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Handwritten Digit Recognition by Multi-Class Support Vector Machines

Yu Wang

Missouri State University, Yu444@live.missouristate.edu

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**HANDWRITTEN DIGIT RECOGNITION BY MULTI-CLASS
SUPPORT VECTOR MACHINES**

A Masters Thesis
Presented to
The Graduate College of
Missouri State University

In Partial Fulfillment
Of the Requirements for the Degree
Master of Science, Mathematics

By
Yu Wang
May 2018

HANDWRITTEN DIGIT RECOGNITION BY MULTI-CLASS SUPPORT VECTOR MACHINES

Mathematics

Missouri State University, May 2018

Master of Science

Yu Wang

ABSTRACT

Support Vector Machine(SVM) is a widely-used tool for pattern classification problems. The main idea behind SVM is to separate two different groups with a hyper-plane which makes the margin of these two groups maximized. It doesn't require any knowledge about the object we are focused on, since it can catch the features automatically. The idea of SVM can be easily generalized to nonlinear model by a mapping from the original space to a high-dimensional feature space, and they construct a max-margin linear classifier in the high dimensional feature space.

This thesis will investigate the basic idea of SVM and apply its application to multi-class case, that is, one vs. all and one vs. one strategies. As an application, we will apply the scheme to the problem of handwritten digital recognition. We show the difference of performance between linear and non linear technology, in terms of confounding matrix and running time.

KEYWORDS: Support Vector Machine; Multi-class Classification; Linear Model; Non-Linear Classifier; Kernel function

This abstract is approved as to form and content

Songfeng Zheng
Chairperson, Advisory Committee
Missouri State University

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Approved:

Dr. Songfeng Zheng, Chairperson

Dr. George Mathew, Member

Dr. Yingcai Su, Member

Dr. Julie J. Masterson, Graduate College Dean

In the interest of academic freedom and the principle of free speech, approval of this thesis indicates the format is acceptable and meets the academic criteria for the discipline as determined by the faculty that constitute the thesis committee. The content and views expressed in this thesis are those of the student-scholar and are not endorsed by Missouri State University, its Graduate College, or its employees.

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TABLE OF CONTENTS

1.	INTRODUCTION	1
1.1.	Background	1
1.2.	Topic	2
1.3.	Thesis Organization	2
2.	STATISTICAL THEORY IN SVM	3
2.1.	Support Vector Machine	4
2.2.	Linear SVM	5
2.3.	Non Linear SVMs	10
3.	EXPERIMENTAL DESIGN	15
3.1.	Sample Space	15
3.2.	One v.s. All	16
3.3.	One v.s One	18
4.	EXPERIMENTAL RESULT	20
4.1.	Accuracy	20
4.2.	Confusion Table	22
4.3.	Time	27
5.	DISCUSSION	28
5.1.	Conclusion	28
5.2.	Advantage and Disadvantage	29
5.3.	Future Work	30
	REFERENCES	31

LIST OF TABLES

Table 1.	The Result of Accuracy from 8 Experiments	20
Table 2.	The Effect of The Parameters	22
Table 3.	Confusion Table of Linear Kernel Based on One vs All	22
Table 4.	Confusion Table of Linear Kernel Based on One vs One	23
Table 5.	Confusion Table of Polynomial Kernel Based on One vs All	23
Table 6.	Confusion Table of Polynomial Kernel based on One vs One	24
Table 7.	Confusion Table of Guassian Kernel based on One vs All	24
Table 8.	Confusion Table of Guassian Kernel based on One vs One	25
Table 9.	Confusion Table of Sigmoid Kernel Based on One vs All	25
Table 10.	Confusion Table of Sigmoid Kernel Based on One vs One	26
Table 11.	The Rank of Algorithm	29

LIST OF FIGURES

Figure 1.	Two Different Cases in SVMs	3
Figure 2.	Best Selection	3
Figure 3.	Key Ideas in SVMs	4
Figure 4.	The effect of removing support vector	5
Figure 5.	The hyperplane	6
Figure 6.	Data Transformation	10
Figure 7.	The hyperplane of Figure 6a	11
Figure 8.	The hyperplane of Figure 1b	11
Figure 9.	One of Sample Unit	16
Figure 10.	The Average Accuracy on Each Experiment	21
Figure 11.	The Time Spend on Training and Recognizing	27

1. INTRODUCTION

Support vector machine(SVM) constructs a hyperplane or set of hyperplanes in a high- or infinite-dimensional space, which can be used for classification, regression, or other tasks like outliers detection[1]. In this thesis, we will apply SVM for the task of classification, which is still a cardinal part in pattern recognition.

