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The Successes And Challenges Of Utilizing Geogebra To Integrate MP5: Use Appropriate Tools Strategically

David Scott Matthews II
Missouri State University

THE SUCCESSES AND CHALLENGES OF UTILIZING GEOGEBRA TO INTEGRATE MP5: USE APPROPRIATE TOOLS STRATEGICALLY

A Masters Thesis

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Of the Requirements for the Degree

Master of Science in Education, Secondary Education

By

David Scott Matthews II

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THE SUCCESSES AND CHALLENGES OF UTILIZING GEOGEBRA TO INTEGRATE MP5: USE APPROPRIATE TOOLS STRATEGICALLY

Mathematics

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Master of Science in Education

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ABSTRACT

The purpose of this action research study was to identify the successes and challenges a teacher experienced when GeoGebra was incorporated into an Algebra II unit of study with the goal of integrating the fifth mathematical practice of the Common Core State Standards, *use appropriate tools strategically*. Data were collected from 20 student participants and the teacher-researcher via the following methods: teacher-researcher self-observations, peer educator observations, student interviews, and video-recordings. Instruments in the form of an observational protocol, utilized by observers, and an interview protocol, utilized by the interviewer, were employed. The data analysis indicated two successes: the teacher-researcher's instruction targeted a deeper level of mathematical understanding by students and a moderately high level of student interest. The data analysis also indicated three challenges: a) challenges with technology, specifically, computer access, internet speed, internet access, and a GeoGebra problem; b) challenges with students, specifically, students being unprepared for class, the time required for students to prepare for a lesson, and the need to monitor student computer usage; and c) challenges with teachers, specifically, with other Algebra II teachers and with the teacher-researcher.

KEYWORDS: GeoGebra, mathematics' software, mathematical tool, MP5, Common Core, Algebra II

This abstract is approved as to form and content

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Dr. Kurt Killion
Chairperson, Advisory Committee
Missouri State University

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Approved:

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Dr. Kurt Killion

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Dr. Gay Ragan

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Dr. Beth Hurst

____
Dr. Julie Masterson: Dean, Graduate College

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I dedicate this thesis to my wife, Erin Matthews. Without her belief in me, I may not have even attempted a graduate degree in mathematics education. Moreover, it was her continual support that allowed me to commit as much energy and time to my studies as I did. Throughout this process she proved numerous times why I consider her to be the most thoughtful and caring person I have ever met and how truly lucky I am to have her in my life, not only as a wife, but as a friend. Thank you Erin.

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CHAPTER I: OVERVIEW OF THE STUDY

As computer technology has advanced, it has had the potential of playing an increasingly larger role in education. In mathematics education, computer technology's potential has been found in two prominent forms, graphing calculators and computers. These two forms have necessitated further research as these forms have become more accessible, with the development of an impetus for schools to provide every student with a computer (one-to-one), and with a call for an inclusion of technology from the Common Core State Standards (CCSS) through mathematical practice five (MP5) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). However, the call for an inclusion of technology into mathematics classrooms is not a new development and has existed before the CCSS and MP5.

In the late 1980s, the National Council of Teachers of Mathematics (NCTM) published their support of the use of calculators and computers in mathematics classrooms in their book *Curriculum and Evaluation Standards for School Mathematics* (1989). In this book NCTM (1989) stated, "Calculators should be available to all students at all times" and "Every student should have access to a computer for individual and group work" (p. 8). This sentiment was then echoed to a greater degree by NCTM (2000) when they published *Principles and Standards for School Mathematics* in which technology was listed as one of the six principles of school mathematics: "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 25).

Though NCTM published *Principles and Standards for School Mathematics* and

called for a higher level of inclusion of technology in the mathematics classroom the problem that mathematics students faced was whether their teachers were willing to incorporate any technology, specifically graphing software and computer algebra systems, in their classrooms. If this was not occurring it was not important whether it was due to a lack of availability, a lack of teacher knowledge of technology, or a lack of financing, as this study discussed an inexpensive alternative to expensive graphing software and computer algebra systems that schools could utilize (Dewey, Singletary, & Kinzel, 2009; Lee & McDougall, 2010; Simonsen & Thomas, 1997). More specifically, in order to fulfill the NCTM's call for an inclusion of technology in the 21st century classroom and to integrate one of the CCSS's mathematical practices, *use appropriate tools strategically* (MP5), this study explored the successes and challenges of a technological alternative, GeoGebra, to graphing software and computer algebra systems.

Rationale and Purpose of the Study

The release of the Common Core State Standards for Mathematics and Mathematical Practices in 2009, has challenged mathematics educators as they continuously struggle with how best to incorporate a curriculum that will address both the standards and practices (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). To aid in this pursuit, this study utilized the implementation of GeoGebra as a way of integrating one of the eight standards of mathematical practice, specifically MP5, *use appropriate tools strategically*. The implementation of GeoGebra with the goal of integrating MP5 allowed for a deeper understanding of the integration process as well as MP5 itself. The school district in

which this study took place had aligned its K-12 mathematics curriculum with the Missouri Learning Standards (MLS) and CCSS, and the Missouri Department of Elementary and Secondary Education (DESE) endorse the MLS (Missouri Department of Elementary & Secondary Education, 2016a, 2016b, National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Thus, integrating MP5 was important to the school district in which this study took place.

GeoGebra was chosen as a means to integrate MP5 for a variety of reasons. Research by Wachira and Keengwe (2011), Dewey et al. (2009), and Simonsen and Thomas (1997) has suggested that part of the reason that graphing calculators are not more widely utilized within mathematics classrooms is their cost and availability. Thus, part of the purpose of this study was to utilize an inexpensive and easy-to-use alternative to graphing calculators, the free computer software package GeoGebra, that could be implemented into a high school level mathematics course in a school in which each student had access to a laptop, a situation known as one-to-one. The software package of GeoGebra possessed the added benefit that not only does it contain the capabilities to supplant graphing calculators in a mathematics classroom but could also offer the ability to act as a substitute for costly computer algebra systems software.

Lastly, this study will guide future studies in the implementation of GeoGebra into a high school mathematics course with the goal of integrating MP5. This study will also guide future preservice teacher development programs or in-service teacher professional development programs for mathematics educators by discussing the successes and challenges experienced by a teacher in his attempt to integrate MP5 utilizing GeoGebra. The aforementioned rationales for the study construct the purpose of the study which is: to identify the successes and challenges a teacher experienced

when GeoGebra was incorporated into an Algebra II unit of study with the goal of integrating the fifth mathematical practice of the CCSS, *use appropriate tools strategically*.

Research Questions

The following research questions guided this study:

1. What successes were experienced by a teacher in his implementation of GeoGebra with the goal of integrating MP5 into an Algebra II unit of study?

2. What challenges were experienced by a teacher in his implementation of GeoGebra with the goal of integrating MP5 into an Algebra II unit of study?

Research Design

The study took place in a suburban area of the southwestern region of Missouri. The participants were 20 students and teacher-researcher and took place during an Algebra II class during the third class period of the day. The students were selected utilizing a convenience sampling and were not selected for any other particular reason with regard to this study (Gay, Mills, & Airasian, 2011). The unit of Algebra II that the study took place during, polynomials and polynomial functions, was also not chosen for any particular reason with regard to this study. The unit of study that was utilized was employed purely because of the time frame in which the Institutional Review Board (Appendix A) approved the study and the close proximity of the beginning of the unit to that time period. To integrate MP5 in an Algebra II unit of study, the technological tool of GeoGebra was chosen due to its lack of expense and possession of graphing capabilities as well as its computer algebra system capabilities. GeoGebra was incorporated into 11 of the 12 lessons developed and taught during the course of this

study. To determine what successes and challenges were experienced during the application of GeoGebra to the Algebra II unit of study, data were collected in the following ways: teacher-researcher self-observations, peer educator observations, interviews with students, and video recordings of lessons taught. To ensure that data were collected in a consistent and organized manner, observations utilized the observational protocol (Appendix B) and interviews utilized the interview protocol (Appendix C). To ensure that the data collected was accurate, the data garnered from the present study was cross-referenced in a process known as data triangulation.

Significance of the Study

This study indicated the successes and challenges of a high school mathematics teacher in pursuit of integrating MP5 by implementing GeoGebra into an Algebra II unit of study. This information is important for other mathematics educators to know in their pursuit of addressing CCSS, and specifically in integrating MP5. This study will help mathematics educators develop a deeper understanding of MP5 and the nuances of its integration. Moreover, this information indicates to other mathematics educators the opportunity, by including both successes and challenges, of GeoGebra to be applied to Algebra II. The findings of this study indicate how GeoGebra can be utilized in a unit of Algebra II and hint at the possibility that it may contain for the class as a whole. Lastly, since GeoGebra is free and easy to access, the results of this study have financial ramifications for school districts as they try to meet 21st century district technology goals. Due to the limited financial circumstances that schools and teachers find themselves operating within, this study offers an understanding of the financial necessities that surround the implementation of GeoGebra.

Assumptions and Limitations

For the purpose of this study, there was only one assumption that was necessary to be made. The one assumption was that students were honest in interviews.

For the purpose of this study, the following limitations were made:

1. The study was limited to 20 students in one southwest Missouri school district in one Algebra II class during the fall of 2016.

2. The three-week period of this study was a limitation. Ideally, many lessons would be conducted with the same group of students over an entire school year so that students could get comfortable with the technology, and long-term data on the teacher's experience could be studied.

3. This study did not collect data to show that the teacher-researcher's goal of achieving a deeper level of mathematical understanding by students and resulting utilization of discovery learning yielded an increase in students' mathematical understanding.

Definition of Terms

For the purpose of this study, the following terms were defined:

1. *Algebra II.* The school district in which the study took place incorporates the following topics into their Algebra II courses: equations and inequalities, linear equations and functions, linear systems, quadratic functions and factoring, polynomials and polynomial functions, rational exponents and radical functions, exponential and logarithmic functions, and rational functions.

2. *CAS.* CAS is an acronym for Computer Algebra System, which is a software program that allows computation over algebraic expressions and equations (Sozcu, Ziatdinov, & Ipek, 2013).

3. *Conceptual Knowledge.* Star and Styliandies (2013) define conceptual knowledge as "to know why something happens in a particular way" (p. 170).

4. *Discovery Learning.* In a discovery learning environment students are presented with a question or problem, as opposed to being presented with established facts, and then allowed time to explore and research the issue so that they further develop their knowledge and attain a solution (Abdi, 2014).

5. *GeoGebra.* GeoGebra is a free internet-based mathematical software package that can be accessed via the internet. Salleh and Sulaiman (2013) described

GeoGebra as a computer algebra system (CAS), meaning it possesses the features of a graphing calculator with algebraic manipulation capabilities, and interactive geometric software (IGS), similar to Geometer's Sketchpad or Cabri Geometry.

6. *MP5.* MP5 is an acronym for Mathematical Practice 5, *use appropriate tools strategically*, and refers to the fifth Standard for Mathematical Practice as listed in the CCSS. MP5 states that "mathematically proficient students consider the available tools when solving a mathematical problem", "are sufficiently familiar with tools appropriate for their grade or course", and "are able to use technological tools to explore and deepen their understanding of concepts" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 7). Furthermore, CCSS states that the tools can include "pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 7).

7. *Procedural Knowledge.* Star and Styliandies (2013) define procedural knowledge as "to know how something happens in a particular way" (p. 170).

Summary

Due to endorsements for an inclusion of technology into the 21st century mathematics classroom from NCTM (2000), CCSS, and MLS, this study was necessary to address those recommendations (Missouri Department of Elementary & Secondary Education, 2016a, National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Thus, the purpose of this study was to identify the successes and challenges a teacher experienced when GeoGebra was incorporated into an Algebra II unit of study with the goal of integrating the fifth mathematical practice of the CCSS, *use appropriate tools strategically*. To integrate MP5 in an Algebra II unit of study, the technological tool of GeoGebra was chosen due to its lack of expense and possession of graphing capabilities as well as its computer algebra system capabilities. To determine what successes and challenges were experienced

during the application of GeoGebra to an Algebra II unit of study, data were collected in the following ways: teacher-researcher self-observations, peer educator observations, interviews with students, and video recordings of lessons taught. The data collected in this study is important for other mathematics educators to know in their pursuit of addressing CCSS, and specifically in integrating MP5. Moreover, this information indicates to other mathematics educators the opportunity, by including both successes and challenges, of GeoGebra to be applied to Algebra II. Lastly, since GeoGebra is free and easy to access, the results of this study have financial ramifications for school districts as they try to meet 21st century district technology goals.

CHAPTER II: REVIEW OF RELATED LITERATURE

This chapter will provide literature to summarize the state of research from 1994-2014 with respect to graphing technology and GeoGebra and help develop a clearer understanding of graphing technology in mathematics classrooms. Specifically in this chapter, the related issues and empirical research to be reviewed are as follows: (a) a discussion of the effect of technology on students' mathematical skill development; (b) the manners in which graphing technology aids or hinders students' mathematical skill development; (c) teachers' beliefs about and attitudes towards the use of graphing technology; and (d) a summary will be provided.

The Effect of Technology on Students' Mathematical Understanding

Ellington (2003, 2006) did a meta-analysis of 54 studies and a meta-analysis of 42 studies to determine the effect of graphing calculators on students' mathematics achievement. The studies evaluated spanned the seventh grade through the first year of college and included courses from pre-algebra through calculus. In both meta-analyses, it was established that graphing calculators, when allowed in the classroom but not allowed to be used on tests, do not help students with procedural understanding or overall mathematical achievement but they do aid with conceptual understanding. When graphing calculators were allowed in the classroom and could be utilized by students on the tests, students showed an increased ability to understand mathematics both procedurally and conceptually, an increase in their operational skills, an increase in their problem-solving skills, and an increase in their mathematical achievement (Ellington, 2003, 2006).

Lee and McDougall (2010) summarized that the majority of research that has been conducted with regard to graphing calculator inclusion in the classroom has shown a positive effect on student learning. Bouck (2009) echoed this finding when she stated, "Research has suggested that graphing calculators can support students in developing a conceptual understanding of mathematics, increase their skill level in problem-solving, and improve test scores on measures of achievement and performance" (p. 207). Moreover, Bouck (2009) also stated, "The use of a graphing calculator has been associated with improvement in mathematics in that the more times a student has used one, the higher their gains have been in developing conceptual understanding and problem-solving" (p. 207).

Rakes, Valentine, McGatha, and Ronau (2010) reviewed 594 research articles discussing algebra instructional improvement strategies. After excluding research articles due to various issues, 82 studies involving 22,424 students were targeted and utilized to produce effect sizes of the algebra instructional improvement strategies. Study inclusion was determined by satisfying three criteria: "the intervention had to target the learning of algebraic concepts", "the intervention had to involve a method for improving learning as measured by student achievement", and "the study had to employ an experimental design with a comparison group" (Rakes et al., 2010, p. 379). For the strategies titled *technology tools,* defined as calculators, graphing calculators, computer programs, and java applets, Rakes et al. found an effect size of 0.304. According to Rakes et al., this would suggest that *technology tools* have a positive effect on learning mathematics and that the effect would be moderate in intensity.

