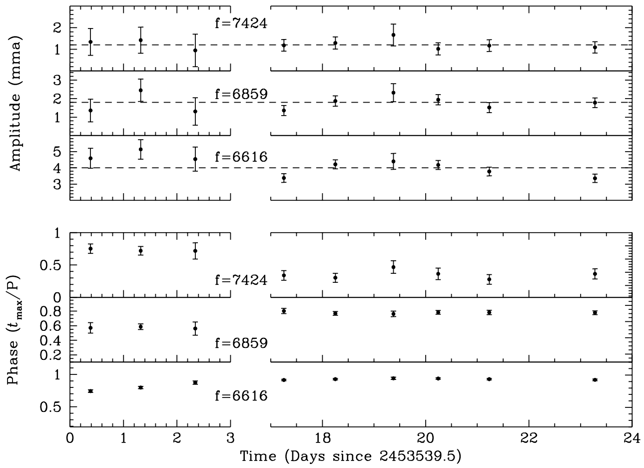

Figure 8. Pulsation amplitudes and phases for frequencies detected in HS 2151. Error bars are the formal least-squares errors.

Table 8. Mean amplitude, standard deviation and their ratio for readily resolvable frequencies.

Frequency (μHz)	$\langle A \rangle$ (mma)	σ (A) (mma)	σ (A)/$\langle A \rangle$
	PG 0154		
9015	0.97	0.38	0.39 ± 0.39
8362	1.37	0.34	0.25 ± 0.22
7688	2.72	0.36	0.13 ± 0.12
7032	4.07	0.49	0.12 ± 0.05
6785	3.90	1.27	0.33 ± 0.06
6090	9.53	0.48	0.05 ± 0.03
	HS 1824		
7190	3.19	1.01	0.31 ± 0.11
	HS 2151		
7424	1.24	0.16	0.17 ± 0.12
6859	1.77	0.40	0.22 ± 0.10
6616	4.18	0.55	0.13 ± 0.04

but ruled it out for the following reasons: the highest amplitude frequency does not show these, the changes are not the same in period (i.e. the same amount of difference in timing), our observations of HS 1824, which occurred during the same nights do not show this shift (though admittedly HS 1824 shows phase variations) and the timing set-up is the same as we have used during all of our other runs[2] and is checked multiple times during the night. As such, we conclude that these shifts that occurred during the 15-night interval between observations are intrinsic to the star.

4.5 The nature of the excitation mechanism

We can also use the criterion outlined in Christensen-Dalsgaard (2004, hereafter JCD04) and first applied to sdBV stars in Pereira & Lopes (2005) to examine the nature of the excitation mechanism. If the frequencies are stochastically excited, we can expect to find the standard deviation of the amplitudes, $\sigma(A)$, divided by the average amplitude $\langle A \rangle$ to have a value near 0.5 (equation 7 of Pereira & Lopes 2005).

Both parameters and their ratios are given in Table 8 for all resolved frequencies in our target stars. For PG 0154, three of the six frequencies have relatively stable pulsation amplitudes, and their ratios reflect this as non-stochastically excited pulsations. The two low-amplitude frequencies, $f1$ and $f2$, have errors too large to constrain the ratio, which for $f5$ is in between. In this case, $f5$ suffers the same problem as it did for mode beating; the time-scale of amplitude variation appears longer than our observations. As such, more data would be required to discern which interpretation is correct for $f5$: stochastically driven pulsations, mode beating or variable amplitudes caused by something else. Surprisingly, the ratio for HS 1824 is nearly the same as for $f5$ of PG 0154. However, we can suggest that these pulsations do not match the JCD04 criteria for stochastic oscillations as our observations are surely sufficient in duration to include several damping lifetimes, yet $\sigma(A)/\langle A \rangle$ is only 0.31 ± 0.11. As such, the rather large variation in amplitude and phase remains unexplained. For the three well-separated frequencies of HS 2151, none of them have ratios indicating stochastically excited pulsations. These results indicate that at most one of our 10 resolved frequencies have indications of being stochastically excited. This can be compared to our previous observations of KPD 2109+4401, where 50 per cent of the resolved pulsations indicated stochastic excitation and a study of PG 1605+072 where *no* indications of stochastic excitation were found from 11 resolvable frequencies (Pereira & Lopes 2005). It is a bit early to interpret the meaning of such results and it is not clear what impact the discovery of many stochastically driven frequencies (should that occur) would mean for the proposed iron driving mechanism of Charpinet, Fontaine & Brassard (2001).

[2] The acquisition computer's clock is synchronized with time servers through Network Time Protocol.

5 CONCLUSIONS

From extensive follow-up data acquired at MDM observatory, we are confident that we have resolved the pulsation spectra of three additional pulsating sdB stars. We are able to confirm all of the previously observed frequencies as well as detect a total of six new frequencies in two of our three targets.

For PG 0154, we have discovered five new frequencies with the closest spacing between frequencies at ≈250 μHz. As such, they are readily resolvable in 1.5 h of data, however, the low amplitude of two frequencies requires high S/N or longer time-base observations to reduce the noise. We notice an enticing common frequency spacing of ≈670 μHz, which could be interpreted as all six frequencies being members of one $\ell = 1$ and one $\ell = 2$ pulsation multiplet. However, this would require a rotation period near 25 min and a rotation velocity of ≈440 km s^{-1} which is clearly ruled out by K04. We applied the Kawaler scheme to the spacings using a purely numerical fit with good results. However, as PG 0154 only has six frequencies, there is no unique solution and so we are not convinced that it should be applied in this case. As such, the cause for the large, nearly equal frequency spacing remains a mystery. Three of the six frequencies are amplitude stable, while one shows variations that could be indicative of a stochastically excited mode. Of course, this would not be possible if it is actually an *m* component of the same degree as other, amplitude-stable frequencies. As such, it seems we still have some to learn about PG 0154 and additional follow-up observations could still be instructive.

In HS 1824, we confirmed the frequency detected by Ø01, but did not uncover any additional frequencies. However, we were able to measure significant amplitude and phase variations over a long span of time. As such, once asteroseismology matures to where it

can interpret such fluctuations, HS 1824 will be an excellent target. However, until then, all we can readily say is that the variability in the amplitude and phase of pulsation seems unrelated to stochastically excited oscillations.

For HS 2151, we confirm the four previously known frequencies and detect an additional oscillation close to a previously observed one. However, none of the frequencies appear to have common spacing indicative of multiplet structure and their amplitudes appear stable, indicating non-stochastically excited pulsations.

These three pulsating sdB stars show the variety, and perhaps the complexity, of oscillations observed in sdB stars. Two of the three have several frequencies, while HS 1824 only had one. Of the 10 total frequencies, several show amplitude variations, but only one has sufficient variability to even consider stochastic excitation. Several of the phases show small variations with those in HS 1824 being much larger, and only one star shows a common frequency spacing that could be used to observationally constrain the pulsation degree, ℓ, except it would require a drastically fast rotation velocity which has been ruled out by previous spectroscopic observations.

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