1.1 Background

In 1936, Ronald Aylmer Fisher invented the first pattern classification algorithm[2], linear discriminant analysis(LDA). It assumes that the data points have the same covariance and normally distributed. Thus, it could have a very well performance only when the assumptions of data are met. However, the assumptions are not always true in the real world, which causes the limitation of LDA.

In 1963, The original linear SVM algorithm was created by Vladimir N. Vapnik and Alexey Ya. Chervonenkis[3], which came into being. since SVM doesn't have any assumptions on data itself, it gives people a lot of freedom to manipulate.

Since SVM is trying to find a hyperplane between the different groups, it doesn't work very well in certain situations. To increasing the ability of SVM, Bernhard E. Boser, Isabelle M. Guyon and Vladimir N. Vapnik[4] suggested a way to create nonlinear classifiers by applying the kernel trick to maximum-margin hyperplanes in 1992, which perfectly solved this issue. There are more explanations in Chapter 2 about linear, nonlinear and kernel.

SVM has been applied to many real-life problems, such as, face detection, handwriting recognition, image classification, Bioinformatics etc[5]. In this thesis, we will apply SVM to handwritten digital recognition with different methods, understanding SVM and solving the problem.

1.2 Topic

This is an era of digital information. Everything is related to data, computer, and Internet. However, we still need to handwrite everyday, for example, doing your math homework, taking a phone number for your colleague and writing a letter to your grandma. Can we just save them as an image? Is there any way we can reformat them word by word into a digital file automatically?

In above scenarios, handwritten digital recognition may not crucially important. But it does help a lot in certain situations. Still a letter, image how many mails your local post office will receive, how much effort the post officer need to spend so that every mail in the right distributor and how depressed the worker will be after doing this job.

If a machine can read the zip code from the envelop, it could send the mail to the right place by the code. So, my goal is helping the computer read and understand the zip code on the envelope.

1.3 Thesis Organization

In the following chapter, we will explain the Statistical Theory in SVM and give some figures to identify the conceptions. Also, we will discuss two kinds of SVMs, Linear and Nonlinear, including the similarity and differences between them.

Chapter 3 gives general information to the experiment, including the data and two methods for multi-class classification. The final experimental results and analysis will be presented in Chapter 4. In the last Chapter, we will draw the conclusion of experiment and expect the future work.

2. STATISTICAL THEORY IN SVM

How to separate two different groups mathematically? Intuitively, we need to find the margin of each group, then draw a line (hyperplane in high dimension) between two closed margins so that it can separate them clearly. It may occur in two cases. Case 1, the line could be a straight line. Case 2, the line has to be a curve. See Fig. 1 for an example. The goal is to find the function of line, which is the classifier we need.

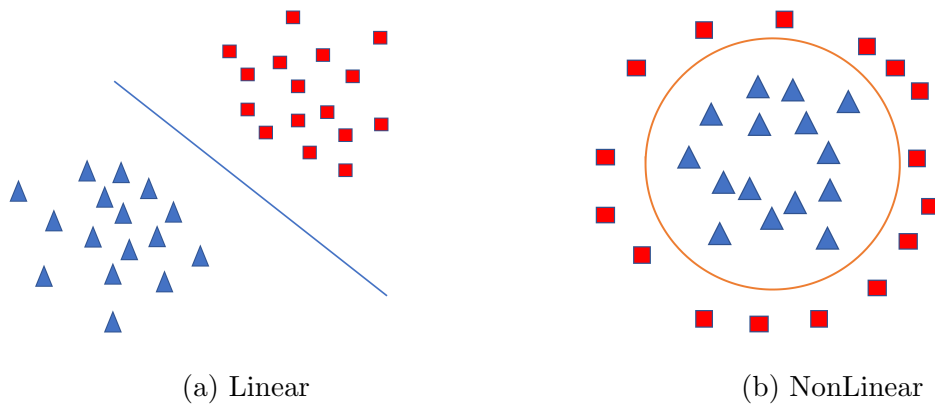


Figure 1: Two Different Cases in SVMs

Usually, there are several possible hyperplanes satisfied this requirement, see the illustration in Fig. 2. Hence, there is naturally a question: which classifier is the best? We will give the answer in the next chapter.