One of the reasons that graphing technology may have such a huge impact on a students' understanding of mathematics may be due to its ability visually to display what

is occurring in a given scenario (Konyalioglu, Aksu, & Senel, 2012). Konyalioglu et al. (2012) cited multiple studies depicting the vital importance that visualization has in mathematics. Stupel and Ben-Chaim (2014) also supported this notion when they referred to graphical visualization as "crucial" to a student properly understanding mathematics (p. 928). More precisely, Stupel and Ben-Chaim (2014) claimed, "that graphical representation permits generalization and better insight into the subject" of mathematics (p. 923). Stupel and Ben-Chaim (2014) indicated that using technology, specifically GeoGebra, allows students "to visualize a wide range of different examples and representations, further elucidating the significance of the solutions" (p. 924).

How Technology Effects Students' Mathematical Understanding

Bouck (2009) hypothesized, as did Graham and Thomas (2000) and Merriweather and Tharp (1999), that the reason that graphing calculators may be superior to four-function calculators was that the visual display of problems was larger which would result in the ability of the user to see multiple steps worked out on the screen. This attribute of graphing calculators would make it easier for students to identify a mistake in their calculations and then fix that mistake. Simonsen and Thomas (1997) provided three main reasons given by teachers as to why graphing calculators were advantageous: students spent less time on computation, received immediate feedback from the calculator, and were better able to visualize the mathematics being taught. Simmit (1997) found that teachers saw a benefit from the graphing calculators in two capacities: they increased students' confidence in the accuracy of their graphs and the calculators motivated the students to a high degree.

Slavit (1996) and Merriweather and Tharp (1999) noted that student interest and motivation were increased while using the graphing calculators. Slavit also noted that students initiated discussions three times more often in lessons that incorporated graphing calculators and the teacher was twice as likely to ask analytical questions during lessons that incorporated graphing calculators. Slavit suggested that these increases may have been due to the fact that the lessons that incorporated graphing calculators possessed the following attributes: the lessons were more analytical in nature, the graphing calculator allowed the teacher to use more real-life examples and thus problems more interesting to the students, and/or because students could investigate problems from a graphical and numerical perspectives while relating it back to the symbolic form. Doerr and Zangor (2000) found that in classes that utilized graphing calculators, a shift occurred where the teacher went from task setter and explainer to consultant, fellow investigator, and resource. This was substantial for them because it meant that the instructional methodology would also shift from a more lecture-based format to a more group work-oriented discovery-based format. Saab, Joolingen, and Hout-Wolters (2005) and Sungur and Tekkaya (2006) support this transition to a discovery-based teaching approach.

Wachira and Keengwe (2011) listed the following as potential problems for teachers to overcome when including graphing calculators: lack of equipment, unreliability of equipment, lack of technical support, lack of training, lack of time, organizational culture of the school, and teachers' openness to change. Lack of adequate time was the main factor that teachers cited as to either why they did not include graphing calculators in their classrooms at all or more than they currently did (Wachira & Keengwe, 2011).

Ruthven, Deaney, and Hennessy (2009) cited the following benefits to using graphing technology in the high school mathematics classroom: improves production, overcomes pupil difficulties and building assurance, enhances the variety and appeal of classroom activity, and fosters pupil independence and peer exchange. After concluding their case study of 11 different schools that utilized either graphing calculators or graphing software, Ruthven et al. (2009) stated, as did Quesada and Maxwell (1994), that teachers mentioned that graphing technology was used because it increased instructional variety, enhanced student motivation, and caused students to have less of a dependence on the teacher. This last benefit lead to the finding that classes could be taught with less teacher direction and more student investigation and group work. Ruthven et al. speculated that the main reason that teachers may be hesitant to include technology might be due to the requirement to modify classroom routines to allow for the incorporation of technology in the classroom. Ruthven et al. pointed out that the schools did have to allow some time for students to become familiar with the graphing technology that they used and that the schools had to teach students how to graph both by hand and by utilizing either computer software or graphing calculators.

With regard to GeoGebra specifically, the benefit lies in the fact that it is easier to illustrate shifts of functions and potential subsequent relationship that could exist with solutions of equations (Stupel & Ben-Chaim, 2014). Furthermore, students are more likely to make conjectures that they can test (Leung, 2006; Mackrell, 2012; Stupel, 2012). Hanna (1998) argued that this attribute, of making students more likely to make conjectures that they can test themselves, would not only lead to informal proofs where students are merely showing that a theorem is correct but would also show students the necessity of a more formal deductive proof. Jones and Gutierrez (2000) and Mariotti

(2000) supported this claim and said that an IGS, which GeoGebra is, causes students to shift their focus to theoretical possibilities. Leung (2006) added that GeoGebra "has the potential of breaking down the traditional separation between action and deduction" (p. 31). Moreover, GeoGebra offers a fast and non-judgmental feedback for students and easily allows them to explore the possibilities of problems presented to them (Leung, 2006). More generally, GeoGebra, with its CAS, IGS, and graphing technology capabilities possesses three essential features: efficiency in mathematics manipulation and communication, multiple representation of mathematics, and interactivity between the learner and the mathematics (Leung, 2006; Anderson & Haciomeroglu, 2013). These features are essential because GeoGebra can bridge the divide that sometimes exists in students' heads between Algebra, Geometry, and numerical representations (Salleh & Sulaiman, 2013).

Little (2009) cited three main barriers to including IGS, including GeoGebra, into a mathematics classroom: teachers' attitudes and beliefs need to shift in a manner that would allow for the inclusion of the IGS, accessibility of computers, and the programs need to be easy to learn. This last barrier mentioned was the biggest worry of the lecturers utilized in Salleh and Sulaiman's (2013) study. The lecturers feared the amount of time and effort they would have to put in to adequately learn a new teaching resource. Hence, these barriers then lead to the necessity of more time for teachers to develop lesson plans that they could include IGS and professional development to instruct teachers on the proper ways to use and not use IGS (Little, 2009). Little also added that the fact that GeoGebra is free would remove the financial barrier for its potential implementation. Salleh and Sulaiman's study did conclude that conceptual understanding of mathematics was improved when lecturers utilized GeoGebra.

In Jonassen, Peck, and Wilson's (1998) work they summarized the benefits of an IGS, which GeoGebra is, as working well with problem or project based learning, information sources for solving problems, cognitive construction tools, and learning with collaboration and social or contextual support. Sozcu, Ziatdinov, and Ipek (2013) continued this list of benefits of GeoGebra to include a simple graphical interface and the possibility of using a shape parameter that would allow the user to modify the shape of a curve using a slider. Sozcu et al. also recorded a couple disadvantages of the software as well which included program bugs and the impossibility of dealing with parametric surfaces.

Hasek (2012) suggested that GeoGebra's most important ability was that it could be used as a tool of investigation for a given problem. This could allow a student access to real-world phenomena that they previously may not have been exposed to and thus may be motivating to the student (Hasek, 2012). Moreover, Hasek stated that "the process of solving such problems in the classroom introduces mathematics as a living and useful science, the application of which crosses boundaries between it and, what at first glance seem distinct disciplines" (p. 228). A specific example and manner that GeoGebra could be employed is by placing a photo on the background of the GeoGebra geometric desktop and then have students analyze that photo mathematically (Hasek, 2012).

Mackrell (2012) identified some potential disadvantages of IGS and particularly GeoGebra. Mackrell noted that if GeoGebra were used to introduce symbolic algebra the fact that the software utilizes letters to label geometric objects would be problematic. Moreover, the symbols for multiplying "*", dividing "/", and exponentiation "^" will require additional training time as these differ from what students typically employ,

especially at the lower grade levels (Mackrell, 2012). Mackrell also noted that dynamic numbers must be represented symbolically and this may cause confusion for students who struggle with symbolic algebra. At the end of the investigation in the potential advantages and disadvantages of GeoGebra, Mackrell calls for a more user-friendly version of GeoGebra, especially considering its potential use in elementary and middle schools.

Ponce-Campuzano (2013) stated that computer software "can be used to help students conceptualize, and construct for themselves, mathematics that has already been formulated by others" (p. 998). Ponce-Campuzano also cited research that implied that students who utilize technology to learn mathematics achieve higher scores than students who do not utilize technology. Additionally, Ponce-Campuzano discussed research that eluded to the idea that students have trouble relating algebraic equations with graphical representations. GeoGebra can help this issue of relating algebraic equations with graphical representations with sliders where students can manipulate the value of a variable and see the subsequent change on the graph (Ponce-Campuzano, 2013). Lastly, Ponce-Campuzano highlighted four attributes that technology possesses that further act as advantages to the student and include the following: technology can reduce the amount of time devoted to boring and repetitive drill, it can increase the amount of time devoted to real-life and thus more interesting problems, it can supply quick feedback to students, and it can offer multiple representations.

The previously discussed studies indicate that graphing calculators, graphing technology, and, more specifically, GeoGebra help students by possessing the following characteristics: a larger viewing screen or computer screen that allows students to view their entire problem solving sequence, reduce the computational demand on students,

provide for the visualization of mathematical concepts, and increase the student's ability to graphically represent an algebraic equation. These characteristics possessed by graphing technology and GeoGebra resulted in an increase in student motivation, an increase in student confidence with graphs, and a decrease in students' dependence on their teacher. The few suggested shortcomings of graphing technology and GeoGebra, include having access to computers, program bugs, and requiring additional instruction into how to utilize the mathematical technology.

Teachers' Beliefs about and Attitudes towards the Use of Technology

While graphing technology is supported by research to be included into the mathematics classroom, research about the beliefs and attitudes of mathematics educators with regard to the inclusion of mathematical technology in their classrooms needs to be reviewed. More specifically, research needs to be reviewed with regard to if mathematics educators possess positive or negative feelings towards mathematical technology, why they possess those feelings, and, most importantly, whether those feelings affect their choice to include or not include mathematical technology in their classrooms.

Simmit (1997) observed six teachers who each taught between 4-10 class periods that covered quadratic equations and found that the teachers used graphing calculators in the manner that agreed with their mathematical philosophies previous to the study being conducted. More specifically, one teacher who believed that students needed a strong foundation in computational ability did not allow students to do any computation work on the graphing calculators. With regard to discovery learning, one teacher had students explore how different parameters affected the graphs of quadratic equations while

another teacher just told the students what the parameter changed on the quadratic graphs (Simmit, 1997). This sentiment was echoed by Goos (2005) who found that between the two teachers that she studied in her case study one used graphing calculators as a tool to introduce and explore transformations of absolute value functions while another only allowed students to utilize graphing calculators after they had discussed the topic and worked through an entire worksheet by hand. Simmt suggested that teachers look at their own beliefs about mathematics and mathematics education before attempting to employ the use of graphing calculators in their rooms and to research the best practices with graphing calculators. Goos suggested professional development for more experienced teachers who may be new to the use of graphing calculators in their classrooms.

Dewey et al. (2009) set out to see what teachers' attitudes were towards graphing calculators and if teachers implemented them in their classrooms. What was established was that while 78% of teachers surveyed stated that they had access to graphing calculators, only 28% used graphing calculators on a regular basis (Dewey et al., 2009). Dewey et al. found that teachers who had a classroom set were more than two times as likely to use graphing calculators in their classroom as opposed to teachers who had access to a department set but did not have the set in their classroom. This would seem to indicate that part of the reason that teachers may not use graphing calculators could be due to how easily graphing calculators are accessible to the teachers. It was also found that Algebra II teachers were four times as likely to use graphing calculators in their classrooms compared to Algebra I teachers (Dewey et al., 2009). The reason identified by most teachers was that Algebra I should focus more on solving Algebra I questions symbolically as opposed to graphically and that teachers believed that an introduction of

the graphing calculators too early would cause students to become dependent upon them. Furthermore, teachers believed that the graphing calculator's main purpose was to supplement the curriculum and not to drive the expansion of it. It was also revealed that older teachers and teachers that are more experienced were more likely to utilize graphing calculators in their classrooms (Dewey et al., 2009). Dewey et al. concluded that teachers were open to the idea of integrating graphing calculators into their instruction but were unsure of how exactly to go about this. This indicates a need for professional development on the specific instances that graphing calculators should be used in the mathematics classrooms.

Simonsen and Thomas (1997) examined why teachers use or do not use graphing calculators as well as suggestions from those teachers of how to alter either situation. The research found that 33% of teachers use graphing calculators at least once a week and the other 67% used graphing calculators once a month or less often (Simonsen & Thomas, 1997). The main reason teachers stated they did not use graphing calculators was due to a lack of access to them. The second main obstacle listed by teachers was that there was not enough time in the school year to include training of students on how to use graphing calculators as well as teach the required mathematical concepts (Simonsen & Thomas, 1997). About 37% of teachers also feared students would become calculator-dependent. Consequently, the majority of this group used calculators the least amount in their classrooms (Simonsen & Thomas, 1997). To improve teachers' ability to utilize graphing calculators, the teachers studied suggested more professional development centered on the best practices and specific lessons that graphing calculators could be used with (Simonsen & Thomas, 1997).

Lee and McDougall (2010) noted that NCTM has technology listed as one of its six principles and thus highly suggests technology inclusion in the mathematics classroom. In their study, Lee and McDougall conducted an observation of three teachers as they utilized graphing calculators, and from that study and their own review of the literature, they made the following conclusions. Some teachers are hesitant to incorporate graphing calculators because they feel a loss of control over their teaching practices when they do. Additionally, there has been shown to be a direct correlation between how teachers themselves were taught mathematics and the way those teachers then teach mathematics (Lee & McDougall, 2010). Moreover, Lee and McDougall also pointed out research that stated that teachers would be more willing to include graphing calculators into their curriculum if they were given access to supplemental teaching materials that included graphing calculators and professional development time.

Lastly, Lee and McDougall (2010) stated that there were two factors that determined whether a teacher used the graphing calculators mainly for mechanical operations or mainly for an exploration of the mathematical concepts. The first factor was how accessible the graphing calculators were to the teachers and the teachers' comfort level with the graphing calculators. The second factor was the mathematical topic that was being covered during the lesson in which the graphing calculators were being utilized. At the end of their study, Lee and McDougall concluded that teachers utilized graphing calculators for "topics where they strongly believed the use of graphing calculators would support and expand student understanding" (p. 868). Thus, the places that teachers choose to integrate graphing calculators needs to be meaningful to teachers (Lee & McDougall, 2010).

