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Pushing the ground-based limit: 14- μ mag photometric precision with the definitive Whole Earth Telescope asteroseismic data set for the rapidly oscillating Ap star HR 1217

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ABSTRACT

HR 1217 is one of the best-studied rapidly oscillating Ap (roAp) stars, with a frequency spectrum of alternating even- and odd- ℓ modes that are distorted by the presence of a strong, global magnetic field. Several recent theoretical studies have found that within the observable

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atmospheres of roAp stars the pulsation modes are magneto-acoustic with significant frequency perturbations that are cyclic with increasing frequency. To test these theories a Whole Earth Telescope extended coverage campaign obtained 342 h of Johnson *B* data at 10-s time resolution for the roAp star HR 1217 over 35 d with a 36 per cent duty cycle in 2000 November–December. The precision of the derived amplitudes is 14 μmag , making this one of the highest precision ground-based photometric studies ever undertaken. Substantial support has been found for the new theories of the interaction of pulsation with the strong magnetic field. In particular, the frequency jump expected as the magnetic and acoustic components cycle through 2π rad in phase has been found. Additionally, comparison of the new 2000 data with an earlier 1986 multisite study shows clear amplitude modulation for some modes between 1986 and 2000. The unique geometry of the roAp stars allows their pulsation modes to be viewed from varying aspect with rotation, yielding mode identification information in the rotational sidelobes that is available for no other type of pulsating star. Those rotational sidelobes in HR 1217 confirm that two of the modes are dipolar, or close to dipolar; based on the frequency spacings and *Hipparcos* parallax, three other modes must be either $\ell = 0$ or 2 modes, either distorted by the magnetic field, or a mix of m -modes of given ℓ where the mixture is the result of magnetic and rotational effects. A study of all high-speed photometric Johnson *B* data from 1981 to 2000 gives a rotation period $P_{\text{rot}} = 12.4572$ d, as found in previous pulsation and photometric studies, but inconsistent with a different rotation period found in magnetic studies. We suggest that this rotation period is correct and that zero-point shifts between magnetic data sets determined from different spectral lines are the probable cause of the controversy over the rotation period. This WET data set is likely to stand as the definitive ground-based study of HR 1217. It will be the baseline for comparison for future space studies of HR 1217, particularly the *MOST* satellite observations.

Key words: stars: individual: HR 1217 – stars: magnetic fields – stars: oscillations – stars: variables: other.

1 INTRODUCTION

The rapidly oscillating Ap (roAp) stars have been observed photometrically since their discovery by Kurtz (1982) over 20 years ago. Frequency analyses of their light curves have yielded rich asteroseismic information on the degrees of the pulsation modes, distortion of the modes from normal modes, magnetic geometries and luminosities. The latter, in particular, are derived asteroseismically and agree well with *Hipparcos* luminosities (Matthews, Kurtz & Martinez 1999).

New theoretical work on the interaction of pulsation with both rotation and the magnetic field by Bigot & Dziembowski (2002) has presented an entirely new look at the oblique pulsator model of these stars: they find that the pulsation axis is inclined to both the magnetic and rotation axes, and the pulsation modes are complex combinations of spherical harmonics that result in modes that, in many cases, can be travelling waves looking similar to (but are not exactly) sectoral m -modes. Bigot & Kurtz (2005) have shown that the improved oblique pulsator model of Bigot & Dziembowski obtains rotational inclination and magnetic obliqueness for the roAp star HR 3831 that are in agreement with those found from a magnetic studies by Bagnulo, Landolfi & degl’Innocenti (1999); however, Kochukhov (2004) finds a different magnetic and pulsation geometry for HR 3831 that is in good agreement with the old oblique pulsator model and is inconsistent with the Bagnulo et al. (1999) geometry. Further magnetic studies should differentiate between these two models.

In a similar study to that of Bigot & Dziembowski, Saio & Gautschy (2004) find modes that are aligned with the magnetic axis and are distorted by the magnetic field so that they cannot be described by single spherical harmonics; they note that horizontal motion can be comparable to vertical motion for these modes. It is the unique geometry of the pulsation modes in roAp stars that allows us to examine their non-radial pulsation modes from varying aspect as can be done with no other type of star.

1.1 High-resolution spectroscopy of roAp stars

The spectra of many roAp stars show a strong core–wing anomaly in the hydrogen lines, particularly the H α line (Cowley et al. 2001; Kochukhov, Bagnulo & Barklem 2002). This indicates abnormal atmospheric structure, as does the fact that consistent abundances for the second and third ionization states of rare-earth elements, particularly Nd and Pr, cannot be found for these stars (Ryabchikova et al. 2002). Until atmospheric models can be found that solve these problems, caution is called for, but new high-resolution spectroscopic results for roAp stars suggest vertical stratification of some ions, particularly the rare earths, and they show the short vertical wavelength of the pulsation modes. It must also be cautioned that there are known horizontal abundance variations with concentration of rare-earth elements towards the magnetic poles. See, for example, studies of γ Equ (HD 201601) (Kochukhov & Ryabchikova 2001a), HD 166473 (Kurtz, Elkin & Mathys 2003), α Cir (Baldry et al. 1999; Kochukhov & Ryabchikova 2001b; Balona & Laney 2003),

HR 3831 (Baldry & Bedding 2000; Kochukhov & Ryabchikova 2001b), HR 1217 (Balona & Zima 2002; Sachkov et al. 2004) and 33 Lib (HD 137949) (Mkrtychian, Hatzes & Kanaan 2003; Kurtz, Elkin & Mathys 2005). The results of these studies are plausibly interpreted in terms of vertical resolution of the pulsation mode structure in the atmospheres of these stars, possibly with stratification of some ions. In general, Fe lines originate near a radial node around an optical depth of $\tau_{5000} = 1$ with little or no radial velocity variations seen; the core of the H α line forms higher in the atmosphere and shows radial velocity variations with amplitudes up to several km s⁻¹; lines from the first and second ionization states of the rare earths Pr and Nd arise from a thin layer around an optical depth of $\tau_{5000} = 10^{-3}$ and can also show amplitudes of several km s⁻¹.

The spectroscopic studies have thus allowed the beginning of a three-dimensional resolution of the pulsation modes, with the vertical stratification giving depth information, and the rotation of the oblique mode providing information on the surface geometry of the modes. While these spectroscopic tools are very powerful, they demand high spectroscopic resolution, high time resolution and high signal-to-noise ratio (S/N) – requirements that can only be met with large telescopes and optimized high-resolution spectrographs. It is thus not possible at present to study in detail the frequencies of many roAp stars spectroscopically because of the need for high duty cycle data sets over extended periods of time.

1.2 Photometry and asteroseismology of HR 1217

To obtain the frequencies that are the basic input data for asteroseismology it is thus necessary to observe roAp stars in multisite ground-based campaigns, as we report in this paper for the Whole Earth Telescope (WET) extended coverage campaign (Xcov20) on the roAp star HR 1217 (DO Eri; HD 24712), or to observe them from space with dedicated asteroseismic satellites. The first of these satellites, *MOST*, was launched on 2003 June 30 and observed HR 1217 for 30 d during 2004 November–December. Hence the discoveries in this work will provide a baseline to compare with the *MOST* results. The data set analysed in this paper from Xcov20 will be referred to throughout the rest of this paper as the ‘2000 data’.

Of the 34 known roAp stars, HR 1217 is one of the best-studied. This star, discovered to be a pulsator by Kurtz (1982), was investigated with an extensive global campaign in 1986 (Kurtz et al. 1989). A key result from that data set (which we will refer to in this paper as the ‘1986 data’) was a list of six principal pulsation frequencies, five of which had alternating spacings of 33.4 and 34.5 μ Hz, the sixth of which was separated by a then inexplicable 50 μ Hz from the fifth frequency.

The asymptotic frequency spacing, ν_0 – known as the ‘large spacing’, is a measure of the sound crossing time of the star, which in turn is determined by the mean density and radius of the star. With a typical mass of Ap stars of approximately $2 M_{\odot}$, ν_0 reflects the radius of the star, with ν_0 scaling as $R^{-3/2}$. In the asymptotic limit, the number of nodes in the radial direction, n , is much larger than the spherical degree ℓ . Assuming adiabatic pulsations in spherically symmetric stars the pulsation frequencies are

$$\nu_{n,\ell} = \nu_0(n + \ell/2 + \epsilon) + \delta\nu,$$

where ϵ is a (small) constant (Tassoul 1980, 1990) and $\delta\nu$, the ‘small spacing’, is a measure of the age of the star as it is sensitive to the central condensation, hence the core H mass fraction. Without precise identification of the degree (ℓ) of the pulsation modes, asymptotic theory allows the frequency spacing to be uncertain by a factor of

2, depending on whether modes of alternating even and odd ℓ are present (producing modes separated by $\nu_0/2$ in frequency), or only modes of the same ℓ with consecutive values of n .

The results of the 1986 campaign were inconclusive as to whether ν_0 was 68 or 34 μ Hz. Fortunately, the ambiguity could be resolved by a precise determination of the luminosity of the star. If ν_0 were 34 μ Hz, then the radius of HR 1217 would be large enough that it would be far removed from the main sequence (i.e. more evolved) and therefore more luminous (Heller & Kawaler 1988). Matthews et al. (1999) used the *Hipparcos* parallax measurement to place HR 1217 unambiguously close to the main sequence, thus determining that ν_0 is indeed 68 μ Hz. This deepened the ‘mystery of the sixth frequency’, lying $\frac{3}{4}\nu_0$ higher than the fifth frequency. It is easy to see from the above asymptotic frequency relation that there is no clear theoretical explanation for this spacing using the above asymptotic relation.

That asymptotic frequency spacing is valid only for linear adiabatic pulsations in spherically symmetric stars. However, the magnetic field, the chemical inhomogeneities and rotation all contribute to breaking the spherical symmetry in roAp stars. It is therefore important to know the effects that these deviations from spherical symmetry have on the theoretical frequency spectra of roAp stars, before comparing those with the observed frequency spectra. The effects of the chemical inhomogeneities have been discussed recently by Balmforth et al. (2001), but those will not concern us further here. The effects of the magnetic field on the oscillations of roAp stars (Dziembowski & Goode 1996; Bigot et al. 2000; Cunha & Gough 2000; Saio & Gautschi 2004), and the joint effect of rotation and magnetic field (Bigot & Dziembowski 2002), have been determined by means of a singular perturbation approach. While generally the magnetic field effect on the oscillations is expected to be small, Cunha & Gough (2000) found that at the frequencies of maximal magneto-acoustic coupling, the latter is expected to become significantly large, resulting in an abrupt drop of the separation between mode frequencies.