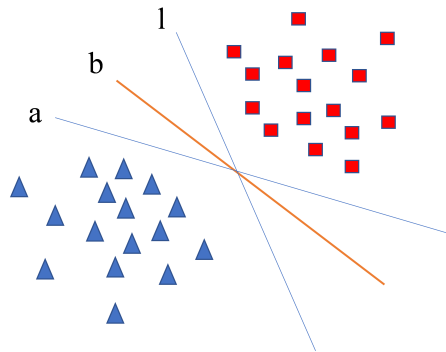


Figure 2: Best Selection

2.1 Support Vector Machine

Support Vector Machines can find the best option to separate two different groups by maximizing the margin between the separating hyperplane. The function of hyperplane, which is called as the classifier, is usually specified by a small subset of training set, which are the support vectors. See Figure 3.

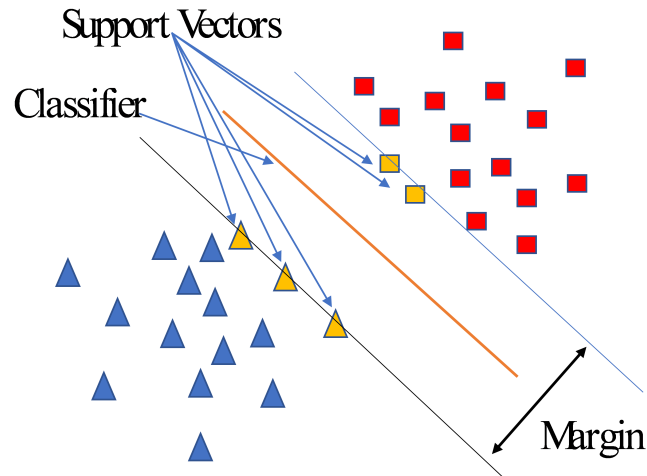


Figure 3: Key Ideas in SVMs

In this thesis, we assume all the training sample come from a determined distribution D . We also assume that there exists a known *Boolean* value corresponding to each sample [6], e.g, $\{x_1, x_2, x_3, \dots, x_n, y\}$, denoted as (\mathbf{x}, \mathbf{y}) . Here, x_i represents the feature of the sample, n is the number of the features, y is the *Boolean* value *True*(1) or *Flase*(-1), \mathbf{x} and \mathbf{y} are vector notations. As such, the units of a group will have a same label, which is the label of the group as well. We don't have any special requirements for the distribution of \mathbf{x} and \mathbf{y} , which is different from with *LDA*. To make our theory easier to accept, we assume the training sample can be separated linearly for now.

2.2 Linear SVM

Linear model is the basis of SVM. Thus, we start with the linear model and explore the idea behind SVM.

Support Vector

Figure 3 shows that the Support Vectors are the data sample lie on (or closed to)¹ the boundary. Here, we can define *support vector* as the elements of the training set that would change the position of the optimal hyperplane if removed, which are the critical elements on training set [7]. Be careful, not every support vector will affect the hyperplane if it is removed. But the other points that are not support vectors will definitely not affect the hyperplane at all even if they are removed.

In Figure 4a, we can clearly see that the hyperplane won't change at all when square p is removed. But the hyperplane m will rotate to m' , and the boundary b_1 and b_2 will follow to b'_1 and b'_2 when the square q is removed in Figure 4b. This illustration explains and proves the definition. On the other hand, non-support vectors don't contribute to the classifier is obviously true from Figure 4.

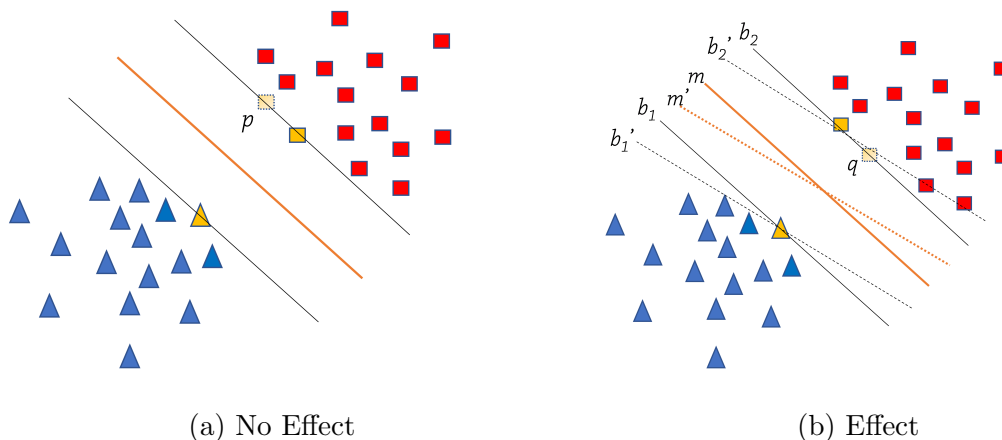


Figure 4: The effect of removing support vector

¹Sometimes the data sample not exactly on but very close to the boundary, and they are also counted as Support Vectors. It will be use in the experiment.