Wachira and Keengwe (2011) referenced a survey from the National Center for Education Statistics from 2005 that indicated that only 44% of mathematics teachers use technology for classroom instruction with the majority of those teachers utilizing computer applications and graphical representations of algebraic concepts. According to Little (2009), this statistic is accurate as he referenced two studies in his study, one from 2000 and the other from 2003, conducted by the Fischer Trust in 373 secondary departments concerning their usage of IGS. Little reported that the studies found low to moderate use of IGS and that based upon his work with schools and universities in the state where he resides, few teachers utilize IGS. Wachira and Keengwe's own survey of 20 mathematics educators in urban school districts reflected a slightly different picture. They found that while 61% of teachers felt they lacked the ability to incorporate technology effectively in their classrooms, 92% were interested in the idea of doing so and felt that, with training, they could do so (Wachira & Keengwe, 2011). Moreover, 77% stated that they realized that technology offered cognitive advantages that could aid students understanding of mathematics, and only 38% felt that technology inclusion would result in a decline of basic fact retention (Wachira & Keengwe, 2011). Wachira and Keengwe's conclusion was that more professional development is needed, more time is needed for educators to create lesson plans that include technology, and all this requires support from administrators.

The research indicates that teachers are resistant, or at the very least hesitant, to the idea of the introduction of graphing calculators and, more generally, technology into their classrooms even though there is evidence that graphing calculators and graphing technology have been effective in raising students conceptual understanding of mathematics and scores on unit tests. From the literature reviewed, the main causes of

teachers’ resistance to the implementation of graphing technology stem from fears of calculator-dependence, a lack of accessibility to graphing calculators, and, seemingly, a lack of understanding of how best to implement them into their classrooms.

Summary

From the research reviewed in the chapter, it can be ascertained that graphing calculators and graphing technology do help students obtain a better conceptual understanding of mathematics and score higher on mathematics exams. It can also be hypothesized that the reason for this increase, other than aiding students in the actual graphing of algebraic equations and understanding of graphing, may be the larger screen for viewing multiple steps, the fact that graphing calculators give error prompts, and decrease the computation demand on the student. However, while the research indicated that graphing calculators and graphing technology have a positive impact on student understanding of mathematics, the research also showed that high school mathematics educators tend to resist the inclusion of technology in their classrooms. The more common reasons cited were a lack of financial support, lack of understanding in the use of graphing technology, and lack of time to properly incorporate a new form of technology.

CHAPTER III: METHODOLOGY

This chapter will supply a detailed account of how this study was conducted in order to answer the research questions and fulfill the purpose of the study. Specifically this chapter will discuss the specifics of each of the following: (a) the research design section will detail the steps that were taken in the order that they occurred; (b) the site of the study will be defined; (c) the participants will be described; (d) ethical considerations will be discussed; (e) data collection procedures will be detailed; (f) the manner in which data analysis transpired will be defined; and (g) a summary will be provided.

Research Design

Due to part of the purpose of this study being to integrate MP5 into an Algebra II unit of study, a mathematical tool to aid in this integration needed to be chosen. This study utilized GeoGebra, since it is a free mathematical software package that can be accessed via the internet. To collect data to determine what successes and challenges were experienced, data collection methods in the form of teacher-researcher self-observations, peer educator observations, interviews with students, and video recordings of lessons taught during the course of this study were utilized. Next, participants were the classroom teacher and the Algebra II students in the third class period of the day.

Following the selection of potential participants the writing of lessons for the unit of study, polynomials and polynomial functions, which incorporated some faucet of GeoGebra, took place. After the lesson plans were written, it was then decided when to incorporate each instrument of measurement, observational protocol (Appendix B) and interview protocol (Appendix C), in the unit of study. It was decided that the

observational protocol would be utilized by the teacher-researcher and peer educators during and after a lesson was observed. The interview protocol would be utilized by the teacher-researcher when conducting student interviews. Next, consent for the study was obtained from the Institutional Review Board (IRB-FY2017-306) (Appendix A), building principal (Appendix D) of the school in which the study took place, and from the parents/guardians (Appendix E) of the students in the third class period. Of the 24 potential student participants 20 gave consent to participate in the study.

Once consent had been obtained, the study commenced. During the course of the study, daily lesson plans were altered as necessary by conducting some daily analysis of collected data. This daily analysis resulted in changing the way in which GeoGebra was incorporated into subsequent lessons. While the 12 lesson plans prepared for this study were written with the intention of students being the ones to utilize GeoGebra, after the first two lessons were taught, it was determined that the teacher modeling to the class as a whole would be more effective. After the conclusion of data collection, the collected data was analyzed and the important data that was identified was confirmed through data triangulation. Data was triangulated by comparing data from the following sources: teacher-researcher self-observations, peer educator observations, student interviews, and lesson plans developed for use in this study.

Site of the Study

The study took place in one high school in a suburban school district in southwest Missouri. According to Missouri Department of Elementary and Secondary Education, the district had one high school, one middle school, one intermediate school, and five elementary schools with approximately 4,560 students enrolled in the district

while the high school had 1,315 students enrolled in 2016 (2016b). In 2016, 91% of high school students were classified as White, 3.7% were classified as Black, 3.3% were classified as Hispanic American, 0.4% were classified as Asian American, and 0.8% were classified as Native American. Also, 35.2% of students received free or reduced lunch (Missouri Department of Elementary & Secondary Education, 2016b). The city had a population in 2015 of 5,454 residents (United States Census Bureau, United States Department of Commerce, 2015). The city had a per-capita income of $34,080 and had an unemployment rate of 5.4% (MERIC: Missouri Economic Research and Information Center, Missouri Department of Economic Development, 2015).

The school district was technologically one-to-one with every student from 5th grade through 12th grade having either a personal laptop or a school-issued laptop they could use at school and at home for the entirety of the school year. It should be noted that some students were not permitted to take their school-issued laptop off school grounds due to prior use in violation of the school technology agreement, having not paid the rental fee, or from not having signed the school technology agreement. Because the majority of the school district was technologically one-to-one, the school district had an emphasis on teaching students to be responsible, productive, and effective with the use of technology.

Participants

Due to an action research design, I was a participant in the study. Since I was a participant in this study, my teaching style and beliefs are necessary to discuss. Prior to the study, my teaching style was primarily direct instruction and thus, centered on a lecture-based format. Furthermore, my approach to teaching a new mathematical topic

was more focused on transmitting procedural knowledge and less focused on transmitting conceptual knowledge. Lastly, while I did believe that technology was beneficial to mathematics education, I utilized graphing technology seldom and almost never utilized a CAS or mathematical software in my classroom. Thus, my instructional knowledge of how to operate graphing technology, CAS, mathematical software, and, specifically, GeoGebra was basic.

Furthermore, since my study took place in my classroom, the students in my third period Algebra II class were also participants. Before the school year in which this study took place, students were assigned to my third period Algebra II class due to scheduling convenience with respect to each of their individual schedules. Hence, my third period Algebra II class was a convenience sampling (Gay et al., 2011).

Of the 24 potential student participants 20 gave consent to participate in the study. These student participants were Algebra II students from a high school in a public school district located in southwestern Missouri. Students were composed of 14 females and six males and with 18 students being White, one student being Black, and one student being Hispanic. Students were in the following grade levels: two sophomores, 16 juniors, and two seniors. The academic grade of participants at the end of the study were evenly distributed between A's to D's with five students earning an A, six students earning a B, five students earning a C, and four students earning a D. In the state of Missouri, students are required to take an End-of-Course (EOC) exam at the conclusion of Algebra I, and their subsequent score on the EOC exam is utilized to classify them into one of four performance levels listed in descending order: advanced, proficient, basic, and below basic. Eighteen students who participated in the study had taken the EOC exam two years prior to their participation in this study, and the EOC exam

indicated that two students had earned an advanced performance level, 13 students had earned a proficient performance level, and three students had earned a basic performance level. Two students did not have an EOC score for Algebra I since they had moved to Missouri after they had completed their Algebra I course in another state.

Ethical Considerations

Overall, I did not anticipate any risk of harm to any students throughout the study since the study involved normal classroom routines and employed a mathematical software package designed for educational use. Despite this anticipation, measures were still put into place to ensure participants were protected. Since the study involved minors, informed consent from the students' parents or guardians (Appendix E) was requested. Of the 24 potential participants, 20 agreed to participate in the study while four did not agree to participate, and thus data was not collected from those four nonparticipants. As an added level of protection, informed consent was also requested from the principal of the school utilized in the study (Appendix D). Moreover, to ensure that all students received the same level of quality education and that their participation or non-participation in the study did not adversely affect their educational experience in the classroom, the following guidelines were employed. Lessons utilized during the course of the study were research-based and deemed, utilizing my professional opinion, to be best practices. Moreover, these lessons were presented in the same manner to all students and not altered in any way for participants or for non-participants. Lastly, in data presentation and publication the study employed the use of pseudonyms to keep the identity of all participants confidential.

Data Collection Procedures

In this study, data were collected in the following ways: teacher-researcher self-observations, peer educator observations, student interviews, and video recordings of the lessons taught. An observational protocol (Appendix B) was utilized by myself for the teacher-researcher self-observations and by peer educators for the peer educator observations. Portions of the observational protocol were developed in 1998 by the University of Wisconsin, in 2003-2006 by the University of Missouri, and in 2015 by the STEAM project (Tarr & Austin, 2015). Furthermore, modifications were made to the observational protocol to address the purpose of this study. The observational protocol was divided into two portions. The first portion, which was to be completed during observation, was for recording and commenting on seven categories. Specifically, comments were requested about students' level of interest, students' level of engagement, students' ability to work in groups while utilizing GeoGebra, students' ability to correctly or incorrectly learn a concept utilizing GeoGebra, the correct and incorrect manners in which students employed GeoGebra, students' ability to translate mathematical concepts between two or more tools, and evidence to either suggest or refute students' ability to choose the appropriate tool for a mathematical task and/or strategically utilize that tool.

The second portion, which was to be completed post-observation, began with having the observer estimate, using his or her professional judgment, the level of interest of the students, the level of engagement of the students, and the level of collaboration of the students. Interest of students was determined by the level of excitement and interest that students exhibited, engagement was determined by the level of on task behavior that students exhibited, and collaboration was determined by the level of work students were

doing by themselves or with others in their groups. Generally, each of these categories could be specified as relatively low, moderate, or relatively high. Specifically, for the level of interest of the students' category, the observer could choose one of the following three choices: relatively few students appeared interested, about one-half of the students appeared interested, or relatively all of the students appeared interested. Specifically, for the level of engagement of the students' category, the observer could choose one of the following three choices: relatively few students appeared to be on task, about one-half of the students appeared to be on task, or relatively all of the students appeared to be on task. Specifically, for the level of collaboration of the students' category, the observer could choose one of the following three choices: most students worked individually, some students worked collaboratively while others worked individually, or most students worked collaboratively. Next, the post-observation form asked the observer to describe the main activities of the class that were observed, how affective those activities were, and why those activities were affective. The post-observation form finished with a request for the observer to provide any suggestions of what could be altered.

I conducted teacher-researcher self-observations of each lesson taught during the course of this study where the observations began by taking quick notes during each lesson utilizing an observational protocol (Appendix B) and then by expanding on those notes when watching the video recording of each lesson taught. I used a video recording device every day during class to capture conversations between students and between students and me that served as evidence of students *using appropriate tools strategically* (MP5) while utilizing GeoGebra. In total, I conducted 12 teacher-researcher self-observations, one for each of the 12 lessons taught during the course of this study.

I was observed five times during five separate lessons taught during the course of this study by the following peer educators: once by a peer mathematics instructor, twice by a vice principal, and twice by an instructional coach. The peer mathematics instructor observed section 5.2B, end behaviors of polynomial functions. The instructional coach observed section 5.5A, polynomial division, and section 5.7C, behavior near zeros. The vice principal observed section 5.7C, behavior near zeros, and section 5.9, write cubic functions. The mathematics instructor was certified to teach 9-12 mathematics in the state of Missouri, had obtained an undergraduate degree in mathematics education at the secondary level, and was working on obtaining a graduate degree in mathematics education at the secondary level. The vice principal was certified to teach K-8 mathematics in the state of Missouri and had obtained an undergraduate and graduate degree in mathematics education at the middle school level. The instructional coach had earned a graduate degree in instructional practices. I informed each peer educator observer ahead of time of the purpose of my study and asked him or her to utilize an observational protocol (Appendix B) to guide his or her focus. The information that an observer was asked to detect and comment on was dependent upon that observer's area of certification. The peer mathematics instructor and vice principal were asked to observe and comment on all previously listed topics in the observational protocol (Appendix B) while the instructional coach was asked to observe and comment on the first three topics of portion one of the observational protocol. Peer educator observations occurred during lessons that were convenient with regard to the peer educators' schedule and were not chosen for any other reason.

I conducted three student interviews with three different students; each student interview was for a separate lesson, which served as evidence of students *using*

appropriate tools strategically (MP5) while utilizing GeoGebra. Student S4 was interviewed after section 5.4A (Appendix F), factoring and solving polynomial equations, student S5 was interviewed after section 5.2B, end behaviors of polynomial functions, and student S6 was interviewed after section 5.7C, behavior near zeros. Each student was chosen using my professional judgement that the student was engaged in the lesson that was taught and would participate in the interview process. The number of students that were interviewed, three, were not chosen for any particular reason. I used an interview protocol (Appendix C) when interviewing students to aid in recording student responses. The interview protocol was the second portion, otherwise known as the post-observation portion, of the observational protocol. The second portion of the observational protocol was chosen for comparison purposes between student responses, peer educator responses, and teacher-researcher responses.

While the course textbook, McDougall Littell's Algebra II textbook, was utilized for section numbers and the titling of each section, to provide definitions, and for example expressions and equations all aspects of the lesson plans that incorporated GeoGebra were produced utilizing other resources (Larson, Boswell, Kanold, & Stiff, 2008). Of the 12 lessons produced for the study, three incorporated applets found on GeoGebra's website, one utilized part of a lesson plan produced by NYS Common Core Curriculum, and one utilized part of a lesson plan found on a mathematics instructor's website. All other incorporations of GeoGebra for this study were created by the teacher-researcher.

It should be noted that while GeoGebra was not utilized in every facet of every lesson that was written it did serve as the impetus to include other mathematical tools. It should also be noted that any teaching approach or mathematical tool mentioned in the

forthcoming paragraph were not ideas utilized from the course textbook. In section 5.3A, adding and subtracting polynomials, Algebra Tiles were initially utilized to help students understand the underlying concept of how to add or subtract two or more polynomials. In section 5.3B, multiplying polynomials, the area model and tabular method were utilized to help students discover the distributive property and how it can be employed for multiplying two or more polynomials of two or more terms. The tabular, or table, method is the utilization of a visual organizer, a table, for multiplying two polynomials (Larson, Boswell, Kanold, & Stiff, 2007). Moreover, in section 5.3A and section 5.3B GeoGebra's CAS application was utilized to simplify expressions.