The observational consequence of the results of Cunha & Gough (2000) suggested that we should see equally spaced modes in roAp stars with an occasional mode much closer to its lower-frequency counterparts. More recently, Cunha (2001) suggested that the explanation of the strange separation between the last two modes observed in HR 1217 in the 1986 data could rest on the occasional abrupt decrease of the large separations predicted by Cunha & Gough (2000). For this prediction to hold, she argued that the observations of Kurtz et al. (1989) must have missed detecting a mode at a frequency 34 μ Hz higher than that of the fifth mode they observed. She predicted that new, more precise measurements would find this ‘missing mode’ if the Alfvénic losses were not large enough to stabilize it. Detailed re-examination of the 1986 data shows no peak at the key position approximately 33 μ Hz above ν_5 at the ~ 0.1 -mmag level.

In the preliminary analysis of the 2000 data we (Kurtz et al. 2002) found that missing mode, giving support to the theory of Cunha. In this paper we have analysed the data in far more detail and report in Section 4 that the new mode is in fact a pair of modes separated by 2.6 μ Hz, a value that is potentially the small spacing, $\delta\nu$. The small spacing has not been unambiguously determined for any roAp star. One of the pair of new modes fits the alternating 33.4, 34.5 μ Hz spacing of the first five modes, and is now separated by nearly precisely $\frac{1}{4}\nu_0$ from the highest detected frequency. The theoretical importance of this new result will be discussed in Section 4.

We also report in Section 4.1 significant amplitude modulation of some of the modes between the 1986 and 2000 data sets, particularly for the frequencies now identified in this paper as ν_3 and ν_4 , but also

for the newly discovered frequencies, ν_7 and ν_8 . This explains why these frequencies were not seen in the 1986 data set – they were below the noise level for that data set, but grew to larger, detectable amplitudes in the Xcov20 2000 data set. Interestingly, we find that the total power for all observed modes is the same in the 1986 and 2000 data sets, even though there have been significant shifts of amplitude between modes, suggesting that the total pulsational energy may be conserved. This is the first time that an roAp star has been observed in enough detail with independent studies to determine this, and it is an important discovery to test against the upcoming *MOST* satellite data set, which will have far higher S/N ratio.

1.3 Astrophysical data for HR 1217

One of the strengths of studying HR 1217 is that there is a wealth of astrophysical information known about it. Its parallax was determined by *Hipparcos* giving a luminosity of $L = 7.8 \pm 0.7 L_{\odot}$ (Matthews et al. 1999). As mentioned earlier, the atmospheres of the most peculiar of the roAp stars show a strong core–wing anomaly (Cowley et al. 2001) in the H lines indicating an extremely abnormal temperature–depth structure to their atmospheres. HR 1217 has this core–wing anomaly. A first attempt at self-consistent atmospheric models that can account for the core–wing anomaly has been made (Kochukhov et al. 2002), but it is not yet even possible to model the Balmer lines fully. The effective temperatures of the roAp stars are thus notoriously difficult to determine. Nevertheless, various photometric and spectroscopic studies conservatively give $T_{\text{eff}} \approx 7300 \pm 200$ K.

The magnetic field has been studied extensively over the years (Preston 1972; Mathys 1991; Bagnulo et al. 1995; Mathys & Hubrig 1997; Leone, Catanzaro & Catalano 2000; Wade et al. 2000). Bagnulo et al. found from broad-band linear polarimetry a polar field strength of approximately 3.9 kG, a rotational inclination of $i = 137^\circ$ and a magnetic obliquity of $\beta = 150^\circ$. The errors on these values are uncertain, but they give us a starting point to discuss the geometry of the pulsation modes.

The rotation period has been controversial, with some (but not all) studies of the magnetic field finding $P_{\text{rot}} = 12.4610$ d and other studies of the magnetic field, the photometric variations and, particularly, the pulsation finding $P_{\text{rot}} = 12.4572$ d. We discuss this problem in detail in Section 4.3 where we find that the pulsation data are definitely best-fitted with $P_{\text{rot}} = 12.4572$ d.

The rotational sidelobes are the key to mode identification in the roAp stars, within the oblique pulsator model (Shibahashi & Takata 1993; Takata & Shibahashi 1995), the improved oblique pulsator model (Bigot & Dziembowski 2002) and the new theory of Saio & Gautschi (2004). We discuss these sidelobes in detail in Section 4.2 where we find they are unchanged between the 1986 and 2000 data sets, and that ν_2 and ν_4 seem to be dipole, or dipole-like modes. Arguments from standard A-star models and the frequency spacings suggests that the even- ℓ modes are probably distorted radial modes, but distorted quadrupole modes cannot be ruled out.

Pulsational radial velocity variations were discovered by Matthews et al. (1988) and have been studied in more detail recently by Balona & Zima (2002) who find rotationally modulated pulsational radial velocity variations that are consistent with the known photometric frequencies. As discussed above, there is a wealth of information in high-precision radial velocity studies of roAp stars. For HR 1217, with its multiple frequencies spaced by $\sim 3 \text{ d}^{-1}$, further progress in this field will need a multisite, 8-m-class telescope campaign. One continuous run of 24 h is all that is needed, so this

is possible – especially as HR 1217 at $\delta = -12^\circ$ is accessible from both hemispheres.

1.4 The importance of the asteroseismic study of HR 1217

HR 1217 is important for both the study of roAp stars and for asteroseismology in general. Cunha, Fernandes & Monteiro (2003) have recently performed seismic tests of the structure of HR 1217. They find that the interior chemical composition has more effect on the theoretical oscillation spectrum than does convection and overshooting. Models with lower abundances of heavy elements in the interior and increased helium come closer to matching the observed frequencies, but they have difficulty finding a plausible model that can yet reproduce the seismic observations of HR 1217. Even though their theoretical predictions can be reconciled with the observations, when the uncertainty in the radius is accounted for, the fact that a systematic shift is found between predicted and observed large separations when different roAp stars are considered (Matthews et al. 1999) might indicate that something is still missing in the theoretical models.

Cunha et al. (2003) emphasize that one important aspect that is not yet well understood is the way in which waves are reflected near the surface of these stars. They argue that if the process of reflection of high-order modes in HR 1217 is frequency dependent, then the large separations could be different from those obtained in their work. Observationally, the complexity of the reflective boundary layer can now be seen for the roAp star 33 Lib (Kurtz et al. 2005). A theory that takes into account the magneto-acoustic nature of the waves near the surface of these stars should be used in order to improve the current models. Thus, the present observations can contribute to the follow-up of this work, not only because of the improvements that they bring to the power spectrum of HR 1217, but also because of their possible contribution to our understanding of the magnetic field of this star.

In this paper we report the frequency analysis of 342 h of high-speed photometry spanning 35 d with a duty cycle of 36 per cent. The resulting amplitude spectra have noise peaks no higher than $80 \mu\text{ma}$,¹ and least-squares errors in amplitude of only $14 \mu\text{ma}$, making this one of the highest precision photometric studies ever undertaken. As outlined in this introduction, we have learned much concerning HR 1217 from WET Xcov20. We have also learned again how complex the roAp stars are. WET Xcov20 is likely to stand as the definitive ground-based study of this star. This is the study that will provide the basis of comparison for higher precision space asteroseismic studies.

2 OBSERVATIONS

WET Xcov20 began on 2000 November 6 and continued for 35 d. Table 1 lists the individual observing runs used in this analysis. Many other runs on marginally photometric nights were rejected because of the demand for very high precision necessary for this study; some other runs were rejected because of equipment problems with particular telescopes in the network. In total

¹ μma denotes micromodulation amplitude and mma denotes millimodulation amplitude. These are in units of 10^{-6} and 10^{-3} of the intensity variation. For amplitudes as low as these they are equal to $-1.087 \mu\text{mag}$ and -1.087mmag , respectively. We use μma and mma throughout this study of the Whole Earth Telescope data newly presented here, but use μmag and mmag for older data where those were the units used in those studies.

Table 1. Observing log of high-speed photometry of HR 1217 from the Whole Earth Telescope Extended Coverage Campaign 20 (WET Xcov20). Columns 2 and 3 list the UT starting date and time of each run. Data were taken with 10-s integrations and then averaged to 40-s integrations; columns 4–6 list the number of 40-s integrations after bad points were removed, the duration of the run in hours and the standard deviation per point of the data in that run after low-frequency sky transparency variations were removed. The BJED start and finish times are for the 40-s integrations that were used in the analysis; in general the first few observations in a run were sky, so the BJED of the first point does not correspond precisely to the UT start time of the run – it begins a short time later. Where the number of points seems few for the duration of the run there are gaps in the data, usually because of cloud, but sometimes because of instrumental problems.