Hyperplane

Now, we define the hyperplane H such that

$$\begin{cases} \mathbf{w}\mathbf{x}_i + b \geq +1, & \text{when } y_i = +1 \\ \mathbf{w}\mathbf{x}_i + b \leq -1, & \text{when } y_i = -1 \end{cases} \quad (2.1)$$

As Figure 5 shows, H_1 and H_2 are the planes

$$H_1 : \mathbf{w}\mathbf{x}_i + b = -1, \quad (2.2)$$

and

$$H_2 : \mathbf{w}\mathbf{x}_i + b = +1. \quad (2.3)$$

H is the classifier:

$$\mathbf{w}\mathbf{x}_i + b = 0. \quad (2.4)$$

In these equations, \mathbf{x}_i is a data point, \mathbf{w} is the coefficients of \mathbf{x}_i . Both \mathbf{x}_i and \mathbf{w} are vectors, and b is a constant.

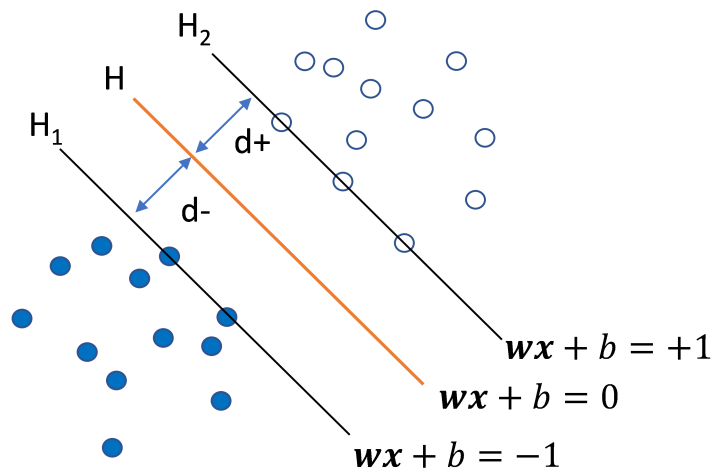


Figure 5: The hyperplane

Distance

Figure 5 is a projection from the high dimension space. H_1 , H_2 and H are the hyperplanes, the solid and empty dots stand for the sampling units. That's why the different sampling units have the same label value (-1 or $+1$) even they are in distinct locations. From Figure 5, we can tell the points on the plane H_1 and H_2 are the support vectors from the previous definition. The points below or on the plane H_1 are considered as one group. Similarly, we regard the points above or on the plane H_2 as another group. d represents the width of the margin, which is expected to be as big as possible, since in this case, H will separate two groups more clearly. To make it fair for both groups, we think H is in the middle of margin area and parallel to the boundaries.

It is well known that the distance from a point (x_0, y_0) to a line $Ax+By+c=0$ is

$$d = \frac{|Ax_0 + By_0 + c|}{\sqrt{A^2 + B^2}}. \quad (2.5)$$

Thus, the distance between the support vectors on H_1 and H_0 is

$$d = \frac{|\mathbf{w}\mathbf{x} + b|}{\sqrt{\mathbf{w}^2}} \quad (2.6)$$

Since the points on plane H_1 satisfy $\mathbf{w}\mathbf{x} + b = -1$, and $\sqrt{\mathbf{w}^2} = \|\mathbf{w}\|$, then

$$d = \frac{|-1|}{\|\mathbf{w}\|} = \frac{1}{\|\mathbf{w}\|} \quad (2.7)$$

Thus, the width of the margin is $\frac{2}{\|\mathbf{w}\|}$.

Optimization

As we stated in the beginning of chapter, our goal is to maximize the width of the margin, equivalently, minimizing $\|\mathbf{w}\|$ such that the discriminant boundary is

obeyed. Notice that the equation (2.1) could combine into a simpler form:

$$y_i(\mathbf{w}\mathbf{x}_i + b) \geq 1 \quad \forall i = 1, 2, 3, \dots, n \quad (2.8)$$

Minimizing $\|\mathbf{w}\|$ is the same as

$$\min \frac{1}{2}\|\mathbf{w}\|^2 \quad (2.9)$$

As such, the problem is converted to solve the minimization problem in (2.9) with the constrained inequality in (2.8). Since polynomial (2.9) is quadratic with positive coefficients, there exist a minimum.