In section 5.5A, polynomial division, two separate GeoGebra applets were used for integer division and polynomial long division, to help students discover the connection between integer long division and polynomial long division, GeoGebra's CAS application was used for factoring, and the graphing calculator application was used for graphing. In section 5.4A and 5.4B, factoring and solving polynomial equations, GeoGebra was utilized to help students discover the sum and difference of two cubes formulas and to help students discover the connection between the factoring method known as the AC method and the factoring method known as factoring by grouping. More precisely, in section 5.4A and 5.4B GeoGebra's CAS application was used to factor polynomials as a means to help students gain a deeper understanding of factoring, to speed up computation, and to check student's work. In section 5.5B, apply the factor theorem, GeoGebra's spreadsheet application was used for synthetic division, the CAS application was used to speed up computation and to check students' answers, and the graphing calculator application was used to graph polynomials. In section 5.7A, apply the fundamental theorem of Algebra, GeoGebra's graphing calculator application

was utilized to help students discover the relationship between a polynomial's degree and the number of solutions that the polynomial possesses. In section 5.7B, apply the fundamental theorem of algebra, Wolfram Alpha was utilized to help students discover the irrational conjugates theorem and the imaginary conjugates theorem.

In section 5.2B, end behaviors of polynomial functions, GeoGebra's graphing calculator application was utilized to help students discover the relationship between a polynomial's degree being even or odd and the polynomial being positive or negative, and the end behavior of the polynomial. In section 5.7C, behavior near zeros, GeoGebra's graphing calculator application was utilized to help students discover the relationship between the multiplicity of a zero of a polynomial and the graph's behavior near that zero. In section 5.8, analyze graphs of polynomial functions, GeoGebra's graphing calculator application was utilized to help students discover the relationship between a polynomial's degree and the number of turning points the graph of that polynomial may possess. In section 5.9, write cubic functions, GeoGebra's graphing calculator application was employed to check students' work.

Data Analysis

To analyze data in a qualitative study it is imperative that data have been collected in multiple manners so that results of the analysis can be viewed as an evidence-based conclusion. In this study, the multiple manners of data collection included teacher-researcher self-observations, peer educator observations, video recordings, and student interviews. During data analysis I specifically looked for evidence of successes and challenges related to utilizing GeoGebra as way to integrate MP5. Next, I grouped information into common findings to gauge whether enough data

existed to support a particular idea. Moreover, for an analysis to be considered an accurate depiction of what transpired during the course of the study, the different methods with which data collection took place must agree with each other and this emerges through a process known as data triangulation. In this study, once major findings were identified, I compared and contrasted how that information was viewed from the different data collection methods utilizing data triangulation.

Summary

The purpose of this study was to identify the successes and challenges a teacher experienced when GeoGebra was incorporated into an Algebra II unit of study with the goal of integrating the fifth mathematical practice of the CCSS, *use appropriate tools strategically*. This study utilized GeoGebra, as it is a free mathematical software package that can be accessed via the internet and due to the school that participated in the study being technologically one-to-one. After consent had been obtained, this study was carried out with 20 high school students from a third hour Algebra II class in a suburban city in the southwestern portion of Missouri. The development of lessons for the unit of study, polynomials and polynomial functions, which incorporated some faucet of GeoGebra, spanned 12 lessons. While the course textbook, McDougall Littell's Algebra II textbook, was utilized for section numbers and the titling of each section, to provide definitions, and for example expressions and equations all aspects of the lesson plans that incorporated GeoGebra were produced utilizing other resources (Larson et al., 2008). Of the 12 lessons produced for the study, three incorporated applets found on GeoGebra's website, one utilized part of a lesson plan produced by NYS Common Core Curriculum, and one utilized part of a lesson plan found on a

mathematics instructor's website. All other incorporations of GeoGebra for this study were created by the teacher-researcher. To collect data to determine what successes and challenges were experienced, data collection in the form of teacher-researcher self-observations, peer educator observations, interviews with students, and video recordings of lessons taught during the course of this study were utilized. Furthermore, an observational protocol (Appendix B), utilized by the teacher-researcher and peer educators, and interview protocol (Appendix C), utilized by the teacher-researcher, were employed as instruments. The collected data was analyzed and the important data that was identified was confirmed through data triangulation.

CHAPTER IV: FINDINGS

After concluding data collection, the data that was collected was examined in search of findings that could be supported through data triangulation. In this chapter, the findings that were supported through data triangulation were presented. The presentation of the data analysis was organized with regard to the two research questions that guided this study and identified five major findings. Research question 1, which asked for the successes I experienced utilizing GeoGebra to integrate MP5 into an Algebra II unit of study, centered around two findings. The first finding was that my instruction targeted a deeper level of mathematical understanding by students. This targeting of a deeper level of understanding also resulted in the utilization of discovery learning. Specifically, my instruction shifted from having never utilized discovery learning towards the incorporation of discovery learning with a goal of helping students see the connection between algebra and its graphical representation. The second finding was with students' having a moderately high level of interest.

Research question 2, which asked for the challenges I experienced utilizing GeoGebra to integrate MP5 into an Algebra II unit of study, centered around three findings. The first finding was challenges with technology, specifically, computer access, internet speed, internet access, and a GeoGebra problem. The second finding was challenges with students, specifically, students being unprepared for class, the time required for students to prepare for a lesson, and the need to monitor student computer usage. The third finding was challenges with teachers, specifically, with other Algebra II teachers and within myself.

Research Question 1

The successes I experienced utilizing GeoGebra to integrate MP5 into an Algebra II unit of study centered around two findings. The first finding was that my instruction targeted a deeper level of mathematical understanding by students. This targeting of a deeper level of understanding also resulted in the utilization of discovery learning. Specifically, my instruction shifted from having never utilized discovery learning towards the incorporation of discovery learning with a goal of helping students see the connection between algebra and its graphical representation. The second finding was with students having a moderately high level of interest.

Deeper Level of Understanding. The first finding related to research question one and success was that my instruction targeted a deeper level of mathematical understanding by students. This finding was obtained from the data collected through data triangulation of lesson plans developed for use in this study, teacher-researcher self-observations, and peer educator observations. Attempting to integrate MP5 through the incorporation of GeoGebra caused me to question the methods I would utilize to include GeoGebra. This inquisitiveness along with the added ability to investigate mathematics that GeoGebra allowed, caused me to question my largely procedural approach to mathematics. Thus, I began looking deeply into the procedures and concepts that mathematics employs and started focusing my lesson plans and teaching around a deeper level of mathematical understanding. Because of this targeting of a deeper level of mathematical understanding, I began utilizing discovery learning. Specifically, my instruction shifted from having never utilized discovery learning towards the incorporation of discovery learning with a goal of helping students see the connection between algebra and its graphical representation.

In the section 5.3A lesson plan, adding and subtracting polynomials, I began the lesson by having students utilize the Frayer model with the term monomial. In the Frayer model students must define the term provided to them, and they must list the term's characteristics, provide examples, and provide non-examples of the term. I used the discussion, along with a subsequent discussion about the term binomial, to introduce students to the notion of adding and subtracting polynomials. Then, to make sure students understood the concept that I was striving for, the following conversation took place, as recorded by a teacher-researcher self-observation:

Mr. Matthews: "I had GeoGebra simplify $(x^2 + 2x + 3) + (3x + 1)$. I want you to explain to me what GeoGebra did to arrive at the answer that it did. Take two minutes and talk it over with your groups and then we will discuss it as a class."

Mr. Matthews: "Time is up, who would like to answer?"

S6: "It combined like terms."

Mr. Matthews: "Class, to challenge you, I want you to explain to me what combine like terms means. You have two minutes; talk it over with your groups."

Mr. Matthews: "Time is up, what does combine like terms mean?"

S12: "To add things that are the same."

Mr. Matthews: "What do you mean?"

S12: "Add the same variables."

Mr. Matthews: "So do this." I wrote on the board: $2x^2 + 2x = 4x^2$. "Is that correct?"

S17: "You need the same variable and exponent to be added together."

Mr. Matthews: "Ok, so walk me through this problem and tell me what I add together."

After finishing that example, we went on to the next example, which dealt with subtracting two polynomials, and a similar discussion took place, as recorded by a teacher-researcher self-observation.

Mr. Matthews: "I had GeoGebra simplify $(3x^2 - 5x + 3) - (2x^2 - x - 4)$. I want you to explain to me what GeoGebra did to arrive at the answer that it did. Take two minutes and talk it over with your groups and then we will discuss it as a class."

S16: "GeoGebra used the distributive property and then combined like terms."

Mr. Matthews: "Class, I want you to describe to me what GeoGebra did without using the terms "distributive property" or "combine like terms". Take two minutes and talk it over with your groups and then we will discuss it as a class."

The two parts of this conversation serve as an example of what my teaching approach was becoming. I wanted students to take a deep look at the procedures they were employing and think about why a particular procedure worked.

Another example of this focus on a deep level of mathematical understanding comes from the section 5.4B lesson plan, factor and solve polynomial equations, where the goal was to have students understand that we can extend the zero product property beyond factored quadratics. I began this conversation by asking students how to factor and solve a quadratic equation, as recorded by a teacher-researcher self-observation.

Mr. Matthews: "Factor the following, $x^2 + 7x + 10 = 0$, utilizing GeoGebra."

Mr. Matthews: "What did GeoGebra use?"

S12: “Factoring.”

Mr. Matthews: “What manner of factoring?”

S12: “AC method.”

Mr. Matthews: “Now have GeoGebra solve it. What did GeoGebra do?”

S12: “Zero property.”

Mr. Matthews: “Zero product property. Can we extend this property to larger polynomials?”

S4: “Yes.”

Mr. Matthews: “Why?”

S9: “Don’t know why.”

Mr. Matthews: “Ok, what does the zero product property state?”

S6: “(x + 2)(x + 5) = 0 then x + 2 =0 or x + 5 = 0.”

Mr. Matthews: “More generally, if a * b = 0, then a = 0 or b = 0. What if a * b * c = 0?”

S17: “a = 0 or b = 0 or c = 0.”

Mr. Matthews: “Can we extend the zero product property further than three factors?”

S17: “Yes.”

This conversation further depicts my desire for students to not just memorize a procedure but to know how it works. Moreover, this conversation demonstrates a more pronounced dedication, as the teacher, to a deeper level of mathematical understanding.

In the section 5.7C lesson plan, behavior near zeros, the lesson began with a coordinate plane and a graph, positioned below the x-axis, approaching the x-axis, drawn on the whiteboard in the front of the classroom. Next, I presented a question for

the students to contemplate. As the instructional coach noted on his/her peer educator observation (Appendix G) stated, "After drawing the xy-coordinate plane the students were asked by the teacher what could occur at the intersection point of the graph and the x-axis?" The goal of the question was for students to consider the ways in which a graph can intersect the x-axis and realize that generally only two scenarios can occur. From that point forward students would be given the opportunity to formulate a theory as to when a graph crosses the x-axis and when it is tangent to the x-axis. This, once more, was an strong focus on the procedure and the targeting of a deeper level of mathematical understanding into why the procedure operates in the manner that it does.

This focus on a deep level of mathematical understanding is further evident in the section 5.3B lesson plan, multiplying polynomials. Section 5.3B was introduced to students by reminding them of the area model and then exploring how that model has extensions to the tabular method of multiplying polynomials. The following conversation transpired, as recorded by a teacher-researcher self-observation (Appendix H):

Mr. Matthews: "How can we multiply two binomials without using the tabular method?"

S1: "Distributive property."

Mr. Matthews: "How do we use that property to do this?"

S2: "We multiply the first term of the first binomial times the first term of the second binomial and then times the second term of the second binomial. Then we multiply the second term of the first binomial times the first term of the second binomial and then times the second term of the second binomial."

Mr. Matthews: “Do these two methods, distributive property and tabular method, agree?”

S4: “Yes, they give the same answer.”

In this example, I was trying to elicit a deeper, more conceptual response to my initial question. While “distributive property” is the correct term, I was trying to determine whether student S1, and the class as a whole, truly understood what the term “distributive property” represents. In the same section, towards the end of the class period, another prime example occurs when I ask students to consider a binomial multiplied by a trinomial, as recorded by a teacher-researcher self-observation.

Mr. Matthews: “Class, give a general description of how to multiply a binomial times a trinomial. You have two minutes to discuss this with your groups and then we will discuss it as a class.”

Mr. Matthews: “What property did we use?”

S4: “Distributive property.”

Mr. Matthews: “Can anyone explain how to do this?”

S17: “Split binomial and multiply it by the trinomial.”

Mr. Matthews: “Class, what is meant by, split the binomial?”

S1: “Take the first term of the binomial and multiply it by all three terms of the trinomial and then take the second term of the binomial and multiply it by all three terms of the trinomial.”

Mr. Matthews: “Could we reverse the order described and get the same answer?”

Once more, I was looking for a deeper level of understanding with my initial question and follow-up questions. I wanted to determine if students truly understood the procedure that was being employed in this scenario.

The existence of a focused effort towards a deeper level of mathematical understanding by students within my teaching is further supported by another example in the section 5.4B lesson plan, factor and solve polynomial equations, in example 2. In this example, I ask students to have GeoGebra factor and solve the following polynomial: $3x^5 + 15x = 18x^3$. I then asked students to fill in the missing steps between the initial polynomial and its subsequent factored form all while explaining what steps they were completing. Next, I asked students to fill in the missing steps between the polynomial's factored form and the polynomial's solutions all while explaining what steps they are completing. This is where we will pick up the conversation, as recorded by a teacher-researcher self-observation:

Mr. Matthews: "What did GeoGebra do first?"

S1: "Move $18x^3$ to the left side of the equation."

Mr. Matthews: "Why do we do this?"

S1: "Because we are supposed to."

Mr. Matthews: "Why are we supposed to?"

S1: "For solving it purposes."

Mr. Matthews: "Why?"

S1: "I am tired of all the critical thinking questions."

S13: "So we can use the zero product property."

Mr. Matthews: "What next?"

S19: "Factor out a common monomial of 3x."

Mr. Matthews: "Now what?"

S1: "AC method."

Mr. Matthews: "The AC method is only for quadratics, why can we use it here?"

S1: “It is in quadratic form.”

Mr. Matthews: “How do we know it is in quadratic form?”

This dialogue from the lesson that transpired illustrates once more a focus on a deep level of mathematical understanding. This was most pronounced by student S1’s comment about all the critical thinking questions. Moreover, students could not just explain the procedures that they employed, they also had to explain why those procedures should be used.

By focusing on integrating MP5 the first finding of success was a shift in the way that my lesson plans were written and the corresponding goal that the lesson plans strived to achieve. By targeting an incorporation of a mathematical tool, GeoGebra, it caused me, as the teacher, to consider how best to utilize GeoGebra in my instruction. This resulted in myself gaining a better understanding of the ability of mathematical tools to allow students the opportunity to discovery portions of mathematics and thus, I developed lesson plans that incorporated discovery learning. This finding was obtained from the data collected through data triangulation of lesson plans developed for use in this study, teacher-researcher self-observations, and student interviews.