| Run name | Date 2000 | UT start | <i>N</i> | ΔT (h) | σ (mma) | BJED start | BJED end | Observatory | Tel (m) |
|----------|--------------|-------------|----------|-------------------|-------------------|---------------|--------------|-------------|------------|
| mdr136 | November 6 | 02:23:30 | 328 | 4.36 | 1.47 | 1854.609 565 | 1854.790 931 | CTIO | 1.5 |
| mdr137 | November 9 | 01:55:20 | 380 | 4.24 | 0.93 | 1857.695 708 | 1857.872 444 | CTIO | 1.5 |
| mdr138 | November 10 | 01:59:50 | 278 | 3.30 | 1.22 | 1858.590 508 | 1858.728 008 | CTIO | 1.5 |
| mdr139 | November 12 | 02:16:00 | 596 | 6.67 | 1.29 | 1860.601 051 | 1860.878 945 | CTIO | 1.5 |
| mdr140 | November 13 | 01:34:30 | 632 | 7.03 | 1.75 | 1861.573 740 | 1861.866 795 | CTIO | 1.5 |
| mdr141 | November 14 | 01:27:40 | 38 | 0.51 | 2.35 | 1862.567 839 | 1862.589 019 | CTIO | 1.5 |
| sa-od044 | November 14 | 21:03:00 | 419 | 4.82 | 2.44 | 1863.383 580 | 1863.584 274 | SAAO | 1.9 |
| mdr142 | November 15 | 01:28:10 | 655 | 7.27 | 1.95 | 1863.567 955 | 1863.870 964 | CTIO | 1.5 |
| sa-od045 | November 15 | 19:20:00 | 597 | 7.03 | 2.43 | 1864.311 936 | 1864.604 760 | SAAO | 1.9 |
| teiden01 | November 16 | 00:42:10 | 317 | 3.65 | 1.36 | 1864.535 778 | 1864.687 861 | OTIAC | 0.8 |
| mdr143 | November 16 | 01:23:00 | 674 | 7.52 | 2.21 | 1864.564 713 | 1864.878 139 | CTIO | 1.5 |
| mdr144 | November 17 | 01:18:20 | 678 | 7.53 | 2.11 | 1865.561 007 | 1865.874 549 | CTIO | 1.5 |
| no1700q2 | November 17 | 07:28:00 | 300 | 3.35 | 2.22 | 1865.817 025 | 1865.956 608 | MKO | 0.6 |
| no1700q3 | November 17 | 12:34:20 | 33 | 0.36 | 2.15 | 1866.030 797 | 1866.045 843 | MKO | 0.6 |
| teiden04 | November 17 | 22:09:10 | 541 | 6.03 | 2.11 | 1866.428 944 | 1866.680 029 | OTIAC | 0.8 |
| no1800q1 | November 18 | 07:22:30 | 401 | 6.26 | 1.78 | 1866.824 862 | 1867.085 655 | MKO | 0.6 |
| teiden05 | November 18 | 22:53:20 | 292 | 3.31 | 1.76 | 1867.459 609 | 1867.597 341 | OTIAC | 0.8 |
| sa-od047 | November 18 | 23:29:00 | 128 | 1.44 | 1.77 | 1867.484 841 | 1867.545 026 | SAAO | 1.9 |
| no1900q2 | November 19 | 10:14:20 | 343 | 3.84 | 1.62 | 1867.934 491 | 1868.094 444 | MKO | 0.6 |
| sa-od048 | November 19 | 18:55:00 | 579 | 6.76 | 1.28 | 1868.294 557 | 1868.576 270 | SAAO | 1.9 |
| teiden06 | November 19 | 22:05:30 | 547 | 6.13 | 1.32 | 1868.426 269 | 1868.681 478 | OTIAC | 0.8 |
| asm-0079 | November 20 | 03:58:40 | 473 | 5.78 | 1.86 | 1868.673 143 | 1868.913 999 | McD | 2.1 |
| no2000q1 | November 20 | 07:37:00 | 550 | 6.14 | 1.72 | 1868.825 340 | 1869.081 243 | MKO | 0.6 |
| sa-od049 | November 20 | 18:51:00 | 630 | 7.33 | 1.43 | 1869.291 656 | 1869.597 096 | SAAO | 1.9 |
| asm0081 | November 21 | 09:06:10 | 73 | 0.84 | 1.82 | 1869.887 947 | 1869.922 900 | McD | 2.1 |
| sa-m0003 | November 21 | 19:26:50 | 566 | 6.71 | 1.84 | 1870.316 357 | 1870.596 044 | SAAO | 0.75 |
| sa-m0004 | November 22 | 18:28:20 | 669 | 7.64 | 1.97 | 1871.275 902 | 1871.594 052 | SAAO | 0.75 |
| teiden08 | November 22 | 22:07:20 | 143 | 1.62 | 1.82 | 1871.427 513 | 1871.494 874 | OTIAC | 0.8 |
| joy-002 | November 23 | 03:59:50 | 455 | 5.22 | 1.58 | 1871.678 668 | 1871.896 029 | McD | 2.1 |
| no2300q1 | November 23 | 07:15:50 | 423 | 5.52 | 1.39 | 1871.810 495 | 1872.040 587 | MKO | 0.6 |
| pvb1123 | November 23 | 12:15:41 | 105 | 1.20 | 1.32 | 1872.057 263 | 1872.107 147 | PO | 0.6 |
| sa-m0005 | November 23 | 18:11:50 | 83 | 1.10 | 2.59 | 1872.265 350 | 1872.311 357 | SAAO | 0.75 |
| teiden10 | November 23 | 22:05:40 | 482 | 5.45 | 1.86 | 1872.426 459 | 1872.653 600 | OTIAC | 0.8 |
| pvb1124 | November 24 | 12:18:40 | 593 | 6.90 | 1.84 | 1873.018 928 | 1873.306 543 | PO | 0.6 |
| sa-m0006 | November 24 | 18:18:00 | 633 | 7.73 | 1.77 | 1873.271 528 | 1873.593 693 | SAAO | 0.75 |
| teiden11 | November 25 | 02:03:30 | 107 | 1.20 | 2.07 | 1873.586 220 | 1873.636 336 | OTIAC | 0.8 |
| no2500q1 | November 25 | 07:03:10 | 593 | 6.66 | 1.84 | 1873.801 439 | 1874.078 899 | MKO | 0.6 |
| teiden12 | November 25 | 22:09:20 | 492 | 5.59 | 2.52 | 1874.428 975 | 1874.662 019 | OTIAC | 0.8 |
| joy-012 | November 26 | 03:55:50 | 363 | 4.07 | 2.00 | 1874.669 481 | 1874.839 272 | McD | 2.1 |
| no2600q2 | November 26 | 06:59:30 | 583 | 6.47 | 2.31 | 1874.798 761 | 1875.068 205 | MKO | 0.6 |
| jxj-0124 | November 26 | 14:14:50 | 340 | 3.79 | 4.42 | 1875.101 533 | 1875.259 404 | BAO | 0.85 |
| sa-m0007 | November 26 | 18:28:40 | 652 | 7.47 | 2.90 | 1875.275 952 | 1875.587 352 | SAAO | 0.75 |
| teiden13 | November 26 | 21:50:30 | 34 | 0.40 | 1.67 | 1875.415 880 | 1875.432 546 | OTIAC | 0.8 |
| joy-016 | November 27 | 04:03:00 | 405 | 4.75 | 3.87 | 1875.674 440 | 1875.872 241 | McD | 2.1 |
| no2700q1 | November 27 | 06:38:00 | 562 | 6.72 | 2.66 | 1875.783 813 | 1876.063 790 | MKO | 0.6 |
| jxj-0127 | November 27 | 13:44:10 | 425 | 4.73 | 2.86 | 1876.079 525 | 1876.276 747 | BAO | 0.85 |
| sa-m0008 | November 27 | 18:27:50 | 668 | 7.56 | 3.57 | 1876.275 065 | 1876.589 880 | SAAO | 0.75 |
| teiden14 | November 27 | 22:28:20 | 330 | 3.70 | 2.94 | 1876.442 134 | 1876.596 185 | OTIAC | 0.8 |
| joy-020 | November 28 | 04:04:20 | 134 | 1.54 | 4.19 | 1876.675 347 | 1876.739 583 | McD | 2.1 |
| no2800q1 | November 28 | 06:45:00 | 44 | 1.84 | 2.97 | 1876.791 317 | 1876.868 053 | MKO | 0.6 |
| sa-h-046 | November 28 | 18:54:30 | 606 | 7.33 | 2.64 | 1877.295 357 | 1877.600 682 | SAAO | 1.9 |
| teiden15 | November 28 | 22:01:50 | 494 | 5.50 | 3.09 | 1877.423 711 | 1877.652 704 | OTIAC | 0.8 |
| joy-025 | November 29 | 04:00:40 | 436 | 5.21 | 2.80 | 1877.672 780 | 1877.889 678 | McD | 2.1 |
| no2900q1 | November 29 | 06:41:00 | 589 | 6.76 | 2.52 | 1877.785 972 | 1878.067 685 | MKO | 0.6 |
| sa-gh465 | November 29 | 20:30:30 | 427 | 5.19 | 2.34 | 1878.277 628 | 1878.493 948 | SAAO | 1.9 |
| teiden16 | November 29 | 21:18:50 | 239 | 2.64 | 2.38 | 1878.518 308 | 1878.628 494 | OTIAC | 0.8 |
| joy-028 | November 30 | 01:19:10 | 466 | 5.23 | 2.09 | 1878.668 360 | 1878.886 069 | McD | 2.1 |
| no3000q1 | November 30 | 06:40:50 | 592 | 6.76 | 2.11 | 1878.785 719 | 1879.067 200 | MKO | 0.6 |

Table 1 – *continued*

| Run name | Date 2000 | UT start | <i>N</i> | ΔT (h) | σ (mma) | BJED start | BJED end | Observatory | Tel (m) |
|----------|--------------|-------------|----------|-------------------|-------------------|---------------|--------------|-------------|------------|
| sa-gh466 | November 30 | 19:30:20 | 140 | 1.67 | 1.16 | 1879.318 577 | 1879.388 137 | SAAO | 1.9 |
| teiden17 | November 30 | 21:17:50 | 404 | 5.92 | 2.03 | 1879.393 112 | 1879.639 929 | OTIAC | 0.8 |
| sa-gh467 | November 30 | 21:19:50 | 67 | 0.75 | 2.00 | 1879.394 616 | 1879.425 982 | SAAO | 1.9 |
| sa-gh468 | November 30 | 22:15:20 | 204 | 2.33 | 1.48 | 1879.432 926 | 1879.530 032 | SAAO | 1.9 |
| sa-gh469 | December 1 | 00:39:00 | 131 | 1.52 | 2.31 | 1879.532 692 | 1879.596 118 | SAAO | 1.9 |
| sa-gh470 | December 1 | 18:57:20 | 294 | 3.45 | 1.46 | 1880.295 983 | 1880.439 849 | SAAO | 1.9 |
| teiden18 | December 1 | 22:04:00 | 338 | 3.85 | 1.46 | 1880.425 147 | 1880.585 390 | OTIAC | 0.8 |
| sa-gh471 | December 1 | 22:30:00 | 310 | 3.65 | 1.56 | 1880.443 549 | 1880.595 633 | SAAO | 1.9 |
| sa-gh472 | December 2 | 18:46:50 | 229 | 2.77 | 1.48 | 1881.289 014 | 1881.404 523 | SAAO | 1.9 |
| sa-gh473 | December 2 | 21:38:30 | 356 | 4.31 | 1.22 | 1881.407 992 | 1881.587 621 | SAAO | 1.9 |
| teiden19 | December 2 | 21:46:50 | 125 | 1.38 | 1.90 | 1881.413 316 | 1881.470 723 | OTIAC | 0.8 |
| sa-gh474 | December 3 | 18:35:20 | 616 | 7.47 | 1.59 | 1882.281 116 | 1882.592 459 | SAAO | 1.9 |
| sa-gh475 | December 4 | 21:51:00 | 329 | 4.20 | 1.22 | 1883.416 385 | 1883.591 500 | SAAO | 1.9 |
| tsm-0087 | December 5 | 05:50:40 | 249 | 2.99 | 1.46 | 1883.749 014 | 1883.873 782 | McD | 2.1 |
| tsm-0089 | December 6 | 03:47:00 | 321 | 4.00 | 2.02 | 1884.663 106 | 1884.829 657 | McD | 2.1 |
| r00-022 | December 6 | 18:55:00 | 201 | 2.22 | 1.82 | 1885.296 998 | 1885.389 591 | ARIES | 1.0 |
| r00-025 | December 7 | 16:53:20 | 202 | 2.24 | 1.96 | 1886.209 392 | 1886.302 563 | ARIES | 1.0 |
| teiden22 | December 8 | 21:37:40 | 121 | 1.40 | 2.42 | 1887.406 650 | 1887.465 099 | OTIAC | 0.8 |
| teiden24 | December 9 | 21:01:50 | 168 | 1.90 | 2.87 | 1888.381 731 | 1888.461 013 | OTIAC | 0.8 |
| teiden27 | December 11 | 00:20:40 | 201 | 2.22 | 2.49 | 1889.526 538 | 1889.619 189 | OTIAC | 0.8 |
| Total | | | | 341.94 | | | 35.01 | | |

The observatories are: CTIO, Cerro Tololo Interamerican Observatory, La Silla, Chile; SAAO, South African Astronomical Observatory, Sutherland, South Africa; OTIAC, Observatorio del Teide de Instituto de Astrofísica de Canarias, Teide, Tenerife, Spain; MKO, Mauna Kea Observatory, Hawaii, USA; McD, McDonald Observatory, Mt Locke, Texas, USA; PO, Perth Observatory, Bickley, Western Australia, Australia; BAO, Beijing Astronomical Observatory, Beijing, China; ARIES, Aryabhata Research Institute of Observational Sciences, Manora Peak, Naini Tal, India.