According to *Karush–Kuhn–Tuckerconditions(KTT)* [8], the solution of problem 2.9 will be the saddle point of

$$L_p = \frac{1}{2}\|\mathbf{w}\|^2 - \sum_{i=1}^n \alpha_i [y_i(\mathbf{w}\mathbf{x}_i + b) - 1]. \quad (2.10)$$

Here, α_i is a nonnegative constant correspond to the nonnegative Lagrange multiplier¹. However, we don't like α too big as to make one vector has too much weight, we say it is less than a positive constant c . Since this is a convex function, the minimal point is the saddle point. At the this point, we take the partial derivative with respect to \mathbf{w} , b and α_i , then we have

$$\frac{\partial L_p}{\partial \mathbf{w}} = \mathbf{w} - \sum_{i=1}^n \alpha_i y_i \mathbf{x}_i = 0, \quad (2.11)$$

$$\frac{\partial L_p}{\partial b} = - \sum_{i=1}^n \alpha_i y_i = 0, \quad (2.12)$$

$$\frac{\partial L_p}{\partial \alpha_i} = y_i(\mathbf{w}\mathbf{x}_i + b) - 1 = 0 \quad \forall i = 1, 2, \dots, n. \quad (2.13)$$

¹The constrains form into multipliers.

From equation 2.13, we can conclude that the sampling units must be on the hyperplane H_1 or H_2 when L_P has the minimal. Also, we can get from equation 2.11 and 2.12

$$\mathbf{w} = \sum_{i=1}^n \alpha_i y_i \mathbf{x}_i \quad \sum_{i=1}^n \alpha_i y_i = 0 \quad (2.14)$$

Next, we substitute equation (2.14) into 2.9 and get

$$\begin{aligned} \min L_p &= \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j \mathbf{x}_i \mathbf{x}_j - \sum_{i=1}^n \alpha_i y_i (\mathbf{x}_i \sum_{j=1}^n \alpha_j y_j \mathbf{x}_j + b) + \sum_{i=1}^n \alpha_i \\ &= \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j \mathbf{x}_i \mathbf{x}_j - \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j \mathbf{x}_i \mathbf{x}_j - b \sum_{i=1}^n \alpha_i y_i + \sum_{i=1}^n \alpha_i \\ &= -\frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j \mathbf{x}_i \mathbf{x}_j + \sum_{i=1}^n \alpha_i \\ &\quad s.t. \quad \alpha_i \geq 0 \quad \forall \quad i = 1, 2, \dots, n. \end{aligned} \quad (2.15)$$

Equivalently, the above minimum can be substituted by

$$\max L_D(\alpha_i) = \sum_{i=1}^n \alpha_i - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j \mathbf{x}_i \mathbf{x}_j \quad (2.16)$$

$$s.t. \quad \sum_{i=1}^n \alpha_i y_i = 0 \quad \text{and} \quad c \geq \alpha_i \geq 0 \quad \forall i = 1, 2, \dots, n. \quad (2.17)$$

Notice that we have removed the dependence from \mathbf{w} and b into α_i . After we find every α_i , we can get \mathbf{w} from the equation 2.14, b from 2.13 with the limitation $c \geq \alpha_i \geq 0$.

The maximal of Eq. (2.16) can be easily found by the quadratic programming. Of course, we don't want to do this problem by hand, since there are tons of simulations, manipulations and iterations. This procedure can be done by the package CVXOPT² in Python.

²CVXOPT is a free software package for convex optimization based on the Python programming language.

After we get \mathbf{w}^* and b^* , the function of classifier will become

$$Boolean = sign(\mathbf{w}^* \mathbf{x} + b^*) \quad (2.18)$$

How to use this classifier when we have a unlabeled test sample unit? First, plug in the unit into equation 2.18. Then, the output of the function is "-1" or "1". Recall the beginning of the chapter, we mentioned each group have a unique *Boolean* value as the label. Then this unit will belong to the group which has the same label.

2.3 Non Linear SVMs

However, the assumption that two groups are linearly separable is not always true in real life. How can we deal with those cases? This section will give the solution. Still, we are going to start with two dimensional cases then generalize the conclusion to the high dimensional space.

Transformation

In this section, the key ideal is the data transformation. In Figure 6a, even though the boundary is not linear, we can use some function $\phi(\mathbf{x}_i)$ so that inputting the original data sample will output the linearly separable data set as Figure 6b.