In Appendix I, a lesson plan developed for use in this study has been provided as just one example of the lesson plans that depicts my transition towards incorporating discovery learning in my to teaching. A student also referenced the discovery learning approach during a student interview. The student, S4, was asked, after they brought up the sum and difference of cubes formulas, what they thought of the lesson design in section 5.4A (Appendix F). Student S4 stated, “It was neat because we found it ourselves.” When the student was then asked if they had any suggestions of how to improve the approach to discovering the sum and difference of cubes formulas, student

S4 stated, “Keep it the same because it was exciting to uncover new information.” These comments would support that student S4 likes discovering new, to student S4, mathematics and that the discovery learning approach was exciting to this student.

Within this transition to a discovery learning approach that I strived for, I had the goal of having students see the connection between the algebra and its graphical representation. This goal was evident within the lesson plans of sections 5.4A, 5.7A, 5.7B, 5.2B, 5.7C, and 5.8 as graphs were directly utilized to help students discover some algebraic aspect of Algebra II. The goal of having students see the connection was also evident in the lesson plan for section 5.5A directly after example 4 where students explore the relationship between x-intercepts, solutions, and factors of a quadratic function. In a similar circumstance, this connection was also evident in the lesson plan for section 5.5B after example 1 where students were again exploring the connection between x-intercepts, solutions, and factors of a quadratic function. The connection that students explored after example 1 was then emphasized again in example 2 and example 3 but this time with cubic equations. Lastly, the connection was emphasized in the lesson plan for section 5.9 where students had to utilize graphs of cubic polynomials to write cubic equations.

During a student interview over section 5.2B, student S5 was asked how effective viewing the graphs was for discovering the patterns for end behaviors of different polynomial functions. Student S5 stated, “Very effective as we could see the patterns and how they related.” When student S5 was then asked about practice problems within the same section, student S5 responded, “The graphs let us see the changes, and the patterns were somewhat easy to see.” These responses are evidence that student S5 liked seeing how the algebraic representation of a polynomial connected

to the graphical representation of the polynomial and that this made the formulating of the pattern much easier. This conclusion is further supported by another student interview in section 5.7C. Student S6 was asked about the effectiveness of utilizing the graphical representation of the algebraic equations in order to discover the behavior of a polynomial's graph near its zeros. Student S6 responded, "The graphs made it obvious what happens at even x-intercepts and at odd x-intercepts." In this specific response, the student was referencing the multiplicity of the x-intercept's corresponding factors being even or odd in value. This connection could also be seen in an exchange between students and me during section 5.7A. To begin this section, students were instructed to factor, solve, and graph five different polynomial functions, each with a different degree, utilizing GeoGebra. The following is the conversation that took place while factoring, solving, and graphing the first polynomial function of the section, a quadratic function, as recorded by a teacher-researcher self-observation:

Mr. Matthews: "Use GeoGebra to factor, solve, and graph each equation."

Mr. Matthews: "Notice if we factor $x^2 + 2x - 8 = 0$ how many factors do we have?"

S12: "Two."

Mr. Matthews: "How many solutions do we have?"

S12: "Two."

Mr. Matthews: "How many x-intercepts do we have if we graph it?"

S18: "Two."

Mr. Matthews: "Why are these three answers the same?"

S4: "They are related."

This conversation hints at the depiction of student S4 making the connection between the number of factors, the x-intercepts, and the solutions of a quadratic equation.

Student Interest. The second finding of research question one with regard to successes experienced, was with students having a moderately high level of interest. This finding was obtained from the data collected through data triangulation of teacher-researcher self-observations, peer educator observations, and student interviews. There were 12 teacher-researcher self-observations completed using the observational protocol (Appendix B), one for each lesson taught during the study, five peer educator observations completed using the observational protocol, and three student interviews completed using the interview protocol (Appendix C), the second portion of the observational protocol. Thus, a total of 20 protocols, specifically the second portion of the observational protocol, were completed. Fifteen of the 20 completed protocols, 75%, indicated that relatively all of the students appeared interested and that five of the 20 completed protocols, 25%, indicated that about one-half of the students appeared interested. Seventeen of the 20 completed protocols, 85%, indicated that relatively all of the students appeared to be on task and that three of the 20 completed protocols, 15%, indicated that about one-half of the students appeared to be on-task. These statistics imply that student interest and on-task behavior was at a moderately high level.

When student S4 was interviewed after the section 5.4A (Appendix F), factor and solve polynomial equations, the student stated that the teaching approach utilized for discovering the sum and difference of cubes formula was neat. This sentiment of students being interested was echoed by comments from my peer observers as well. The fellow mathematics instructor that observed section 5.2B, end behavior of polynomials, stated, “Two students were off-task but everyone else was either actively listening and

watching, or drawing". The vice-principal that observed section 5.7C, behavior near zeros, noted, "high interest" by the students and "all students seemed interested and engaged". Later in the vice-principal's observation he/she estimated that "95% of students were engaged and working." The instructional coach that observed section 5.7C, behavior near zeros, noted, "Lots of discussion about problems at tables while the instructor circled and answered questions." On the post-observation form the instructional coach noted, "The chance to work collaboratively allowed students a chance to discuss ideas, evaluate, and correct misconceptions before sharing." The observer information garnered from observational protocols and interview protocols is depicted in Appendix J.

Research Question 2

The challenges I experienced utilizing GeoGebra to integrate MP5 into an Algebra II unit of study centered on three findings. The first finding was challenges with technology, specifically, computer access, internet speed, internet access, and a GeoGebra problem. The second finding was challenges with students, specifically, students being unprepared for class, the time required for students to prepare for a lesson, and the need to monitor student computer usage. The third finding was challenges with teachers, specifically, within myself and with other teachers. At the outset of this study, I wanted students to utilize their personally owned computing devices or school-issued laptops to perform the explorations I had planned with GeoGebra. However, that goal was quickly made difficult to achieve.

Challenges with Technology. Due to the fact that GeoGebra is a mathematical software package and is accessed via the internet or can be downloaded onto a computer,

technological challenges can arise. This study was not immune to that potential challenge. I discovered my first technological challenge before data collection had officially begun. The school's internet was designed to block certain websites on students' laptops that were deemed potentially inappropriate for students to access. Knowing that this could be a challenge I had one of my students attempt to access GeoGebra on their laptop and I discovered that GeoGebra happened to be a blocked website. After discussing the challenge with the technology department, the challenge was able to be resolved. If the challenge had not been detected before the first lesson plan of the study was to be implemented none of my students would have been able to access GeoGebra on their laptops.

I discovered my second technological challenge also before data collection had officially begun. The day prior to the study beginning, I was scheduled to give my students an online quiz through my class website. However, when the class period in which the quiz was to take place begun, and I requested that students access the quiz through my website, some students were either unable to gain access to the internet or the speed with which the web pages were downloading was incredibly slow. While these challenges only affected six students the challenges did persist until other students began finishing their online quiz and exiting their internet browsers. Moreover, though the quiz was designed to be completed within 10-20 minutes the internet challenges necessitated the entire 47 minute class period for all students to be able to complete the online quiz.

After the previous two potential challenges to accessing GeoGebra, I thought it prudent to have students attempt to download GeoGebra to their laptops, and thus a week prior to the study beginning I requested that students download GeoGebra to their

laptops. Coincidently, during the first scheduled day for GeoGebra lesson plan implementation and data collection the school lost internet connection. While I and a small handful of students had previously downloaded GeoGebra, other students had not so they would not have been able to participate in the planned activities. Thus, for the students who had not downloaded GeoGebra, I altered my lesson plan at the beginning of the class period so that GeoGebra would be displayed via my projector. In utilizing my school computer, I was able to teach the entire class at one time rather than allowing all the students to explore GeoGebra individually, as was originally the intention that day.

To access my school computer, I must log in each morning. This log in is automatically reset for various reasons, and during what would have been the ninth day of GeoGebra lesson plan implementation, my log in was automatically reset. Due to this dilemma, I was not able to gain access to my computer and subsequently was not able to access GeoGebra to display it via my projector. While I did regain access to my school computer that day, it was after my third period Algebra II class, the class participating in this study, had occurred. Hence, due to this predicament I was forced to alter my lesson plan for that class period and to review previously taught material. The planned GeoGebra lesson plan was moved to the next scheduled class period.

Lastly, when presenting section 5.5A, polynomial division, I was attempting to graph $x^2 - 2x - 15$ but GeoGebra would not graph the quadratic equation. I attempted to input the expression multiple times but each attempt ended with the lack of a graph. After ensuring that I had not inserted an incorrect expression I decided to open another tab in my internet browser and access GeoGebra again. This yielded the same result of no graph for the given expression. Lastly, to fix the challenge I completely closed out of

my internet browser, reopened my internet browser, reopened GeoGebra, and reentered the expression. This resulted in the graph of the expression. While this technological challenge did not waste a lot of time it was not immediately obvious to me the fix that needed to occur in order obtain the graph of the given expression.

Challenges with Students. The second finding of research question two of challenges that I experienced was with challenges with the students. Due to the fact that my initial goal was for each student to utilize his or her own school-issued laptop, my ability to instruct in this manner hinged upon whether my students could fulfill what I asked and expected of them. The first challenge that arose was that some of the students did not carry their laptops with them to class. Some simply forgot it at home or in their lockers. While others intentionally did not bring their laptops, as either none of their other classes required them to use their school-issued laptops, the student would rather have a fellow student share with them, or the student had a general lack of interest in school and did not care. In the case that the student laptop was in their locker a simple solution to this challenge would have been for students to get the laptop from their lockers. However, this would have been a potential disruption and require time out of class. Also, though students not having their laptops was a challenge in the beginning of this study I merely had students share laptops and kept the planned GeoGebra lesson plan the same.

A second challenge was that some students either had completely depleted the battery life of their computer or had done so to a great extent prior to their Algebra II class. The battery life of the students' school-issued laptops varied greatly. Some would last a meager couple of hours while others could persist for up to six hours. Since the class period that was included in this study was the third period of the school day,

students could deplete their battery life in the previous two class periods before arriving in my classroom. This could be done through academic use or recreational use, where the students had used their laptops to listen to music, watch movies, play games, or access social media sites. Lastly, students did not always remember to charge their laptops in other classrooms or after each school day and would routinely return to school the next day with a dead battery and in need of a charge. In either case these students, with greatly depleted or dead batteries, would have to sit next to an electrical outlet in order to operate their computer. While my classroom has 14 potential electrical outlets for students to use to recharge their laptops, only eight are accessible by the students as the others are in use by myself or reside behind my desk. Additionally, though eight electrical outlets are accessible by the students, any student wanting to recharge their laptop would most likely have to change seats or move their desk away from their group to do so. Since I maintain a seating chart to minimize disruptions and to maximize the cohesiveness of group members, an alteration to that seating chart may not be advisable or acceptable. In hindsight, the logical solution to this problem would have been to provide each group with an extension cord and a power strip.

The third challenge was with the amount of time it took students to prepare for a class period with which GeoGebra was going to be incorporated. Specifically, the amount of time it took students to get their laptops out, turn on their computers, logon to the laptops, and open GeoGebra. Though this would necessitate a minimum of one to two minutes per class period this would amount to 12-24 minutes over the course of the study, which would equate to one-fourth to one-half of a class period. Of course, this time would be longer if they had not downloaded GeoGebra and needed to gain access to the internet prior to accessing GeoGebra. Though I attempted to meet students at the

entrance to my classroom and remind them upon entry that they needed to begin the process of accessing GeoGebra on their laptops, the time it required seemed considerably inefficient. Futhermore, this inefficiency was only perceived as worse when compared to my ability to pre-input data into GeoGebra and have it ready for class before the class period had even begun.

The fourth challenge was the need to monitor students' computer usage. My vice-principal highlighted this potential challenge when he or she, during his or her observation of section 5.7C, behavior near zeros, stated, "engagement could be higher if students had their own computer but then you would have to monitor the technology use." While I, as the teacher, should be moving about my classroom and monitoring student progress, maintaining proper technology usage could be burdensome. This is due in part to the seating arrangement necessary for students to work in groups and thus, my physical position within the classroom will not remain ideal for monitoring student computer usage. It is also due in part to students desire to access aspects of their computers and the internet that are unrelated to the topic at hand and the ease with which they can hide this behavior from me.

Challenges with Teachers. The third finding of research question two of challenges that I experienced was with challenges with other Algebra II teachers and within me. The first challenge was with the limit of my knowledge with technology in general, and specifically technology within the classroom setting. More precisely, with regard to this study, my ability to understand what GeoGebra offers, how to use GeoGebra, and how to incorporate GeoGebra into lesson plans. Though I had previous experience with technology in the classroom, and with operating GeoGebra, the experience was not extensive. Hence, during the preparatory phase for this study, I had

to learn how to navigate through and operate the various applications of GeoGebra. This discovery period of the abilities of GeoGebra was followed by acquainting myself with how GeoGebra could be employed within my classroom. That is, I had to gain an understanding of how GeoGebra's abilities could be put to use within my lesson plans. However, through the experience of this study, I have a much better understanding of GeoGebra and how to utilize it within my classroom. The fact that I began the preparatory phase of this study as a novice, with regard to my understanding of GeoGebra, would imply that my lack of experience was a challenge.

This challenge logically leads to the second challenge within myself of having sufficient time to prepare for the lessons to be taught within this study. Specifically, having the necessary time to create lesson plans that utilize GeoGebra appropriately and affectively. While creating lesson plans is always time-consuming, creating lesson plans that utilize an unfamiliar piece of technology and that has caused me to adopt a more conceptual and discovery-centered approach to my lesson plans was greatly time-consuming. Additionally, though I was aware of GeoGebra, a conceptual approach to teaching, and a discovery-centered approach to teaching before the study had begun, incorporating all three faucets required an abnormally large amount of consideration and contemplation of how to do so with each lesson plan. For each lesson I would attempt to recognize the underlying concept that needed to be targeted, if and how that concept could be discovered through an inquiry-based approach, and then if and how GeoGebra could be utilized to aid in this process, or if I needed to find another use for GeoGebra somewhere in the lesson plan. Once more, since this was an unfamiliar approach to take with my lesson planning the time to do so was enlarged.

A third challenge was with teaching students to operate a new software package that they were not accustomed to using. As student S5 stated during their interview over section 5.2B, end behaviors of polynomials, it "would be fun to do by ourselves but I don't know how to use it." The student was referring to the lack of knowledge in the general use of GeoGebra. Before the commencement of this study, students had previous experience with operating their laptops and accessing websites. However, students had not used, and had been supplied with very little exposure to, previous to this study, graphing technology or CAS. Moreover, students had not experienced any exposure to GeoGebra specifically. Due to this lack of experience with mathematical technology, and specifically the technology utilized in this study, students required training in both the capabilities and the general operation of GeoGebra. While I could have provided an introductory GeoGebra lesson to students prior to the study commencing, this singular lesson would not have adequately prepared students to integrate MP5 through the use of GeoGebra, though it would have aided the implementation of GeoGebra. Thus, the exposure to and training with mathematical technology, or at least technology similar to the capabilities of GeoGebra, of my students should have taken place in previous mathematics classes to Algebra II to have adequately prepared my students to integrate MP5 through the use of GeoGebra. Additionally, I should have been utilizing these same forms of mathematical technology with my students prior to the study commencing.