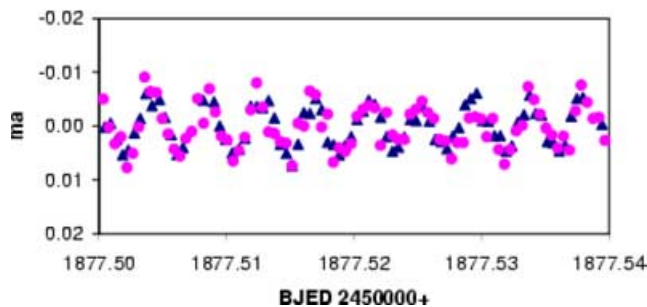


Figure 1. An example of a 1-h section of the light curve of HR 1217 with overlapping data from SAAO (triangles) and OTIAC (circles). The excellent agreement between the two observatories is apparent.

341.94 h of observations were analysed and are listed in Table 1. They span 35.01 d. There was overlap between observatories for 40.65 h of the 341.94 h total, so the duty cycle for the entire run was 36 per cent = $(341.94 - 40.65) / (35.01 \times 24)$. The overlapping light curves from pairs of observatories show good agreement, as can be seen in Fig. 1.

The telescopes used range in aperture from 0.6 to 2.1 m. Data from all sites were obtained using photoelectric photometers, with 10-s individual integrations. At Beijing Astronomical Observatory, McDonald Observatory, Mauna Kea Observatory, State Observatory at Naini Tal and Observatorio del Teide, the observers used three-channel photometers that are functionally similar to the equipment described in Kleinman, Nather & Phillips (1996). The South African Astronomical Observatory and Perth Observatory observations were made with single-channel photometers, and the observations at CTIO with a two-channel photometer. At all sites, observations were made through a Johnson *B* filter, along with neutral

density filters when needed to keep the count rates below 10^6 s^{-1} . Following the procedures described in Kleinman et al. (1996), the sky background was continuously monitored with the three-channel instruments. At sites using two- and single-channel photometers, the sky was obtained several times during the night at irregular intervals, and then interpolated during reduction. Dead-time corrections were measured and applied for all photometers. After bad points were removed and the data reduced using standard procedures, low-frequency sky transparency noise was removed by successive pre-whitening of low-frequency peaks until the low-frequency noise was of the same amplitude as that at higher frequencies; in general this procedure was applied to frequencies below 0.6 mHz (i.e. on time-scales longer than approximately 30 min), but for no run does the low-frequency filtering affect the pulsation frequencies near 2.7 mHz. The principal reason for removing low-frequency noise is to obtain white noise across the frequency spectrum for appropriate estimates of least-squares errors. The 10-s integrations were co-added to 40-s integrations to reduce the computing time and smooth high-frequency noise in the visual inspection of the light curves.

During a WET campaign data are reduced at headquarters by a variety of people to a high standard. Nevertheless, it is WET policy to re-reduce all data in a campaign uniformly. This is best performed by one person working diligently to the highest standard of precision. In the case of Xcov20 and the HR 1217 data analysed in this paper, all reductions were uniformly carried out by one of us (CC), and then double-checked by another of us (DWK).

3 FREQUENCY ANALYSIS

Fourier analysis of the data listed in Table 1 was performed using a discrete Fourier transform for unequally spaced data. Fig. 2 shows

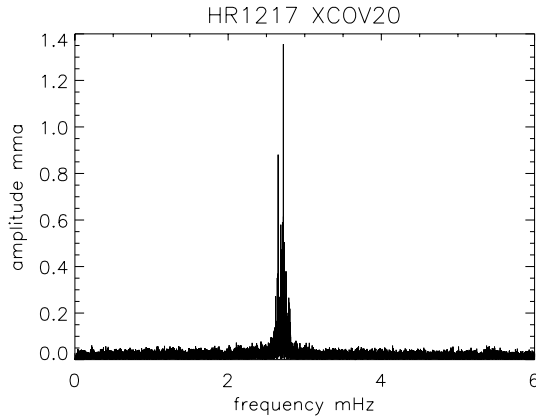


Figure 2. An amplitude spectrum of the entire data set showing that all of the signal is near 2.7 mHz. The first harmonic of the unresolved frequencies can be seen in some of the individual runs when the amplitude is highest, but not in the entire data set.

the amplitude spectrum over the range 0–6 mHz, where it can be seen that all of the signal is concentrated near 2.7 mHz.

In some of the individual data sets the first harmonic of the unresolved frequencies can be seen. Harmonics are commonly observed in roAp stars, so it is no surprise to find evidence of them for HR 1217, although they remain unexplained theoretically. Recently, Kurtz et al. (2005) have found that the 4.030-mHz harmonic in HD 137949 has a higher amplitude than the 2.015-mHz mode at some atmospheric depths; it is clear now that the harmonics of the modes in roAp stars can be studied in much more detail spectroscopically than photometrically. In any case, at the low S/N ratio available for the harmonics for HR 1217, there is nothing useful that we can do with them here.

In Fig. 3 we expand the region where there is signal to show the amplitude spectrum at higher resolution. The level of the highest noise peaks in the amplitude spectrum is approximately 80 μ ma making this one of the highest precision ground-based data sets ever obtained. The bottom panel of Fig. 3 shows the spectral window for our 36 per cent duty cycle data set. While there is still some cross-talk between the spectral windows of the real frequencies, it is possible with sequential pre-whitening to extract the component frequencies with considerable confidence.

We show some of this sequential pre-whitening in Fig. 4 for the three highest peaks. The techniques we used were: to identify the highest peak in the amplitude spectrum; fit the frequency of that peak and all previously determined peaks by linear least squares to the data with their rotational sidelobes fixed using the known rotation frequency (from $P_{\text{rot}} = 12.4576$ d – Kurtz & Marang 1987 and Section 4.3 below); then use the non-linear least-squares method to optimize the frequencies, and the amplitudes and phases, and to determine formal errors on the frequencies; then to revert to the linear least-squares method using the mode frequencies from the non-linear least-squares fit, but again fixing the rotational sidelobes using the known rotational frequency spacing. The frequencies of the rotational sidelobes did not shift significantly in the non-linear least-squares fits, but it is important to keep them at exactly the rotational frequency spacing to examine their phases in the final interpretation of the data, as is seen in Section 4.2 below. (From our knowledge of roAp stars it seems a reasonable assumption that the rotational sidelobes are spaced at the rotational frequency. Tests of this assumption with the current data set are consistent with it.) The data were then pre-whitened by the solution and the next highest

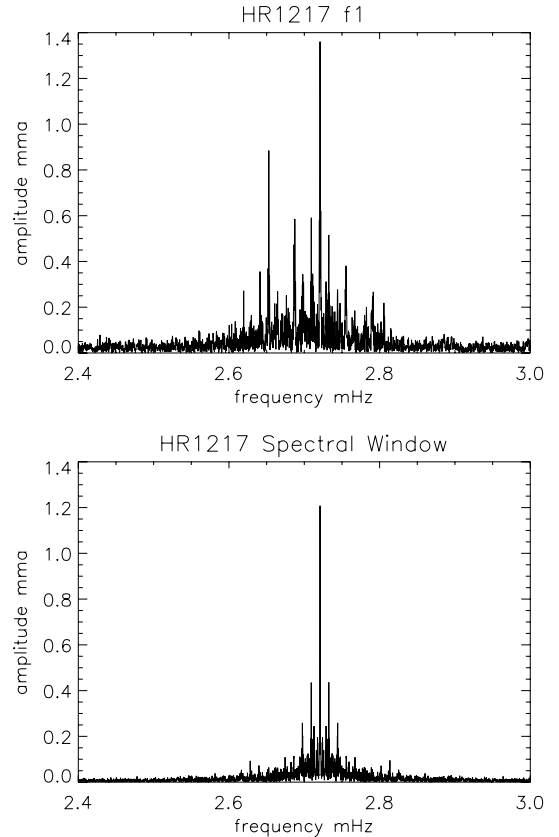


Figure 3. The top panel shows the amplitude spectrum for the entire data set over the frequency range where there is significant signal. The bottom panel shows the spectral window for the highest peak in the top panel. This has been generated by sampling a noise-free sinusoid with the same frequency and amplitude as the highest peak at the actual times of the observations. The amplitude used is taken from the final determination of the frequencies given in Table 2. The difference between the amplitudes of the highest peak in the top panel and in the spectral window in the bottom panel gives a good indication of the amount of cross-talk between the spectral windows of the real frequencies. In a careful examination of both panels the presence of the rotational sidelobes can be seen in the top panel, but not in the spectral window in the bottom panel.

significant peak searched for. This was continued until no further significant peaks could be extracted.

From the panels in Fig. 4 it can be seen that the duty cycle is sufficient that crosstalk among the pulsation frequencies and their rotational sidelobes is not a major problem. Fig. 5 shows a schematic amplitude spectrum of the determined frequency solution and Table 2 gives the frequencies, amplitudes, phases and formal internal errors for that solution. Eight independent pulsation modes were found with significant first rotational sidelobes for most of them. The second rotational sidelobes did not have amplitudes higher than the highest noise peaks, so are not included in the solution, although for ν_4 they are formally significant. The lowest-amplitude rotational sidelobes fitted in Table 2 are only formally significant; as they have amplitudes lower than the highest noise peaks, they cannot be considered to be really detected, but they are included in the fit for completeness and to put a limit on their possible amplitudes.