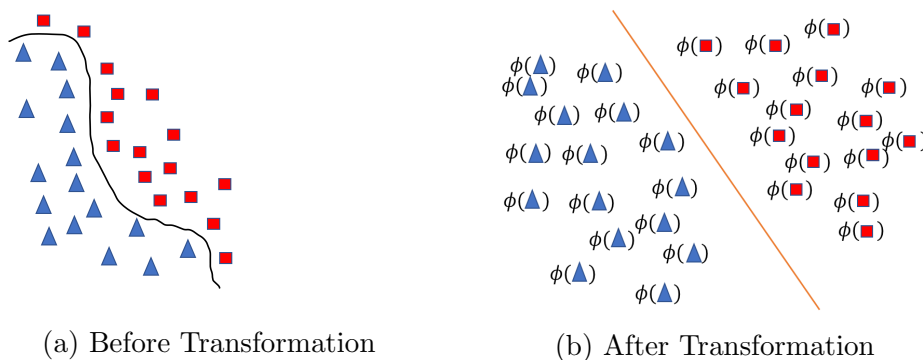


Figure 6: Data Transformation

So, what's the function of $\phi(\mathbf{x}_i)$ then? Particularly, we can set the hyperplane in a X - Y coordinate system in Figure 7, X and Y are the data features. Then, we obtain

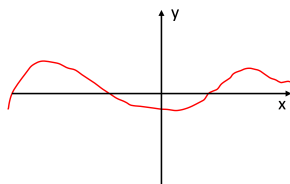


Figure 7: The hyperplane of Figure 6a

There are 4 intersection point between the separating line and X -axis. So, we can assume

$$x_2 = (x_1 - r_1)(x_1 - r_2)(x_1 - r_3)(x_1 - r_4) \quad (2.19)$$

Here, r_1, r_2, r_3, r_4 are the roots of $\phi(x_1)$.

Recall the Figure 1b, we can have the separating hyperplane as Figure 8.

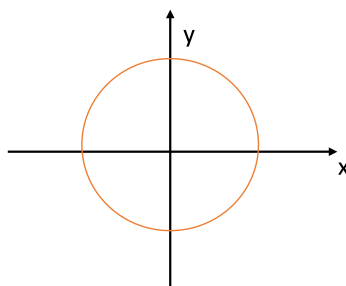


Figure 8: The hyperplane of Figure 1b

Since the hyperplane is a circle, more general, we can consider it as an ellipse. So, we can assume

$$1 = \sqrt{\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2}} \quad (2.20)$$

Optimization

Recall, the Lagrange Function 2.10 where $\mathbf{x}_i\mathbf{x}_j$ is the dot product of two feature vectors. Since we transfer the feature vector \mathbf{x} to $\phi(\mathbf{x})$, we need to substitute $\mathbf{x}_i\mathbf{x}_j$ to $\phi(\mathbf{x}_i)\phi(\mathbf{x}_j)$ as well, which is a function of \mathbf{x}_i and \mathbf{x}_j . There exists a “kernel function” K [9] such that $K(\mathbf{x}_i, \mathbf{x}_j) = \phi(\mathbf{x}_i)\phi(\mathbf{x}_j)$.

There are three popular kernel functions used in SVMs as the following.

First, *polynomial* Kernel:

$$K(\mathbf{x}_i, \mathbf{x}_j) = (\mathbf{x}_i^T \mathbf{x}_j + 1)^p \quad (2.21)$$

Second, *radial-basis* Kernel:

$$K(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|}{2\sigma^2}\right) \quad (2.22)$$

Third, *Sigmoid* Kernel:

$$K(\mathbf{x}_i, \mathbf{x}_j) = \tanh(\beta_0 \mathbf{x}_i^T \mathbf{x}_j + \beta_1) \quad (2.23)$$

Here, T stands for the transpose of a vector, p , σ , β_0 and β_1 are all user defined parameters. We can adjust the parameters in order to get a better result during the actual experiment. In particular, the function 2.21 is associated with equation 2.19 and the function 2.22 is correspond to 2.20.

Follow the procedure like what we did in the linear case, we just substitute \mathbf{x} to $\phi(\mathbf{x})$, then we will get the similar Lagrangian function:

$$L_p = \frac{1}{2} \|\mathbf{w}\|^2 - \sum_{i=1}^n \alpha_i [y_i(\mathbf{w}\phi(\mathbf{x}_i) + b) - 1]. \quad (2.24)$$