The fourth challenge that I faced was with the extent to which I was permitted to employ GeoGebra within my classroom. At the high school in which I am employed, teachers who teach common classes are required to utilize the same unit tests. This is a requirement so that test scores can be compared between teachers in an attempt to

identify the better teaching approaches to specific topics. The present unit test for the unit of study in this study, polynomials and polynomial functions, was only permitted to be taken by students utilizing a scientific calculator, and no other forms of technology were permitted. Due to the this lack of an incorporation of technology, specifically GeoGebra, into the unit test I requested permission to alter the unit test or develop a new unit test, one which would allow students to utilize GeoGebra or other mathematical tools. During the course of the study this request was denied, as it would have required the other teachers, common to the subject of Algebra II, to integrate technology into their lesson plans in a similar fashion to the lesson plans that I developed for this study, and result in an increase to their workload.

Furthermore, teachers who teach common classes are also required to be on a similar teaching schedule. That is, common teachers should assess each unit taught within one week of other common teachers. The reason for this requirement is to ensure that students who must transfer between teachers do not face a large deficit in material that has been taught to them. In the case of this study, I had to be careful to not spend a large amount of time on any singular concept or lesson, and to continue to progress towards the unit test within a similar timeframe to other common teachers of Algebra II. This restricted timeframe stayed in the forefront of mind when planning activities, especially with the amount of time that I wanted to allow students to utilize discovery learning.

Summary

The presentation of the data analysis was organized with regard to the two research questions that guided this study and identified five major findings. Research

question 1, which asked for the successes I experienced utilizing GeoGebra to integrate MP5 into an Algebra II unit of study, centered around two findings. The first finding was that my instruction targeted a deeper level of mathematical understanding by students. This targeting of a deeper level of understanding also resulted in the utilization of discovery learning. Specifically, my instruction shifted from having never utilized discovery learning towards the incorporation of discovery learning with a goal of helping students see the connection between algebra and its graphical representation. The second finding was with students' having a moderately high level of interest.

Research question 2, which asked for the challenges I experienced utilizing GeoGebra to integrate MP5 into an Algebra II unit of study, centered around three findings. The first finding was challenges with technology, specifically, computer access, internet speed, internet access, and a GeoGebra problem. The second finding was challenges with students, specifically, students being unprepared for class, the time required for students to prepare for a lesson, and the need to monitor student computer usage. The third finding was challenges with teachers, specifically, with other Algebra II teachers and within myself.

CHAPTER V: DISCUSSION

The rationale for this study stemmed from the advent of the Common Core State Standards for mathematics and suggested Mathematical Practices in 2009 (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The release of CCSS naturally left mathematics educators curious about how best to incorporate a curriculum that will address both the standards and practices. To aid in this pursuit, this study utilized the implementation of GeoGebra as a way of integrating one of the eight standards of mathematical practice, specifically MP5, *use appropriate tools strategically*. This chapter will have: (a) discussion of the findings; (b) suggestions for future research; (c) suggestions for future practice; and (d) conclusion of the study.

Discussion of the Findings

According to CCSS in order to integrate MP5 students must “consider the available tools when solving a mathematical problem” and must be “sufficiently familiar with the tools” so that they are “able to use technological tools to explore and deepen their understanding of concepts” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 7). This study was successful at integrating a new, to students, mathematical tool, GeoGebra, into an Algebra II unit of study that contributed to students’ understanding of mathematics. More precisely, GeoGebra was utilized in a manner that provided students with the opportunity to explore and extend their understanding of mathematical concepts. Furthermore, utilizing anecdotal evidence of a comparison with past students that I have

taught in Algebra II, students in this study were having discussions, observations, and questions that implied a growth in their understanding of mathematical concepts in Algebra II. This study was also successful at incorporating GeoGebra in 11 of the 12 lessons developed for use in this study. Additionally, while this study targeted the integration of MP5, I noticed that I unintentionally integrated, to some extent, other mathematical practices. Specifically, I partially integrated MP1, *make sense of problems and persevere in solving them*, MP3, *construct viable arguments and critique the reasoning of others,* and MP7, *look for and make use of structure.* This result highlighted the intertwining of the mathematical practices and how integration of one mathematical practice will, most likely, lead to some integration of other mathematical practices.

However, this study was not completely successful, as it did not integrate all aspects of MP5. Specifically, students were not given a choice between technological, mathematical tools and students were not given the opportunity for the strategic use of the technological tool, GeoGebra, they were provided. For students to have the choice between mathematical tools, this study should have included the study of multiple mathematical tools. In order for the incorporation of multiple tools into this study to be successful and in order for students to be sufficiently familiar with the mathematical tools available to them, I should have incorporated the training of students in how to use multiple mathematical tools previous to this study commencing. Moreover, in order for students to truly consider their available tools, I should have incorporated scenarios where finding a solution was less directed from the teacher. In these scenarios, I would envision students being provided with a question or situation to analyze and then empowered to choose which mathematical tool to employ in order to find the answer.

More precisely, the students would not be given a suggestion from the teacher of what approach to take or what tool or tools to utilize with the goal that students would get comfortable with analyzing new situations and deciphering an appropriate approach and an appropriate tool to use.

The reason I believe that I did not successfully integrate all aspects of MP5 was due to my teaching approach before the study. Prior to the study, my teaching approach was primarily direct instruction and thus, centered on a lecture-based format. Furthermore, my approach to teaching a new mathematical topic was more focused on transmitting procedural knowledge and less focused on transmitting conceptual knowledge. Lastly, while I did believe that technology was beneficial to mathematics education, I utilized graphing technology seldom and almost never utilized a CAS or mathematical software in my classroom.

Though I made strides in the direction of a conceptually-based instructional approach that utilizes technological tools, my teaching style was still traditional in nature. My teaching style still involved a lot of direct instruction along with very regimented and controlled lesson plans. I maintained this teaching style because I felt somewhat uncomfortable utilizing technology in a manner that I did not experience in my K-12 mathematical education. Furthermore, I felt somewhat uncomfortable with allowing students time to analyze and decipher a situation, as visible work was not necessarily taking place, which I perceived incorrectly to be downtime. Thus, the result was that my students were not given sufficient training prior to this study with multiple mathematical tools in my classroom and my students were not provided with the opportunity to strategically apply the use of the few technological tools that they had at their disposal.

Before discussing research question one's findings of successes it should be noted that, with regard to the study as a whole, the incorporation of technology, and specifically GeoGebra, into a mathematics classroom is consistent with current literature (Ellington, 2003, 2006, Konyalioghu et al., 2012, NCTM, 1989, 2000, Rakes et al., 2010, Stupel & Ben-Chaim, 2014). The first finding of successes, that my instruction targeted a deeper level of mathematical understanding by students and that this lead to the utilization of discovery learning, was consistent with current literature. Ponce-Campuzano (2013) and Salleh and Sulaiman (2013) found that a teacher's teaching style became more conceptual in nature due to the inclusion of graphing technology. While my instruction did not become conceptually focused, it did target a deeper level of mathematical understanding, which is a move from a purely procedural instructional approach towards a conceptual approach. This targeting of a deeper level of understanding also resulted in the utilization of discovery learning, which according to Saab et al. (2005) and Sungur and Tekkaya (2006) is a success. A shift in instruction from a lecture-based format to a discovery-based format due to the inclusion of graphing utilities into a mathematics classroom is similar to work by Doerr and Zanger with graphing calculators (2000). While Doerr and Zanger's study focused on graphing calculators, their result should hold some significance to this study as graphing calculators are a form of graphing technology. Moreover, utilizing graphing utilities for the purpose of connecting the algebraic representation of mathematics with the graphical representation is also consistent with current literature (Ponce-Campuzano, 2013, Salleh & Sulaiman, 2013, Stupel & Ben-Chaim, 2014).

However, there is an aspect of the first finding of successes discussed that were not addressed by current literature, and consequently will necessitate future studies. The

first finding of successes identified deal with how the decision to incorporate one form of technology caused me to not only question the best way to do so, but also whether my general teaching philosophy needed to be altered. This resulted in changing my teaching style and approach. Thus, the inclusion of technology, the ensuing deep analysis of my teaching philosophy, and the resulting change to my teaching style and approach, was a finding that was not addressed by current literature.

The second finding of successes was with students having a moderately high level of interest. This finding of successes is in support of current literature. The high levels of interest and on-task behavior because of either introducing graphing calculators or graphing technology into a mathematics classroom is consistent with current literature (Merriweather & Tharp, 1999, Quesada & Maxwell, 1994, Ruthven et al., 2009, Slavit, 1996). It seems that the novelty of GeoGebra and the increased rigor that accompanies a targeting of a deeper level of mathematical understanding are the driving force for why students' level of interest were at a high level (Ruthven et al., 2009).

Research question two, the challenges I experienced utilizing GeoGebra to integrate MP5 into an Algebra II unit of study, centered around three findings. Two of the three findings of challenges appear to be consistent with current literature. The first finding of challenges of unreliable technology, computer access, internet access, internet speed, and program bugs, is consistent with current literature (Sozcu et al., 2013, Wachira & Keengwe, 2011). The third finding of challenges of a lack of training and understanding in how to operate an unfamiliar form of technology and in how to incorporate the technology into the mathematics classroom is consistent with current literature (Wachira & Keengwe, 2011). The third finding of challenges of a lack of sufficient time to learn about an unfamiliar form of technology and to prepare for

incorporating that technology into one's classroom is consistent with current literature (Little, 2009, Salleh & Sulaimen, 2013, Wachira & Keengwe, 2011). The third finding of challenges of a need for time to acclimate students to a new mathematical technology is consistent with current literature (Mackrell, 2012, Ruthven et al., 2009).

The second finding of challenges and one aspect of the third finding of challenges are not addressed by current literature. Specifically, the second finding of challenges of students being unprepared for class, the time required for students to prepare for a lesson, and the need to monitor student computer usage is not addressed by current literature. While this finding is important to note for a teacher that wants to incorporate mathematical technology in their classroom, these challenges can each be addressed and should not represent a significant hurdle. The aspect of the third finding of challenges not addressed by current literature, that other Algebra II teachers did not permit me to alter my unit test, could represent a significant hurdle. It is important to note that in schools where department members work closely together and common classes are expected to be approached in a similar fashion, this could represent a potential barrier. In my specific case, I could still alter lesson plans and thus this challenge did not restrict my teaching, only my assessment.

Recommendations for Future Research

After the data obtained in this study was analyzed two findings warrant a deeper investigation and subsequent future studies. Since this study did not successfully integrate all aspects of MP5, the logical recommendation for future research would be to remedy the underlying causes of this failure and to conduct the study again. The first remedy would be either to include multiple technological tools in the study or to ensure

that students had been provided with a sufficient number of opportunities with multiple technological tools before the study began. The second remedy would be to have provided opportunities to students prior to the study with scenarios where they were given a problem to solve and then no more instruction into how to solve the problem, with the goal of students truly considering different approaches and different tools to use. The third remedy would be for the instructor(s) involved in the future study to be fully aware of their teaching practices, and how that could affect the study prior to the study beginning. For example, in this study my desire for a regimented approach and control of the activities taking place decreased my ability to integrate all aspects of MP5 into my classroom.

One finding of successes identified by this study was that the incorporation of GeoGebra into my lesson plans caused me alter my approach to the classroom and to target a deep level of mathematical understanding by students. Thus, the recommendation for future research would be to uncover what circumstances need to be in place in order to cause a teacher to identify his or her teaching beliefs, analyze whether his or her beliefs and practices align, and to deeply analyze his or her teaching beliefs and practices. Furthermore, this analysis should be of a sufficient depth that a teacher's beliefs and practices are not only considered, but that those beliefs and practices also have the potential to be altered. This recommendation for future research should be desirable as altering a teacher's beliefs and practices could have massive effects on student mathematical understanding.

Recommendations for Future Practice

The biggest finding from this study is that integrating MP5 requires careful thought, planning, and, preferably, the cooperation of many teachers to ease the workload on any individual teacher. More precisely, students will need to have been exposed to multiple mathematical tools, provided with opportunities to learn about those mathematical tools, and provided with opportunities across multiple grade levels to utilize those mathematical tools. Furthermore, the understanding that students should have of these mathematical tools must be in-depth enough that students know how to operate each tool, the capabilities of each tool, and the limitations of each tool. Students need to be provided with opportunities, during both instruction and assessment, in which they have the option of choosing which mathematical tool is appropriate to utilize and then to employ their choice to attempt to obtain a solution.

Another finding from this study is figuring out what can be done to inspire teachers into identifying their teaching beliefs, analyzing whether their beliefs and practices align, and then deeply analyzing their teaching beliefs and practices to such an extent that it becomes obvious when a change is prudent. An extension of that idea for individual teachers is to begin the process of deeply analyzing their beliefs and practice. This recommendation for future practice should be desirable as altering a teacher's beliefs and practices could have massive effects on student mathematical understanding.

Conclusion of the Study

The purpose of this study was to identify the successes and challenges a teacher experienced when GeoGebra was incorporated into an Algebra II unit of study with the goal of integrating the fifth mathematical practice of the CCSS, *use appropriate tools*

strategically. Utilizing the two research questions that guided this study five major findings were identified during data analysis. The first finding of successes was that my instruction targeted a deeper level of mathematical understanding by students. This targeting of a deeper level of understanding also resulted in the utilization of discovery learning. Specifically, my instruction shifted from having never utilized discovery learning towards the incorporation of discovery learning with a goal of helping students see the connection between algebra and its graphical representation. The second finding of successes was with students' having a moderately high level of interest.

The first finding of challenges was with technology, specifically, computer access, internet speed, internet access, and a GeoGebra problem. The second finding of challenges was with students, specifically, students being unprepared for class, the time required for students to prepare for a lesson, and the need to monitor student computer usage. The third finding of challenges was with teachers, specifically, with other Algebra II teachers and within myself.

Those findings of successes and challenges yielded two findings that should guide future research and practice. The first finding that this study highlighted was that integrating MP5 requires careful thought, planning, and, preferably, the cooperation of many teachers to ease the workload on any individual teacher. More precisely, students will need to have been thoroughly provided with opportunities to utilize multiple mathematical tools and provided with opportunities, during both instruction and assessment, in which they have the option of choosing which mathematical tool is appropriate to utilize. The second finding that this study highlighted was the need to identify what circumstances must occur in order to inspire teachers into identifying their teaching beliefs, analyzing whether their beliefs and practices align, and then deeply

analyzing their teaching beliefs and practices to such an extent that it becomes obvious when a change is prudent. Through the information identified, the impetus has been set for future research and an alteration to current practice.

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APPENDICES

Appendix A

Institutional Review Board Acceptance Letter

Missouri State
UNIVERSITY

To:
Kurt Killion
Beth Hurst, Gay Ragan

RE: Notice of IRB Approval
Submission Type: Initial
Study #: IRB-FY2017-306
Study Title: Utilizing GeoGebra in an Algebra II Classroom to Use Appropriate Tools Strategically
Decision: Approved

Approval Date: Oct 31, 2016
Expiration Date: Oct 30, 2017

This submission has been approved by the Missouri State University Institutional Review Board (IRB) for the period indicated.