Additional peaks were found near ν_2 and ν_3 and are listed in Table 2. Similar peaks were seen in the 1986 data (Kurtz et al. 1989) and are suspected to be caused by frequency and/or amplitude modulation over the time-span of the data set. As can be seen in Fig. 5

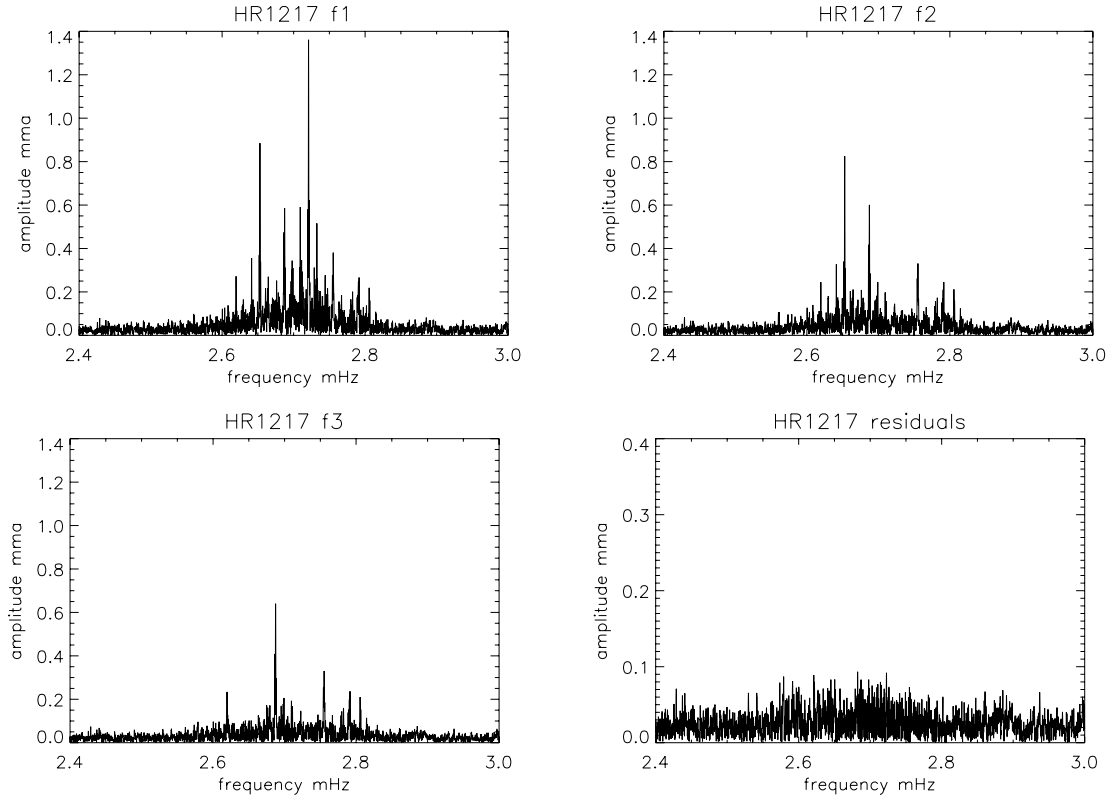


Figure 4. The upper left-hand panel shows the amplitude spectrum for the entire data set over the frequency range where there is significant signal. This is the same as the top panel in Fig. 3 and is reproduced here for easy comparison with the amplitude spectra in the other panels; the highest peak is ν_4 (where the frequencies are labelled in order of increasing frequency; see Table 2). The upper right-hand panel shows the amplitude spectrum after the highest peak, ν_4 , and its rotational sidelobes have been pre-whitened; the highest peak here is ν_2 . The lower left-hand panel shows the same after ν_4 and ν_2 and their rotational sidelobes have been pre-whitened; the highest peak is ν_3 . The text explains the use of linear and non-linear least squares in the procedure. The lower right-hand panel skips many steps to show the residuals after ν_1 to ν_8 , their rotational sidelobes and two other significant frequencies have been pre-whitened. Note the change of ordinate scale in the bottom right-hand panel. There is some amplitude left above the noise level. This could be caused by further, low-amplitude, undetected frequencies; by amplitude or frequency variation over the 35-d data set; or by imperfect pre-whitening because of some cross-talk among the spectral windows of the real frequencies.

and discussed in Section 4.1, there is clear amplitude modulation between the 1986 and 2000 data, making it believable that small amounts of such modulation may be detected over a 35-d time-span. These additional peaks, named ν_{2+} and ν_{3-} in Table 2 are not fully resolved from the peaks they are close to, so they are quoted to much lower precision in frequency. They should only be taken to be indicative of additional amplitude left in the amplitude spectrum after pre-whitening by the eight-frequency solution with rotational sidelobes.

4 RESULTS

In their preliminary analysis of the Xcov20 2000 data Kurtz et al. (2002) discovered the ‘missing’ mode in HR 1217 predicted by Cunha (2001). The most striking new result we find here in the analysis of the full data set is that the new frequency is actually two modes very closely spaced. In the determination of these frequencies we are working near the S/N ratio limit in even this outstandingly low-noise data set, so that confirmation of our results will have to await the analysis of the *MOST* data set. However, the frequencies determined have spacings with respect to the other frequencies that give us some confidence in their reality. We show the frequency spacings in Table 3 and compare them to those determined in the 1986 data set, where the errors on frequency for the 1986 data are

from a new non-linear least-squares fit we have performed for those data.

Table 3 suggests that the frequency separations are variable by up to few tenths of a μHz . Given that it is known that there is frequency variability in roAp stars, with an amplitude of 0.12 μHz on a 1.6-yr time-scale in the case of HR 3831 (Kurtz et al. 1997), this variability in the frequency separations in HR 1217 seems to be real. The cause of this frequency variability in roAp stars is not known.

As can be seen from Table 3, and in Fig. 5, there is an alternating frequency spacing of approximately 33.4 and 34.5 μHz , consistent with an interpretation of alternating even and odd ℓ -modes, although, for HR 1217 the modes cannot be pure spherical harmonic modes. The spacing of $\nu_8 - \nu_6$ is, within the errors, exactly half of the 33.4- μHz spacing, and the spacing of $\nu_7 - \nu_6 = 2.6 \mu\text{Hz}$ is possibly the ‘small spacing’.

Cunha (2001) suggested that in HR 1217 an optimal magneto-acoustic coupling, associated with large energy losses, takes place at a frequency – which we will call ν_{jump} – lying between the last two modes observed in the 1986 data. If this suggestion is correct, then any mode in HR 1217 with an unperturbed frequency (i.e. the frequency that the mode would have if there were no magnetic field) above ν_{jump} should have its frequency substantially reduced by the magnetic field. As two new modes were found with frequencies close to ν_{jump} , namely ν_6 and ν_7 , we should ask whether these new

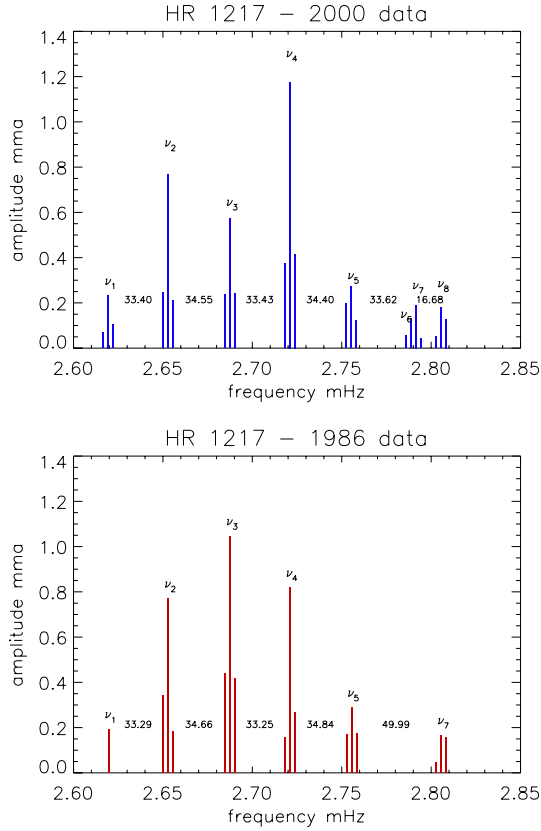


Figure 5. Top panel: a schematic amplitude spectrum of the adopted frequency solution given in Table 2. The spacing of the rotational sidelobes is exaggerated to make them easier to see. Bottom panel: the same for the 1986 data, again with exaggerated rotational sidelobe separations to make them easier to see.

modes have corresponding unperturbed frequencies below or above ν_{jump} . The simplest interpretation, in the light of Cunha’s suggestion, is that ν_6 is the ‘missing mode’ already referred to in her work. In this case its unperturbed frequency is smaller than ν_{jump} and is only slightly changed (enlarged) by the presence of the magnetic field. Assuming this interpretation holds, then ν_7 can either be a mode with an unperturbed frequency very close to that of ν_6 , and also smaller than ν_{jump} , or it can be a mode where the frequency, in the absence of the magnetic field, would be above ν_{jump} , but has been greatly reduced due to the effect of the magnetic field.

In the first case, the simplest interpretation is that ν_6 and ν_7 are modes with consecutive orders and degrees differing by two, hence differing in frequency by the ‘small spacing’. In that case only ν_8 would have its frequency greatly reduced by the magnetic field. Unfortunately, the theory developed by Cunha & Gough (2000) has to be improved before one can quantify with precision the amount by which the frequency of ν_8 is decreased. Preliminary results (Cunha, in preparation) indicate that modes with frequencies larger than that at which the maximum coupling takes place have their frequencies decreased by a fraction of the large separation, that fraction being determined by an integral that depends on the degree of the mode and the geometry of the magnetic field. However, in order to know whether that fraction is the 25 per cent implied by the observations, under this interpretation, it is necessary first to improve the theoretical model.

The second possibility considered here is more interesting, in the sense that it puts additional constraints on Cunha’s interpretation.