Here, α_i is a nonnegative constant correspond to the nonnegative Lagrange multi-

plier and less than a positive constant c . Since this is a convex function, the minimal point is the saddle point. At the this point, we take the partial derivative with respect to \mathbf{w} , b and α_i , yielding

$$\frac{\partial L_p}{\partial \mathbf{w}} = \mathbf{w} - \sum_{i=1}^n \alpha_i y_i \phi(\mathbf{x}_i) = 0, \quad (2.25)$$

$$\frac{\partial L_p}{\partial b} = - \sum_{i=1}^n \alpha_i y_i = 0, \quad (2.26)$$

$$\frac{\partial L_p}{\partial \alpha_i} = y_i(\mathbf{w}\phi(\mathbf{x}_i) + b) - 1 = 0 \quad \forall i = 1, 2, \dots, n. \quad (2.27)$$

From equation 2.27, we can easily conclude that the sampling units must be on the hyperplane H_1 or H_2 when L_P has the minimal. We can get from equation 2.25 and 2.26 that

$$\mathbf{w} = \sum_{i=1}^n \alpha_i y_i \phi(\mathbf{x}_i) \quad \sum_{i=1}^n \alpha_i y_i = 0 \quad (2.28)$$

Next, we substitute equation 2.14 into 2.9 and get

$$\begin{aligned} \min L_p &= \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j \phi(\mathbf{x}_i) \phi(\mathbf{x}_j) - \sum_{i=1}^n \alpha_i y_i (\phi(\mathbf{x}_i) \sum_{j=1}^n \alpha_j y_j (\phi(\mathbf{x}_j) + b) + \sum_{i=1}^n \alpha_i \\ &= \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j \phi(\mathbf{x}_i) \phi(\mathbf{x}_j) - \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j \phi(\mathbf{x}_i) \phi(\mathbf{x}_j) - bn \sum_{i=1}^n \alpha_i y_i + \sum_{i=1}^n \alpha_i \\ &= \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j K(\mathbf{x}_i, \mathbf{x}_j) - \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j K(\mathbf{x}_i, \mathbf{x}_j) + \sum_{i=1}^n \alpha_i \\ &= -\frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j K(\mathbf{x}_i, \mathbf{x}_j) + \sum_{i=1}^n \alpha_i \end{aligned}$$

$$s.t. \quad c \geq \alpha_i \geq 0 \quad \forall i = 1, 2, \dots, n$$

Equivalently, the above minimization problem can be rewritten as

$$\max L_D(\alpha_i) = \sum_{i=1}^n \alpha_i - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j K(\mathbf{x}_i, \mathbf{x}_j) \quad (2.29)$$

$$s.t. \sum_{i=1}^n \alpha_i y_i = 0 \text{ and } c \geq \alpha_i \geq 0 \forall i = 1, 2, \dots, n$$

So, the non linear SVMs is similar to linear SVM, and the only difference is that nonlinear SVM uses different kernel functions rather than the original inner product $\mathbf{x}_i \mathbf{x}_j$. Hence, we can still find every α_i by the CVXOPT package.

As equation 2.18 shows, we can determine the label of the testing unit \mathbf{z} in a way similar to linear SVM. First of all, we need transform (\mathbf{z}) to $\phi(\mathbf{z})$, then the classifier would be

$$Boolean = sign(\mathbf{w}^* \phi(\mathbf{z}) + b^*) \tag{2.30}$$

$$= sign\left(\sum_{i=1}^n \alpha_i y_i \phi(\mathbf{x}_i) \phi(\mathbf{z}) + b^*\right) \tag{2.31}$$

$$= sign\left(\sum_{i=1}^n \alpha_i y_i K(\mathbf{x}_i, \mathbf{z}) + b^*\right) \tag{2.32}$$

After we have the *boolean* value, the same determination will be applied here with the linear case.

3. EXPERIMENTAL DESIGN

In this chapter, we are going to apply the technology we have introduced in the last chapter into a real world problem, handwritten digital recognition. The theory of last chapter can only apply for two-classes recognition, but the digital recognition is a matter of ten classes. Hence we have to divide this multi-classes problem into several two-classes cases, then solve them by some inner algorithm. Two main technologies will be demonstrated in the following, they are similar in the ideal but different in the performance.

3.1 Sample Space

First of all, we need to learn the property of the data in order to get a better result. The sample comes from the zip code on the envelopes³, separated into two subsets, training set and testing set. All of them have been converted from the image with dimension 16*16 pixel as Figure 9 shows into a min-max normalization[10] vector with its label as the first element of this vector. e.g.

$$[5., -1., -1., -1., -0.813, \dots, -0.671, -0.095, -0.671, -0.828, -1.]$$

Here, “5” means that this vector is the representation of digit 5, all the numbers behind are the gray-scale values of a pixel after min-max normalization, which have the minimum 0 and maximum 255 so that we can apply this normalized technology here. In addition, “0” corresponds to all black color and “255” stands for all white color. That’s why the length of this vector is $257 = 1 + 16 \times 16$.