Federal regulations require that all research be reviewed at least annually. It is the Principal Investigator's responsibility to submit for renewal and obtain approval before the expiration date. You may not continue any research activity beyond the expiration date without IRB approval. Failure to receive approval for continuation before the expiration date will result in automatic termination of the approval for this study on the expiration date.

You are required to obtain IRB approval for any changes to any aspect of this study before they can be implemented. Should any adverse event or unanticipated problem involving risks to subjects or others occur it must be reported immediately to the IRB.

This study was reviewed in accordance with federal regulations governing human subjects research, including those found at 45 CFR 46 (Common Rule), 45 CFR 164 (HIPAA), 21 CFR 50 & 56 (FDA), and 40 CFR 26 (EPA), where applicable.

Researchers Associated with this Project:
PI: Kurt Killion
Co-PI: Beth Hurst, Gay Ragan
Primary Contact: Kurt Killion
Other Investigators: David Matthews

Appendix B

Observational Protocol
Part 1 – During Observation

Observer:___________________________ Date of Observation:_______________

As you observe the lesson, record in Column 3 the events of the students as they relate to the following topics:

1. Students' level of interest
2. Students' level of engagement
3. Students' ability to work in groups while utilizing GeoGebra
4. The manner in which students employ GeoGebra
5. The manner in which students correctly or incorrectly employ GeoGebra
6. Students' ability to correctly or incorrectly learn a concept utilizing GeoGebra
7. Students' ability to translate mathematical concept between two or more mathematical tools
8. Evidence to either suggest or refute a students' ability to choose the appropriate mathematical tool for a mathematical task and/or strategically use that mathematical tool

Provide a time stamp in Column 2 to correspond with the events. If you are a certified mathematics educator then fill out the protocol with regard to items 1-8, all others fill out the protocol with regard to items 1-4. After the lesson, assign an appropriate topic number for the events described in Column 3 (e.g., 1, 3, and 7) in Column 4. More than one line can be used to discuss any event(s).

Line:	Time:	Event:	Activity Number:
1			
2			
3			
4			
5			
6			
7			
8			

Portions of this instrument were developed in 1998 at the University of Wisconsin, in 2003-2006 at the University of Missouri, and in 2015 by the STEAM project. Modifications have been made to reflect the goals of this study.

Observational Protocol
Part 2 – Post Observation

Observer:__________________________ Date of Observation:_______________

Seated (circle one): Individually Pairs Groups of four

Use your notes from Part 1 of the Observational Protocol to summarize the classroom observation and complete the remainder of this form.

1. Indicate the overall level of student interest during the class period (mark the descriptor that best applies):
   ____ Relatively few students appeared interested
   ____ About one-half of the students appeared interested
   ____ Relatively all of the students appeared interested

2. Indicate the overall level of student engagement during the class period (mark the descriptor that best applies):
   ____ Relatively few students appeared to be on task
   ____ About one-half of the students appeared to be on task
   ____ Relatively all of the students appeared to be on task

3. Indicate the dominant level of student collaboration for the class period (mark only one):
   ____ Most students worked individually
   ____ Some students worked collaboratively while others worked individually
   ____ Most students worked collaboratively

4. Describe the following: a.) the main activities that occurred during the class period, b.) how affective you feel those activities were, and c.) why you feel those activities were affective.

Portions of this instrument were developed in 1998 at the University of Wisconsin, in 2003-2006 at the University of Missouri, and in 2015 by the STEAM project. Modifications have been made to reflect the goals of this study.

5. Please list any suggestions of what could be altered and if so, why?

Appendix C

Interview Protocol

Interviewer:______________________________ Date of Interview:_______________
Interviewee:______________________________

1. Indicate the overall level of student interest during the class period (mark the descriptor that best applies):
   ____ Relatively few students appeared interested
   ____ About one-half of the students appeared interested
   ____ Relatively all of the students appeared interested

2. Indicate the overall level of student engagement during the class period (mark the descriptor that best applies):
   ____ Relatively few students appeared to be on task
   ____ About one-half of the students appeared to be on task
   ____ Relatively all of the students appeared to be on task

3. Indicate the dominant level of student collaboration for the class period (mark only one):
   ____ Most students worked individually
   ____ Some students worked collaboratively while others worked individually
   ____ Most students worked collaboratively

4. Describe the following: a.) the main activities that occurred during the class period, b.) how affective you feel those activities were, and c.) why you feel those activities were affective.

Portions of this instrument were developed in 1998 at the University of Wisconsin, in 2003-2006 at the University of Missouri, and in 2015 by the STEAM project. Modifications have been made to reflect the goals of this study.

5. Please list any suggestions of what could be altered and if so, why?

Portions of this instrument were developed in 1998 at the University of Wisconsin, in 2003-2006 at the University of Missouri, and in 2015 by the STEAM project. Modifications have been made to reflect the goals of this study.

Appendix D

Principal Informed Consent Form
Missouri State University

Dear XX. XXXX XXXXXX:

As part of the final requirements for a Master of Science in Education, Secondary Education: Mathematics Area of Emphasis degree from Missouri State University, I would like to conduct a study to determine what successes and challenges are faced by mathematics educators as they utilize GeoGebra in an Algebra II classroom. The purpose of this letter is to request your permission to utilize archival data that I previously collected as part of my regular classroom instruction and assessment.

For my study, I would like to analyze the archival data from regular classroom interactions that utilized GeoGebra to assist in classroom instruction in an Algebra II classroom. The information I collect from this study will be kept confidential. No names of individuals or the school will be used. Students' scores will be assigned a number to protect their identity. You may withdraw from this study at any time.

The study will not interfere with the mathematics curriculum, nor will it disrupt the learning process. Since the lessons utilizing GeoGebra were already given as part of the regular routine classroom instruction, parent consent will not need to be obtained.

Please complete the lower portion of this letter, and return it to me by October 19, 2016. Thank you for your consideration. If you have any questions, or require more information, please do not hesitate to contact my university supervisor, XX. XXXX XXXXXXX at XXXXXXXXXXX@missouristate.edu, or myself.

Sincerely,

David S. Matthews II

As principal of XXXXXXX High School, I give my formal consent for David Matthews to conduct his study titled *Utilizing GeoGebra in an Algebra II Classroom to Use Appropriate Tools Strategically* where he will be examining archival data. I understand that I may withdraw my school from the study at any time.

____________________________ ________________________
Principal Date

Appendix E

Parent/Guardian Informed Consent Form
Missouri State University

Dear Parents,

I will be conducting a study in our classroom to investigate what successes and challenges are experienced when a teacher implements GeoGebra, a mathematics software, into an Algebra II unit of study. The study will only last for one unit of study (approx. 2-3 weeks). I will teach using research-based strategies showing students how strategically to utilize the mathematics software as an appropriate tool in mathematics. When student work highlights the challenges or successes that occur throughout the unit, I would like to use their work as evidence. I will also video record each lesson taught within the unit so that I have an accurate account of what transpired and can analyze it in more detail after class.

I am writing to ask permission to use the data I collect from your child during this process. Participation in this study involves only regular classroom activities and thus partaking in this study will not contain any risk or inconvenience to your child. Furthermore, your child's participation is strictly voluntary and you may withdraw their participation at any time without penalty. Your child's participation or nonparticipation will not affect their Algebra II grade. All information collected will be used only for my research and will be kept confidential. There will be no connection to your child specifically in the results or in future publication of the results. Once the study is completed, I would be happy to share the results with you if you desire. You may contact me at any time regarding your child's participation. My phone number is XXX-XXX-XXXX ext. XXXX and my email address is XXXXXXXXXXXX@XXXXX.XXX The principal, XX. XXXXXX, has approved this study.

Please check the appropriate box below, sign the form, and return to Mr. Matthews by October 19, 2016:

- ☐ I give permission for my child's data to be used in this study. I understand that I will receive a signed copy of this consent form. I have read this form and understand it.
- ☐ I do not give permission for my child's data to be included in this project.

____________________________ ______________________________
Student's Name Signature of Parent/Guardian

____
Date

Appendix F

Student Interview Example

Interview Protocol

Interviewer:_David Matthews______________ Date of Interview:__11/18/16_______
Interviewee:_Student S4 (Section 5.4A)______

1. Indicate the overall level of student interest during the class period (mark the descriptor that best applies):
   ____ Relatively few students appeared interested
   ____ About one-half of the students appeared interested
   _X__Relatively all of the students appeared interested

2. Indicate the overall level of student engagement during the class period (mark the descriptor that best applies):
   ____ Relatively few students appeared to be on task
   ____ About one-half of the students appeared to be on task
   _X_ Relatively all of the students appeared to be on task

3. Indicate the dominant level of student collaboration for the class period (mark only one):
   ____ Most students worked individually
   ____ Some students worked collaboratively while others worked individually
   _X__Most students worked collaboratively

4. Describe the following: a.) the main activities that occurred during the class period, b.) how affective you feel those activities were, and c.) why you feel those activities were affective.

__a.) 1.) Talked about what equations were factorable and how to determine if they ___

_________are.

_____2.) Found sum and difference of cubes formulas.

_____3.) Went over factor by grouping.

__b.) I didn't understand how to determine if equations were factorable but I thought __

_____that finding the formulas were neat and factor by grouping seemed easy.________

__c.) I thought that finding the formulas was neat because we found it ourselves. The__

_____factor by grouping was easy because it wasn't anything new._________________

Portions of this instrument were developed in 1998 at the University of Wisconsin, in 2003-2006 at the University of Missouri, and in 2015 by the STEAM project. Modifications have been made to reflect the goals of this study.

5. Please list any suggestions of what could be altered and if so, why?

__Try to make finding if equations are factorable easier.__________________________

__Keep finding the formulas the same because it was exciting to uncover new________

__information.____

__Keep factor by grouping the same because it was easy to understand.______________

Appendix G

Instructional Coach Observation Example

Observational Protocol
Part 1 – During Observation

Observer:_Instructional Coach (Section 5.7C)_ Date of Observation:_1/9/17_________

As you observe the lesson, record in Column 3 the events of the students as they relate to the following topics:

1. Students' level of interest
2. Students' level of engagement
3. Students' ability to work in groups while utilizing GeoGebra
4. The manner in which students employ GeoGebra
5. The manner in which students correctly or incorrectly employ GeoGebra
6. Students' ability to correctly or incorrectly learn a concept utilizing GeoGebra
7. Students' ability to translate mathematical concept between two or more mathematical tools
8. Evidence to either suggest or refute a students' ability to choose the appropriate mathematical tool for a mathematical task and/or strategically use that mathematical tool

Provide a time stamp in Column 2 to correspond with the events. If you are a certified mathematics educator then fill out the protocol with regard to items 1-8, all others fill out the protocol with regard to items 1-4. After the lesson, assign an appropriate topic number for the events described in Column 3 (e.g., 1, 3, and 7) in Column 4. More than one line can be used to discuss any event(s).

Line:	Time:	Event:	Activity Number:
1	10:02	Drawing of graph intersecting axis – students asked what would occur at intersection point.	1,2
2	10:03	Students respond with possibilities.	1,2
3	10:04	Presents options and prepares students to make connections to next activity.	1,2
4	10:05	Students given handout (students seated in groups)	1,2
5	10:06	Students log in to GeoGebra (displayed via projector at front of the room).	1,2,3
6	10:07	Students identify zeros looking at equations.	1,2,3

Portions of this instrument were developed in 1998 at the University of Wisconsin, in 2003-2006 at the University of Missouri, and in 2015 by the STEAM project. Modifications have been made to reflect the goals of this study.

Line:	Time:	Event:	Activity Number:
7	10:08	Students introduced to multiplicity – definition	1,2,3
8	10:08	Students work through one problem together	1,2,3
9	10:09	Identified tangent, recalling past lessons.	1,2,3
10	10:10	Students asked question – clarified	1,2,3
11	10:11	Students as a class work through another problem – students taking notes, call for questions.	1,2,3,4
12	10:13	Class works through problem identifying zeros, multiplicities, and behavior.	1,2,3,4
13	10:14	Students call out answers and take notes	1,2
14	10:15	Last example	
15	10:16	Students compare multiplicities to behavior and determine explanation, discuss in groups	1,2,3,4
16	10:18	Students share explanations and others evaluate	1,2,3,4,5
17	10:20	Consensus, take notes	1,2,3,4,5
18	10:20	Introduces main topic (explanation given)	1,2,3,4,5
19	10:21	Students turn page for individual/group work	1,2,3,4
20	10:22	Explains past concept – imaginary zero by asking students to give examples and define/clarify.	1,2,3,4
21	10:25	Students begin work on own – list zeros, multiplicities, is it real or imaginary, describe behavior. Lots of discussion about problems at tables whole instructor circled and answered questions.	1,2,3,4
22	10:27	Students compare answers to board, ask (?)	1,2,5,6
23	10:29	Move onto next two problems – they look different, but	1,2,3,4

Line:	Time:	Event:	Activity Number:
24	10:29	Students asked to try.	1,2,3,4
25	10:32	Work together: list zeros, multiplicities, identify real or imaginary, behavior	1,2,3,4
26	10:33	point about graphing imaginary number from student	5,6
27	10:34	Questions – none / cover next problem	1,2,3,4
28	10:34	Explanation of challenging zero	1,2,3,4
29	10:36	Students explain why numbers are real or imaginary and then behaviors	1,2,5,6,7
30	10:37	No questions from students	
31	10:37	Students work on last two equations in groups while instructor circles (progressively harder/more complex)	1,2,3,4
32	10:40	Students all working individually and discussing if stumped.	1-4
33	10:43	Students check their answers with board and find two mistakes on "F"	1,2,5,6,7
34	10:45	Students identify mistakes – fix and explain	1,2,5,6,7

Observational Protocol
Part 2 – Post Observation

Observer:_Instructional Coach (Section 5.7C)_ Date of Observation:_1/9/17_________

Seated (circle one): Individually Pairs Groups of four

Use your notes from Part 1 of the Observational Protocol to summarize the classroom observation and complete the remainder of this form.

1. Indicate the overall level of student interest during the class period (mark the descriptor that best applies):
   ____ Relatively few students appeared interested
   ____ About one-half of the students appeared interested
   _X__Relatively all of the students appeared interested

2. Indicate the overall level of student engagement during the class period (mark the descriptor that best applies):
   ____ Relatively few students appeared to be on task
   ____ About one-half of the students appeared to be on task
   _X__Relatively all of the students appeared to be on task

3. Indicate the dominant level of student collaboration for the class period (mark only one):
   ____ Most students worked individually
   _X__Some students worked collaboratively while others worked individually
   ____ Most students worked collaboratively

4. Describe the following: a.) the main activities that occurred during the class period, b.) how affective you feel those activities were, and c.) why you feel those activities were affective.