Table 2. The frequency solution for HR 1217.

| ID | ν (μHz) | A (mma) | ϕ (rad) |
|----------------------------|-----------------------------|-------------------|------------------|
| $\nu_1 - \nu_{\text{rot}}$ | 2618.617 | 0.069 ± 0.014 | 2.42 ± 0.20 |
| ν_1 | 2619.546 ± 0.015 | 0.235 ± 0.014 | 2.24 ± 0.06 |
| $\nu_1 + \nu_{\text{rot}}$ | 2620.475 | 0.106 ± 0.014 | 2.49 ± 0.13 |
| $\nu_2 - \nu_{\text{rot}}$ | 2652.014 | 0.246 ± 0.014 | 1.09 ± 0.06 |
| ν_2 | 2652.943 ± 0.005 | 0.767 ± 0.014 | 1.08 ± 0.02 |
| $\nu_2 + \nu_{\text{rot}}$ | 2653.872 | 0.209 ± 0.014 | 1.02 ± 0.07 |
| $\nu_3 - \nu_{\text{rot}}$ | 2686.566 | 0.238 ± 0.015 | 1.18 ± 0.06 |
| ν_3 | 2687.495 ± 0.006 | 0.573 ± 0.014 | 0.53 ± 0.03 |
| $\nu_3 + \nu_{\text{rot}}$ | 2688.424 | 0.241 ± 0.014 | 0.86 ± 0.06 |
| $\nu_4 - \nu_{\text{rot}}$ | 2719.998 | 0.374 ± 0.014 | -2.54 ± 0.04 |
| ν_4 | 2720.927 ± 0.003 | 1.176 ± 0.014 | -2.77 ± 0.01 |
| $\nu_4 + \nu_{\text{rot}}$ | 2721.856 | 0.416 ± 0.014 | -2.85 ± 0.03 |
| $\nu_5 - \nu_{\text{rot}}$ | 2754.402 | 0.196 ± 0.014 | -2.27 ± 0.07 |
| ν_5 | 2755.331 ± 0.014 | 0.273 ± 0.014 | -2.04 ± 0.05 |
| $\nu_5 + \nu_{\text{rot}}$ | 2756.260 | 0.123 ± 0.014 | -1.87 ± 0.11 |
| $\nu_6 - \nu_{\text{rot}}$ | 2788.017 | 0.056 ± 0.014 | 1.12 ± 0.25 |
| ν_6 | 2788.946 ± 0.037 | 0.105 ± 0.014 | 1.23 ± 0.14 |
| $\nu_6 + \nu_{\text{rot}}$ | 2789.875 | 0.100 ± 0.014 | 0.37 ± 0.14 |
| $\nu_7 - \nu_{\text{rot}}$ | 2790.624 | 0.130 ± 0.014 | 0.38 ± 0.11 |
| ν_7 | 2791.553 ± 0.023 | 0.191 ± 0.014 | 0.26 ± 0.08 |
| $\nu_7 + \nu_{\text{rot}}$ | 2792.483 | 0.042 ± 0.014 | 0.53 ± 0.34 |
| $\nu_8 - \nu_{\text{rot}}$ | 2804.692 | 0.051 ± 0.014 | 1.23 ± 0.28 |
| ν_8 | 2805.621 ± 0.021 | 0.180 ± 0.014 | 2.44 ± 0.08 |
| $\nu_8 + \nu_{\text{rot}}$ | 2806.550 | 0.129 ± 0.014 | 1.51 ± 0.11 |
| ν_{2+} | 2653.4 | 0.114 ± 0.014 | 2.52 ± 0.13 |
| ν_{3-} | 2686.2 | 0.201 ± 0.015 | 0.66 ± 0.07 |

$$\sigma_{\text{fit}} = 1.65 \text{ mma. } t_0 = \text{BJD}2451876.93.$$

If the unperturbed frequency of ν_7 is above ν_{jump} , it is necessary to explain simultaneously the abrupt decrease in the frequencies of two modes, namely ν_7 and ν_8 . In her preliminary calculations Cunha found that for a dipole field, the decrease in the frequency of $\ell = 0$ modes is considerably larger than the decrease in the frequencies of $\ell = 1$ and 2 modes. Thus, with the current models, the only combination that she found for which both ν_7 and ν_8 would have unperturbed frequencies above ν_{jump} requires ν_7 to be an $\ell = 1$ mode and ν_8 to be an $\ell = 0$ mode. Moreover, this interpretation is possible only if ν_6 has even degree, which, due to the alternation between even and odd degrees in the power spectrum, would also imply even degrees for ν_2 and ν_4 . As the latter two modes are more likely to be dipole modes, at the moment the interpretation that both new modes have unperturbed frequencies below ν_{jump} and that their frequencies differ by an amount that is equal to the small spacing is favoured by Cunha’s theory. We emphasize, however, that due to the strong dependence of Cunha’s results on the geometry of the magnetic field, this conclusion might be modified if the magnetic field geometry of HR 1217 is not dipolar. We also note that in the work of Bagnulo et al. (1995) the authors state that their polarimetric study cannot distinguish between a dipolar magnetic field and one described by a dipole plus a co-linear quadrupole, i.e. a centred dipole field, or a displaced dipole field.

Recently, Saio & Gautschi (2004) have also studied the magnetic perturbations to the eigenfrequencies, and found results which are qualitatively in agreement with those of Cunha & Gough (2000). Rather than applying a variational principle, as Cunha & Gough did, Saio & Gautschi expanded the solutions in sums of spherical harmonics and determined, simultaneously, the perturbed

Table 3. Comparison of the HR 1217 frequency spacings between the 2000 WET data and the 1986 data.

| ID | $\Delta\nu$ 2000 | $\Delta\nu$ 1986 | $\Delta\Delta\nu$ |
|-------------------------------------|---------------------|---------------------|-------------------|
| $\nu_2 - \nu_1$ | 33.397 ± 0.016 | 33.268 ± 0.020 | 0.13 ± 0.03 |
| $\nu_3 - \nu_2$ | 34.552 ± 0.008 | 34.712 ± 0.008 | -0.16 ± 0.01 |
| $\nu_4 - \nu_3$ | 33.432 ± 0.007 | 33.204 ± 0.007 | 0.23 ± 0.01 |
| $\nu_5 - \nu_4$ | 34.404 ± 0.014 | 34.936 ± 0.018 | -0.53 ± 0.02 |
| $\nu_6 - \nu_5$ | 33.615 ± 0.039 | | |
| $\nu_7 - \nu_6$ | 2.607 ± 0.044 | | |
| $\nu_7 - \nu_5$ | 36.222 ± 0.027 | | |
| $\nu_8 - \nu_6$ | 16.675 ± 0.042 | | |
| $\nu_8 - \nu_7$ | 14.068 ± 0.031 | | |
| $2(\nu_8 - \nu_6)$ | 33.350 ± 0.085 | | |
| $\nu_8 - \nu_5$ | 50.290 ± 0.025 | 49.889 ± 0.029 | 0.40 ± 0.03 |
| $(\nu_2 - \nu_1) - (\nu_4 - \nu_3)$ | -0.035 ± 0.018 | 0.04 ± 0.11 | |
| $(\nu_3 - \nu_2) - (\nu_5 - \nu_4)$ | 0.148 ± 0.016 | -0.18 ± 0.11 | |

eigenfrequencies and eigenfunctions. This expansion in spherical harmonics had to be truncated in order to obtain the solutions, and generally no convergence was found for the frequencies at which the perturbation is most significant.

According to the suggestion of Cunha (2001), ν_6 lies precisely in the frequency region where the perturbation is greatest. Thus, just as in the case of Cunha & Gough (2000), caution should be applied when using Saio & Gautschi's frequency perturbations at the frequencies around ν_6 . Despite the existence of quantitative differences in the absolute frequency perturbations found, the ratio between the 'frequency jumps' suffered by modes of different degrees is very similar in the two works. Thus, the above discussion regarding the interpretation of ν_6 , ν_7 and ν_8 is maintained if the results of Saio & Gautschi (2004) are used, rather than those of Cunha & Gough (2000).

4.1 Amplitude variations between the 2000 and 1986 data sets

A question that arose in our preliminary study of the 2000 WET data (Kurtz et al. 2002), where the 'missing' frequency was discovered, was 'why was this frequency not seen in the 1986 data?' The amplitude found by Kurtz et al. was high enough that it should have been detected in the 1986 data, if it were there at the same amplitude. It is clear from the comparison of the amplitudes in the 2000 and 1986 data sets shown in Table 4 and in Fig. 5 that significant amplitude variability does occur for HR 1217. In particular, look at the amplitudes of the highest peak, ν_4 , the mode next to it at ν_3 and their rotational sidelobes. Their amplitudes have changed by several tenths of an mma – much more than the amplitude of the newly discovered frequencies now called ν_6 and ν_7 . We conclude that these newly discovered frequencies had amplitudes below the detection limit in the 1986 data; for the 2000 data the noise level was reduced and the amplitudes grew.

Interestingly, there is little difference in the sum of the power between the two data sets (given at the bottom of Table 4), indicating that the total pulsational energy has possibly been conserved, but that there has been transfer of some energy from one mode to another – particularly, from ν_3 to ν_4 . It is not possible to be certain about this until we can characterize the modes completely, as distorted modes of differing ℓ have different projection factors, precluding relating pulsation power (amplitude squared) directly to pulsation energy. Some singly periodic roAp stars are known to have very

Table 4. Amplitude variation between the 2000 and 1986 data. Only frequencies detected in both data sets are listed; i.e. only the frequencies found in the 1986 data, all of which are also found in the 2000 data, are listed. The values of the frequencies from the 2000 data set are given, and the identification labels are the same as those in Table 2.

| ID | ν (μHz) | A 2000 (mma) ± 0.014 | A 1986 (mma) ± 0.018 | ΔA (mma) ± 0.023 |
|--------------------------------|-----------------------------|-----------------------------------|-----------------------------------|------------------------------------|
| ν_1 | 2619.546 | 0.235 | 0.193 | 0.042 |
| $\nu_2 - \nu_{\text{rot}}$ | 2652.014 | 0.246 | 0.343 | -0.097 |
| ν_2 | 2652.943 | 0.767 | 0.770 | -0.003 |
| $\nu_2 + \nu_{\text{rot}}$ | 2653.872 | 0.209 | 0.183 | 0.026 |
| $\nu_3 - \nu_{\text{rot}}$ | 2686.566 | 0.238 | 0.442 | -0.204 |
| ν_3 | 2687.495 | 0.573 | 1.046 | -0.473 |
| $\nu_3 + \nu_{\text{rot}}$ | 2688.424 | 0.241 | 0.418 | -0.177 |
| $\nu_4 - \nu_{\text{rot}}$ | 2719.998 | 0.374 | 0.156 | 0.218 |
| ν_4 | 2720.927 | 1.176 | 0.818 | 0.358 |
| $\nu_4 + \nu_{\text{rot}}$ | 2721.856 | 0.416 | 0.266 | 0.150 |
| $\nu_5 - \nu_{\text{rot}}$ | 2754.402 | 0.196 | 0.170 | 0.026 |
| ν_5 | 2755.331 | 0.273 | 0.288 | -0.015 |
| $\nu_5 + \nu_{\text{rot}}$ | 2756.260 | 0.123 | 0.176 | -0.053 |
| $\nu_8 - \nu_{\text{rot}}$ | 2804.692 | 0.051 | 0.048 | 0.003 |
| ν_8 | 2805.621 | 0.180 | 0.166 | 0.014 |
| $\nu_8 + \nu_{\text{rot}}$ | 2806.550 | 0.129 | 0.157 | -0.028 |
| Power = A^2 mma ² | | 3.1 ± 0.5 | 3.2 ± 0.7 | 0.1 ± 0.7 |

stable amplitudes, such as HR 3831 (Kurtz et al. 1997) where there has been minimal amplitude change over 18 yr of observations. Other roAp stars, such as HD 60435 (Matthews, Wehlau & Kurtz 1987) have many more pulsation modes than HR 1217 and show significant amplitude modulation on a time-scale of only a few days. We conjecture that the multimode roAp stars transfer energy between modes, whereas the singly periodic stars cannot do that and have stable amplitudes. The results in Table 4 are the first data that can test whether total pulsation power is conserved in multimode stars where energy exchange occurs. Great caution is called for here, however, as so little is known concerning pulsation power stability. A counter-example to the above suggestion is the star HD 217522 (Kreidl et al. 1991) where an entirely new frequency appeared

between data sets in 1982 and 1989 and power does not seem to have been conserved in a star with only two modes.

4.2 The rotational sidelobes

The oblique pulsator model (OPM) for the roAp stars (Kurtz 1982) has been the standard interpretation for the rotational amplitude and phase modulation for these stars since it was introduced. In this picture the pulsation modes are axisymmetric modes aligned with the magnetic axis which is itself inclined to the rotational axis by an angle β . The rotational axis is inclined to the line of sight by an angle i . The most developed theory of the standard OPM can be found in Takata & Shibahashi (1995) and Shibahashi & Takata (1993).

Recently, Bigot & Dziembowski (2002) have introduced the ‘improved oblique pulsator model’ (IOPM) which presents a very different picture. They find that the centrifugal distortion of the star has a greater effect on the pulsations than the Coriolis force, and that the combination of rotational and magnetic effects on the pulsation modes couples the $2\ell + 1$ magnetic levels for each given ℓ . In particular, for $\ell = 1$ they find three eigenmodes for which the axis of symmetry precesses in a plane that does not generally contain the magnetic axis.

Bigot & Kurtz (2005) find a geometry for the roAp star HR 3831 of $(i, \beta) = (84, 7)^\circ$ from the IOPM. This is in good agreement with $i = 90^\circ \pm 1^\circ$ and $\beta = 8^\circ \pm 1^\circ$ found by Bagnulo et al. (1999) from magnetic measurements. The old oblique pulsator model with a dipole-like pulsation aligned with the magnetic axis is not plausible with this geometry. The observed semi-amplitude for HR 3831 is approximately 5 mmag in B ; with this geometry, if the mode could be seen from pole-on, then it would have a semi-amplitude of 36 mmag. This is vastly larger than the highest amplitude seen for any of the 34 known roAp stars of 8 mmag, in the case of HD 60435, and that is for multiple modes beating with each other. The highest-amplitude individual modes are for HD 101065 and HR 3831 and are approximately 5 mmag. However, Kochukhov (2004) finds a different magnetic and pulsation geometry for HR 3831 that is in good agreement with the old oblique pulsator model and is inconsistent with the Bagnulo et al. (1999) geometry. Further magnetic studies should differentiate between these two models.

Most recently, Saio & Gautschi (2004) have looked at the interaction of pulsation and the magnetic field in roAp stars and confirmed, as first found by Dziembowski & Goode (1996), that the pulsation modes cannot be characterized by a single spherical harmonic. They are distorted dipole and quadrupole modes that are aligned with the magnetic axis.

In all of these models the pulsation modes are inclined to the rotation axis so that they are viewed at varying aspect with the

rotation of the star. This is a unique aspect of the roAp stars that allows their mode geometries to be studied in more detail than for any other kind of pulsating star. The information concerning the rotational modulation of the modes is contained in the amplitudes and phases of the rotational sidelobes to the mode frequencies. Those amplitudes and phases are listed in Table 2 and shown schematically in Fig. 5. Given that there is residual amplitude in the amplitude spectrum of the residuals to our frequency solution with highest peaks around $80 \mu\text{ma}$ (see Fig. 4), we advise caution in interpreting the rotational sidelobes with amplitudes less than that, even though they are formally significant.

First, we will examine the phases of the rotational sidelobes in Table 2. We do not consider the rotational sidelobes for ν_6 and ν_7 to be significant enough to discuss further, and, as was found in the 1986 data by Kurtz et al. (1989), ν_8 seems to be a rotational doublet, rather than a triplet – or at least it has very different amplitudes for the rotational sidelobes. This has been found in two independent multisite data sets (1986 and 2000), so is significant, but we do not know the cause of it. So we will look at the rotational sidelobes for the first five modes, ν_1 to ν_5 .

The zero point of the time-scale for the frequency solution in Table 2 is $t_0 = \text{BJED}2451876.93$ and it was selected to set the phases of the rotational sidelobes of ν_1 to ν_5 equal to each other. It can be seen that the phases of the modes and their rotational sidelobes for each of the five modes under consideration here are nearly equal, in most cases they are not statistically significantly different within the formal errors, and those errors neglect the cross-talk problems that we know are still present in this 36 per cent duty cycle data set. This near equality of the phases of the frequency triplets means that the pulsation modes are being amplitude modulated with rotation, but not phase modulated. This significantly simplifies the interpretation of the rotational triplets; pure amplitude modulation is what we expect for axisymmetric normal modes seen from varying aspect.

All of the oblique pulsator models characterize the rotational triplets for dipole modes using two parameters. We will use the notation of Bigot & Dziembowski in the IOPM and call these two parameters γ^+ and γ^- . They are:

$$\gamma^+ = \frac{A_{+1} + A_{-1}}{A_0} \quad \text{and} \quad \gamma^- = \frac{A_{+1} - A_{-1}}{A_{+1} + A_{-1}},$$

where A_{+1} and A_{-1} refer to the amplitudes of the higher- and lower-frequency rotational sidelobes, respectively, and A_0 refers to the amplitude of the central frequency of the triplet. In Table 5 we list these parameters for the all eight modes in the 2000 data and compare them to the values found by Kurtz et al. (1989) for five of those modes in the 1986 data. The frequencies and frequency labels are those from the 2000 WET data set – this work. There is no difference in the values of γ^+ and γ^- greater than 3σ between the

Table 5. The values of γ^+ and γ^- that characterize the rotational amplitude modulation of the pulsation modes.

| ID | Frequency | γ^+ | γ^- | γ^+ | γ^- |
|---------|-----------|-----------------|------------------|-----------------|------------------|
| | | 2000 | | 1986 | |
| ν_1 | 2619.546 | 0.74 ± 0.10 | 0.21 ± 0.12 | | |
| ν_2 | 2652.943 | 0.59 ± 0.03 | -0.08 ± 0.04 | 0.68 ± 0.04 | -0.30 ± 0.05 |
| ν_3 | 2687.495 | 0.84 ± 0.04 | 0.01 ± 0.04 | 0.82 ± 0.03 | -0.03 ± 0.03 |
| ν_4 | 2720.927 | 0.67 ± 0.02 | 0.05 ± 0.03 | 0.52 ± 0.03 | 0.26 ± 0.06 |
| ν_5 | 2755.331 | 1.17 ± 0.09 | -0.23 ± 0.06 | 1.20 ± 0.12 | 0.02 ± 0.07 |
| ν_6 | 2788.946 | 1.49 ± 0.27 | 0.28 ± 0.13 | | |
| ν_7 | 2791.553 | 0.90 ± 0.12 | -0.51 ± 0.13 | | |
| ν_8 | 2805.621 | 1.00 ± 0.13 | 0.43 ± 0.12 | 1.23 ± 0.20 | 0.53 ± 0.14 |

2000 and 1986 data sets, except for γ^+ for ν_4 which is technically at the 4σ level, but with some cross-talk between the frequencies in both data sets, we do not interpret that to be significant.

Kurtz et al. (1989) concluded that ν_2 and ν_4 are very nearly dipole modes within the standard OPM, but found that the other modes could not be purely spherical harmonic modes. Their argument came from the fact that within the OPM

$$\gamma^+ = \frac{A_{+1} + A_{-1}}{A_0} = \tan i \tan \beta.$$

In that model i and β are the same for a dipole pulsation mode and a dipole magnetic field. The most recent and best values for those parameters come from a broad-band linear polarization study by Bagnulo et al. (1995) who find $i = 137^\circ$ and $\beta = 150^\circ$. Those give $\gamma^+ = \tan i \tan \beta = 0.54$, close to the values of 0.59 and 0.67 found in Table 6 for ν_2 and ν_4 , hence the suggestion that those modes are dipole pulsation modes, or at least something close to normal dipole modes. Matthews et al. (1999) showed from *Hipparcos* parallaxes and standard A-star models that the large spacing, $\Delta\nu_0$ equals 68 μHz for HR 1217, hence the modes are alternating even and odd ℓ -modes, or distorted alternating even and odd ℓ -modes. Shibahashi & Saio (1985) long ago argued from standard A-star models that the alternating spacing seen in HR 1217 (see Fig. 5) suggests that the even- ℓ modes should be radial modes, rather than quadrupole modes. Of course, those even- ℓ modes are amplitude modulated, so cannot be simple radial modes which would not vary with aspect, but neither can they be quadrupole modes – they do not modulate as expected for quadrupole modes, given the known i and β from magnetic measurements (see Kurtz et al. 1989). Hence the conclusion within the standard OPM is that the even- ℓ modes must be distorted by the magnetic field. The values of γ^+ for ν_1 , ν_3 and ν_5 are close to each other in Table 5, so whatever the interpretation of them, they can be considered to have the same mode identification – i.e. they modulate in the same way with rotation.

The new theoretical work of Bigot & Dziembowski (2002) addresses the rotational and magnetic distortions directly; that of Saio & Gautschi (2004) addresses only the magnetic distortion. In the latter study the modes are truly distorted from simple spherical harmonic modes; in the former (IOPM) the modes are still described by spherical harmonics, but are combinations of azimuthal m -modes for a given ℓ . In the IOPM γ^+ depends on the rotational inclination, i , the inclination of the normal of the pulsation plane to the rotation axis, δ , and the ellipticity of the pulsation mode motion in the pulsation plane, ψ , all of which depend ultimately on the rotational and magnetic distortions and have to be modelled.