___Students were asked to identify familiar components of an equation, and were also__ introduced to a new concept, multiplicities. Students worked through four practice____ equations together and were given ample opportunity to ask questions. Then they were_ asked to complete progressively difficult equations on their own/in groups while the___ instructor answered individual questions and perused student work. These activities were effective because they allowed students a chance to recall past information needed, apply new learning, and look for patterns that explained behavior, which allows them a_

Portions of this instrument were developed in 1998 at the University of Wisconsin, in 2003-2006 at the University of Missouri, and in 2015 by the STEAM project. Modifications have been made to reflect the goals of this study.

a chance to apply behaviors/rules to other problems. The chance to work collaboratively allowed students a chance to discuss ideas, evaluate, and correct misconceptions before sharing. Students asked questions, which showed trust in classroom environment. ☺

5. Please list any suggestions of what could be altered and if so, why?

Consider having groups explain so it isn't always voluntary in order to hit all students in the class – or use cold call.

Instead of asking if anyone has questions give students a chance to respond all at once with traffic signal color cards or thumb/fist-to-five regarding understanding – it's a fast way to see how the class as a whole is responding to content.

Appendix H

Teacher-Researcher Self-Observation Example

Observational Protocol
Part 1 – During Observation

Observer:_David Matthews (Section 5.3B)_ Date of Observation:_11/14/16__________

As you observe the lesson, record in Column 3 the events of the students as they relate to the following topics:

1. Students' level of interest
2. Students' level of engagement
3. Students' ability to work in groups while utilizing GeoGebra
4. The manner in which students employ GeoGebra
5. The manner in which students correctly or incorrectly employ GeoGebra
6. Students' ability to correctly or incorrectly learn a concept utilizing GeoGebra
7. Students' ability to translate mathematical concept between two or more mathematical tools
8. Evidence to either suggest or refute a students' ability to choose the appropriate mathematical tool for a mathematical task and/or strategically use that mathematical tool

Provide a time stamp in Column 2 to correspond with the events. If you are a certified mathematics educator then fill out the protocol with regard to items 1-8, all others fill out the protocol with regard to items 1-4. After the lesson, assign an appropriate topic number for the events described in Column 3 (e.g., 1, 3, and 7) in Column 4. More than one line can be used to discuss any event(s).

Line:	Time:	Event:	Activity Number:
1	10:44	Mr. Matthews – Used area model to prove the answer is correct.	
2	10:46	Mr. Matthews – "What numbers represent each side?"	
3	10:46	S4 – Answered correctly.	1,2
4	10:47	Mr. Matthews – "What are the areas of each rectangle?"	
5	10:47	S3 – Gave the areas of each rectangle.	1,2
6	10:47	S4 – Also supplied the areas of each rectangle.	1,2
7	10:48	Mr. Matthews – "What property was just depicted?"	

Portions of this instrument were developed in 1998 at the University of Wisconsin, in 2003-2006 at the University of Missouri, and in 2015 by the STEAM project. Modifications have been made to reflect the goals of this study.

Line:	Time:	Event:	Activity Number:
8	10:48	S12 – “Distributive property.”	1,2
9	10:48	Mr. Matthews – “How is this similar to the last problem?”	
10	10:49	S8 – “Substitute 20 in.”	1,2
11	10:49	Mr. Matthews – Used tabular method and said to students, “Take a guess as to how to do this?”	
12	10:50	Mr. Matthews – “What are the side lengths?”	
13	10:50	S3 – Answered	1,2
14	10:50	S4 – “The tabular method is like the Punnett Square.”	1,2,7
15	10:51	Mr. Matthews – “Great recognition!”	
16	10:52	Mr. Matthews – “What polynomial does this sum to?”	
17	10:53	S12 – Answered	1,2,7
18	10:53	S6 – “They are equal.”	1,2,7
19	10:54	Mr. Matthews – I related the present problem to the previous problem.	
20	10:55	Mr. Matthews – Couldn’t be, explained why	
21	10:56	S5 – “Substitute 20 in then.”	1,2,7
22	10:57	Mr. Matthews – “How can we multiply two binomials without using the tabular method?”	
23	10:57	S1 – “Distributive property.”	1,2,7
24	10:58	Mr. Matthews – “How do we use that property to do this?”	
25	10:58	S2 – “We multiply the first term of the first binomial times the first term of the second binomial and then times the second term of the second binomial. Then we multiply the second term of	1,2,7

Line:	Time:	Event:	Activity Number:
		the first binomial times the first term of the second binomial and then times the second term of the second binomial."	
26	11:00	Mr. Matthews – "Do these two methods, distributive property and tabular method, agree?"	
27	11:01	S4 – "Yes they give the same answer."	1,2,7,8
28	11:01	Mr. Matthews – "What property do they use?"	
29	11:01	S4 – "Distributive property."	1,2,7
30	11:02	Mr. Matthews – "GeoGebra gives the answer on problem now you explain to me what GeoGebra did to get answer."	
31	11:03	S12 – Explained the process.	1,2,3,6,8
32	11:04	Mr. Matthews – "What generally is occurring?"	
33	11:04	S17 – "Distributive property."	1,2,3,6,8
34	11:05	Mr. Matthews – "What specifically is occurring?"	
35	11:05	S11 – Answered	1,2,3,6,8
36	11:06	Mr. Matthews – "Now verify answer with the tabular method."	
37	11:07	S4 – Showed the table filled out.	1,2,7
38	11:09	Mr. Matthews – "Now what?"	
39	11:09	S1 – "Combine like terms."	1,2,7
40	11:09	Mr. Matthews – "Give me the specifics of how that occurred."	
41	11:10	S15 – Gave answer.	1,2,7
42	11:10	S16 – Gave detailed response.	1,2,7

Line:	Time:	Event:	Activity Number:
43	11:10	Mr. Matthews – "How did GeoGebra do this next problem?"	
44	11:11	S17 – "Distributive property."	1,2,3,6,8
45	11:12	Mr. Matthews – "Now what?"	
46	11:12	S11 – "Distributive property again."	1,2,3,6,8
47	11:12	Mr. Matthews – "Do we have another option?"	
48	11:13	S17 – "Combine like terms."	1,2,3,6,8
49	11:13	Mr. Matthews – "Give me a general description of how to multiply a binomial times a trinomial?"	
50	11:16	Mr. Matthews – "What property did we use?"	
51	11:17	S4 – "Distributive property."	1,2,7
52	11:17	Mr. Matthews – "Can anyone explain how to do this?"	
53	11:19	S17 – "Split the binomial and multiply it by the trinomial."	1,2,7
54	11:19	Mr. Matthews – "Class what is meant by split the binomial?"	
55	11:20	S1 – "Take the first term of the binomial and multiply it by all three terms of the trinomial and then take the second term of the binomial and multiply it by all three terms of the trinomial."	1,2,7
56	11:21	Mr. Matthews – "Could we reverse the order described and get the same answer?"	
57	11:22	S6 – "Yes"	1,2,7
58	11:24	Mr. Matthews – "On number four what is your first step?"	
59	11:24	S4 – "Distributive property."	1,2,7

Line:	Time:	Event:	Activity Number:
60	11:25	Mr. Matthews – “Are we allowed to do this?”	

Observational Protocol
Part 2 – Post Observation

Observer: David Matthews (Section 5.3B) Date of Observation: 11/14/16

Seated (circle one): Individually Pairs Groups of four

Use your notes from Part 1 of the Observational Protocol to summarize the classroom observation and complete the remainder of this form.

1. Indicate the overall level of student interest during the class period (mark the descriptor that best applies):
   ____ Relatively few students appeared interested
   ____ About one-half of the students appeared interested
   _X__ Relatively all of the students appeared interested

2. Indicate the overall level of student engagement during the class period (mark the descriptor that best applies):
   ____ Relatively few students appeared to be on task
   ____ About one-half of the students appeared to be on task
   _X__ Relatively all of the students appeared to be on task

3. Indicate the dominant level of student collaboration for the class period (mark only one):
   ____ Most students worked individually
   ____ Some students worked collaboratively while others worked individually
   _X__ Most students worked collaboratively

4. Describe the following: a.) the main activities that occurred during the class period, b.) how affective you feel those activities were, and c.) why you feel those activities were affective.

____

____

____

5. Please list any suggestions of what could be altered and if so, why?

____

____

____

Portions of this instrument were developed in 1998 at the University of Wisconsin, in 2003-2006 at the University of Missouri, and in 2015 by the STEAM project. Modifications have been made to reflect the goals of this study.

Appendix I

Lesson Plan Example

Section 5.5B – Apply the Factor Theorem (Teacher Copy)

Recall:

What are the two methods for dividing polynomials?

1.) Polynomial Long Division 2.) Synthetic Division

When can synthetic division be employed?

Answer: Only when the divisor is a binomial.

What does it mean to say, “7 is a factor of 42”?

Answer: then 42 is divisible by 7

How could we use this information to find the other prime factors of 42 and then do so?

Answer: 42 can be divided by 7 and then the quotient (6) can be further divided to 2 and 3, resulting in the fact that 2 x 3 x 7 = 42 (verify this for students using www.geogebra.org so that students can see the connection between factors of a number and factors of a polynomial)

Example 1.) A.) What does it mean to say that “x + 2 is a factor of $f(x) = 3x^3 - 4x^2 - 28x - 16$"?

Answer: the polynomial f(x) is divisible by x + 2

B.) Factor $f(x) = 3x^3 - 4x^2 - 28x - 16$ completely given that x + 2 is a factor. (Check your answer using www.geogebra.org [by factoring] and using the synthetic division applet https://www.geogebra.org/m/JrsTw2rt)

Recall:

Factor, solve, and then graph the expression $x^2 + 2x - 3$ and its equation using www.geogebra.org .

How do the factors compare to the solutions?

Answer: the factors set equal to zero and solved for x will obtain the solutions

How do the solutions and x-intercepts compare?

Answer: they are the same numerical value

What is another name for x-intercepts of a graph?   Answer: zeros

Example 2.) One zero of $f(x) = x^3 - 2x^2 - 23x + 60$ is x = 3. What are all the zeros of f(x)? Verify the zeros using www.geogebra.org (make sure to show students the connection between the solutions and the zeros/x-intercepts of the graph of f(x))   Note – this means x – 3 is a factor of f

Recall:

What are the five different manners in which we can solve a quadratic equation?

Answer: 1. square root 2. factoring 3. completing the square 4. quadratic equation 5. graphing

Example 3.) One solution of $g(x) = x^3 + 2x^2 - 9x - 18$ is x = -2. Find the other solutions. Verify the solutions using www.geogebra.org (make sure to show students the connection between the solutions and the x-intercepts of the graph of f(x))   Note – this means that x + 2 is a factor of g

Section 5.5B – Apply the Factor Theorem (Student Copy)

Recall:

What are the two methods for dividing polynomials?

1.) 2.)

When can synthetic division be employed?

What does it mean to say, "7 is a factor of 42"?

How could we use this information to find the other prime factors of 42 and then do so?

Example 1.) A.) What does it mean to say that "x + 2 is a factor of $f(x) = 3x^3 - 4x^2 - 28x - 16$"?

B.) Factor $f(x) = 3x^3 - 4x^2 - 28x - 16$ completely given that x + 2 is a factor. (Check your answer using www.geogebra.org and using the applet https://www.geogebra.org/m/JrsTw2rt)

Recall:
Factor, solve, and then graph the expression $x^2 + 2x - 3$ and its equation using www.geogebra.org .

How do the factors compare to the solutions? ______________________________

____

How do the solutions and x-intercepts compare? ___________________________

What is another name for x-intercepts of a graph? __________________________

Example 2.) One zero of $f(x) = x^3 - 2x^2 - 23x + 60$ is x = 3. What are all the zeros of f(x)? Verify the zeros using www.geogebra.org

Recall:
What are the different manners in which we can solve a quadratic equation?

1.)________________ 2.)___________________ 3.)______________________

4.)________________ 5.)___________________

Example 3.) One solution of $g(x) = x^3 + 2x^2 - 9x - 18$ is x = -2. Find the other solutions. Verify the solutions using www.geogebra.org

Appendix J

Data from Observational and Interview Protocols

Data Collection Method and Topic	N	Percentage
Student Interviews		
Overall level of student interest during the class period.		
Relatively few students appeared interested.	0	0%
About one-half of the students appeared interested.	0	0%
Relatively all of the students appeared interested.	3	100%
Total	3	100%
Indicate the overall level of student engagement during the class period.		
Relatively few students appeared to be on task.	0	0%
About one-half of the students appeared to be on task.	0	0%
Relatively all of the students appeared to be on task	3	100%
Total	3	100%
Indicate the dominant level of student collaboration for the class period.		
Most students worked individually.	0	0%
Some students worked collaboratively while others worked individually.	0	0%
Most students worked collaboratively.	3	100%
Total	3	100%
Peer Educator Observations		
Overall level of student interest during the class period.		
Relatively few students appeared interested.	0	0%
About one-half of the students appeared interested.	1	20%
Relatively all of the students appeared interested.	4	80%
Total	5	100%
Indicate the overall level of student engagement during the class period.		
Relatively few students appeared to be on task.	0	0%
About one-half of the students appeared to be on task.	0	0%
Relatively all of the students appeared to be on task.	5	100%
Total	5	100%
Indicate the dominant level of student collaboration for the class period.		

Data from Observational and Interview Protocols continued

Data Collection Method and Topic	N	Percentage
Most students worked individually.	0	0%
Some students worked collaboratively while others worked individually.	3	60%
Most students worked collaboratively.	2	40%
Total	5	100%
Teacher-Researcher Self-Observations		
Overall level of student interest during the class period.		
Relatively few students appeared interested.	0	0%
About one-half of the students appeared interested.	4	33%
Relatively all of the students appeared interested.	8	67%
Total	12	100%
Indicate the overall level of student engagement during the class period.		
Relatively few students appeared to be on task.	0	0%
About one-half of the students appeared to be on task.	3	25%
Relatively all of the students appeared to be on task.	9	75%
Total	12	100%
Indicate the dominant level of student collaboration for the class period.		
Most students worked individually.	0	0%
Some students worked collaboratively while others worked individually.	8	67%
Most students worked collaboratively.	4	33%
Total	12	100%
Totals from Observations and Interviews		
Overall level of student interest during the class period.		
Relatively few students appeared interested.	0	0%
About one-half of the students appeared interested.	5	25%
Relatively all of the students appeared interested.	15	75%
Total	20	100%
Indicate the overall level of student engagement during the class period.		
Relatively few students appeared to be on task.	0	0%
About one-half of the students appeared to be on task.	3	15%
Relatively all of the students appeared to be on task.	17	85%

Data from Observational and Interview Protocols continued

Data Collection Method and Topic	N	Percentage
Total	20	100%
Indicate the dominant level of student collaboration for the class period.		
Most students worked individually.	0	0%
Some students worked collaboratively while others worked individually.	11	55%
Most students worked collaboratively.	9	45%
Total	20	100%