The values of γ^- are all less than unity. Within the standard OPM (Takata & Shibahashi 1995) this suggests that the rotational perturbation is small compared with the magnetic perturbations, as

$$\gamma^- = \frac{A_{+1} - A_{-1}}{A_{+1} + A_{-1}} = \frac{C_{n\ell}\Omega}{\omega_1^{(1)\text{mag}} - \omega_0^{(1)\text{mag}}},$$

where the numerator is a measure of the rotational perturbation and the denominator of the magnetic perturbation.

Within the IOPM of Bigot & Dziembowski (2002) the relative importance of the rotational and magnetic effects is described by the parameter

$$\mu = \frac{\omega_0^{\text{mag}} - \omega_1^{\text{mag}}}{D},$$

where ω_0^{mag} and ω_1^{mag} are the magnetic eigenfrequencies for modes with $m = 0$ and 1, respectively, and D is a measure of the centrifugal distortion. For HR 1217, with its relatively long 12.4572-d rotation period (see Section 4.3) and strong polar magnetic field strength of

3.9 kG (Bagnulo et al. 1995), we would expect a large value of μ . In that case, the IOPM predicts that one of modes should be nearly aligned with the magnetic field, independently of the magnetic obliquity, implying that for that mode $\psi \approx 0$ and $|\delta| \approx |\pi/2 - \beta|$.

Having fixed the inclination of the observer, $i = 137^\circ$ (from Bagnulo et al. 1995), the observed values of γ^\pm from Table 5 lead to the common value of $\delta \approx 56^\circ$ for both eigenmodes, ν_2 and ν_4 . This value of δ is relatively well determined with an error of approximately 2° – 3° . The ellipticities of the modes ν_2 and ν_4 are small, $\psi \sim 1^\circ$ – 3° . These two modes are then almost standing modes pulsating in the direction $\pi/2 + \delta \approx 146^\circ$, which is the same as that found by Bagnulo for the inclination of the magnetic axis, within the observational error bars. We can conclude that, from the IOPM, the modes ν_2 and ν_4 are essentially $m = 0$ modes in the magnetic reference system. That these two frequencies are well represented by a single spherical harmonic could be in contrast with the expectations that the modes should be distorted (as found by Dziembowski & Goode 1996; Cunha & Gough 2000; Bigot & Dziembowski 2002; and Saio & Gautschi 2004). However, the degree of distortion depends on the entire geometry, on the strength of the coupling and on the averaging effect. For dipole modes, in particular, it is likely that the distortion seen is very small, as the averaging by cancellation of the higher ℓ components over the stellar disc would be greater than that of the $\ell = 1$ components. At present we can only say for ν_1 , ν_3 and ν_5 that if they are radial modes, then they are distorted, as they are modulated with rotation; if they are quadrupole modes, then further study is needed to understand them within the OPM, the IOPM and the theory of Saio & Gautschi.

4.3 The rotation period

There is a controversy over the rotation period of HR 1217. Kurtz & Marang (1987) used a combination of measurements of magnetic field variability, mean light variability (i.e. rotational light variability) in Johnson and Strömgren colours and Eu II line strength variability – all from different authors and different studies – to derive a rotational ephemeris of

$$\text{HJD (extremum)} = 2440578.23 \pm 0.09 + 12.4572 \pm 0.0003 \text{ E.}$$

This derivation was made under the assumption that all of those parameters vary with exactly the same period, and that period is the rotation period. This is consistent with what is known of oblique rotation in many magnetic Ap stars.

Kurtz et al. (1989) found in their multisite study of the pulsation of HR 1217 that the times of magnetic and pulsation maximum are the same within 0.05 ± 0.03 rotation periods, but that the time of mean light extremum is slightly different by -0.031 ± 0.010 rotation periods to the pulsation maximum. These small differences over the many rotation periods studied in this star are not the source of the controversy over the rotation period. They are simply too small to matter at the precision that the rotation period is known. Kurtz et al. (1989) used the rotation period of Kurtz & Marang and gave an ephemeris for pulsation amplitude maximum of

$$\text{HJD (amplitude maximum)} = 2446743.54 \pm 0.38 + 12.4572 \text{ E.}$$

Mathys (1991) found this period to be incompatible with his and Preston's (1972) combined magnetic measurements. He derived a rotation period of 12.4610 ± 0.0025 d; although that value is formally compatible with the period of 12.4572 ± 0.0003 d found by Kurtz & Marang, Mathys found that the latter did not phase his and Preston's combined magnetic observations satisfactorily. He suggested that the assumed phase equality of the photometry in

different colours and the Eu II line strengths may have caused Kurtz & Marang to derive a wrong period. For the problem to be in the magnetic measurements there must be a systematic shift between the observations of Mathys and those of Preston larger than is expected. Mathys & Hubrig (1997) obtained five new measurements of the magnetic field of HR 1217 to add to the two measurements of Mathys (1991) and compared with the measurements of Preston. They confirm the 12.4610-d rotation period and show that the 12.4572-d period cannot correctly phase their measurements with those of Preston.

Bagnulo et al. (1995) used broad-band linear polarimetry to derive measures of the longitudinal magnetic field of HR 1217 on 14 different dates between 1991 September and 1994 January. It is from their study that we quoted the best values of i and β in the last section. They obtained a rotation period of 12.4610 ± 0.0011 d, consistent with that of Mathys and inconsistent with that of Kurtz & Marang. They performed further error analysis, however, and concluded that the period of Kurtz & Marang cannot be ruled out by their study.

Wade et al. (2000) obtained four new measurements of the longitudinal field strength. They found that the 12.4572-d period phases all observations except those of Mathys & Hubrig (1997) well, whereas the 12.4610-d period does not. They suggest that a zero-point shift is needed to bring the measurements of Mathys & Hubrig and others into agreement.

Leone et al. (2000) obtained eight new measurements of the longitudinal magnetic field of HR 1217 over a time-span of 123 d. They concluded that the best rotation period is that of Kurtz & Marang, not that of Mathys. They found agreement between their measurements and those of Mathys, but suggested that those of Preston need to be shifted by -0.75 kG. They noted that 12.4572-d period phases the *Hipparcos* photometry and the single filter photometry of Wolff & Morrison (1973) well, while the 12.4610-d period is shifted by 0.15 periods.

We discussed in the introduction the clear spectroscopic observations of roAp stars that show horizontal and/or vertical abundance distributions in these stars that are obvious in the amplitudes and phases of the pulsational radial velocities. It is therefore possible from the conclusions of Leone et al. (2000) and Wade et al. (2000) above that the use of different sets of spectral lines to determine the magnetic field strengths in the different data sets has sampled the magnetic field differently, giving rise to a zero-point shift between data sets. A good test of this is to use one set of data to measure the magnetic field strengths using Preston's (1972) chosen lines and Mathys's (1991) chosen lines to see if there is a systematic shift in the measured magnetic field values.

We can calculate the rotation period from studies of the pulsation alone. We have collected all high-speed B photometric observations of HR 1217 dating back to 1981. We have fitted the highest amplitude frequency in our solution, $\nu_4 = 2.7209$ mHz to sections of the data five pulsation cycles long (30.63 min) and used a least-squares fit to derive the amplitude for that section. Of course, for sections that short the many modes beat against each other, but all the modes are rotationally modulated, so the highest-amplitude sections do vary with rotation clearly. This procedure gave us 1614 5-cycle amplitudes spanning 20 yr of observations. A Fourier analysis yielded a best value of the rotation period of 12.4572 d – the same as that derived by Kurtz & Marang, so we conclude that, at least for the pulsation data, that is the best value for the rotation period. Fig. 6 shows the relevant section of the amplitude spectrum.

We conclude that the photometric pulsation rotation period is probably correct, implying that some of the magnetic measurements

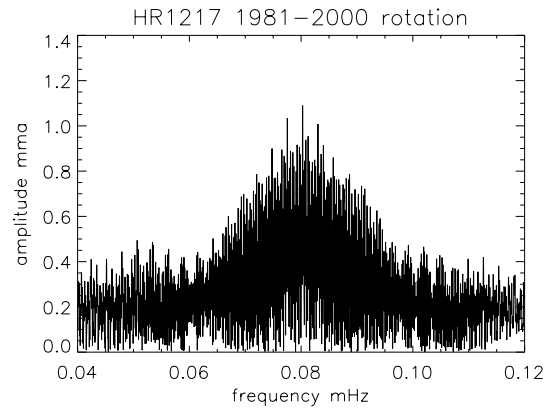


Figure 6. Amplitude spectrum of the pulsation amplitude determined by least squares for sections of the data 5 cycles, or 30.63-min long, for $\nu_4 = 2.7209$ mHz. The data span 20 yr from 1981. The highest peak is at $\nu_{\text{rot}} = 0.080\,272\,\text{d}^{-1}$, corresponding to a best period of 12.4572 d.

need zero-point shifts larger than has heretofore been thought reasonable. It is not plausible that the magnetic period and photometric period may be different, as that would cause a drift in the relative times of maxima between the magnetic field and pulsation, making their present coincidence improbable.

5 CONCLUSIONS

The alternating frequency spacing of HR 1217 makes its amplitude spectrum one of the most interesting known after that of the Sun for the study of non-degenerate stars asteroseismically. For this reason it is one of the best-studied roAp stars. The WET Xcov20 campaign on HR 1217 is likely to stand as the definitive ground-based study with a photometric precision of $14\,\mu\text{m}$. Future observations and studies will be from space; the first space data set has already been obtained with the *MOST* satellite. Observations with *MOST* have a precision many times better than in the WET data set; the data presented in this paper and those from the 1986 campaign have set the baseline to which the space data will be compared.

We have found the new frequency predicted by Cunha (2001), and also expected in the theory of Saio & Gautschi (2004), giving strong support to these studies. The results of our frequency analysis are so detailed, especially with the discovery of both ν_6 and ν_7 and the precise determination of their spacings from the previously known frequencies, that more theoretical development is needed to confront these results.

We have clearly shown that there is amplitude modulation for some of the modes in HR 1217 between the 1986 and 2000 multisite campaigns. This explains why the new frequencies were not detected in the 1986 data. It appears that the pulsational energy has been conserved between the 1986 and 2000 data sets, suggesting that energy is transferred between modes, although the case for this is not strong.

The rotational sidelobes confirm that the modes giving rise to ν_2 and ν_4 are close to dipolar, while the modes giving rise to ν_1 , ν_3 and ν_5 are distorted even- ℓ modes, but whether $\ell = 0$ or 2 cannot be determined, for either the old oblique pulsator model or the new improved oblique pulsator model.

In the controversy over the rotation period we confirm that $P_{\text{rot}} = 12.4572$ d fits *all* the photometric B pulsation data from 1981 to 2000. This suggests that some zero-point shifts between some of the magnetic measurements are needed.

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