Secondly, how to use this sample space? As the name suggests, the training sample is for the purpose of training algorithm and find the classifiers. And testing

³These data were kindly made available by the neural network group at AT&T research labs (thanks to Yann Le Cunn).

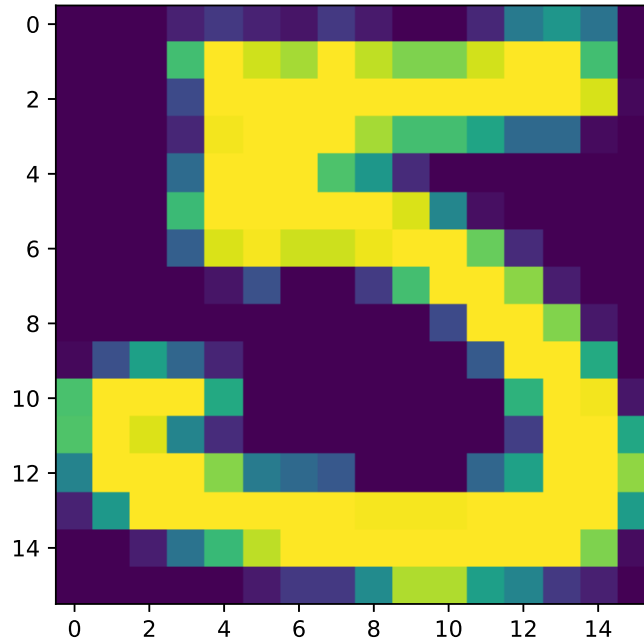


Figure 9: One of Sample Unit

sample is for testing if our classifiers are sufficient and how well it perform. Usually, the training sample has the bigger size than the testing one. In our case, they have 7291 and 2007 sample units, respectively.

3.2 One v.s. All

As two technologies we mentioned at the beginning of this chapter, let's start with an easier, common and more acceptable one, One v.s. All. Here are the following steps to establish this whole process:

Step 1: Classify the sampling units by their labels into 10 subsets;

Step 2: Randomly⁴ select 100 sample units from each subset;

Step 3: Make the first two-class recognition, one class is the subset including all the

⁴The uniform distribution, every sample unit has the equal probability to be selected.

units with label “0” and reset the label as “+1”; another one (not 0) is the combination of 100 sample units from the rest of subsets and reset the label of combination as “-1”;

Step 4: Run the Linear SVMs algorithm with two classes from step 3 and get the parameter \mathbf{w}_0 and b_0 ;

Step 5: Make the second two-classes recognition, one class is the subset including all the units with label “1” and reset the label as “+1”; another one (not 1) is the combination of 100 sample units from the rest of subsets and reset the label of combination as “-1”;

Step 6: Run the Linear SVMs algorithm between two classes from the last step and get the parameter \mathbf{w}_1 and b_1 ;

Step 7: Similarly, run the last two step for “2” and “not 2”, “3” and “not 3”, ..., “9” and “not 9”, and get \mathbf{w}_2 and b_2 , ..., \mathbf{w}_9 and b_9

Step 8: Obtain 10 classifiers if the testing unit is \mathbf{x} ,

$$y_0 = \mathbf{w}_0\mathbf{x} + b_0 \tag{3.33}$$

$$y_1 = \mathbf{w}_1\mathbf{x} + b_1 \tag{3.34}$$

$$\vdots \quad \vdots \quad \vdots \tag{3.35}$$

$$y_9 = \mathbf{w}_9\mathbf{x} + b_9 \tag{3.36}$$

Step 9: Plug in a testing sample unit into the 10 classifiers, we will get one output from each one;

Step 10: Compare among all 10 outputs and find the maximum of them. Then the testing unit will belong to the group with the label with the subscript of the classifier who give the largest output;

Step 11: Change Linear SVMs into Non Linear SVMs with three different kernel function and run the same algorithm three times;

Step 12: Summarize the accuracy of recognizing of each digit from each algorithm.

The above steps complete the One v.s. All process of SVMs. Nobody knows if the two-classes are linearly separable specially when they are in high dimensional space, so we will try them all and find the better one. In addition, the above experiment involved some parameters need to be specified by the designer ahead. As such, some experience may required when we adjust the parameters.

3.3 One v.s One

One v.s One technology will involve another two mathematical tricks, combinatorics and voting. Before we go to the main topic, let's get familiar with them first. The number of ways we can chose two objects from 10 is $\binom{10}{2} = \frac{10(10-1)}{2} = 45$, this is the combinatorics we need to use later.

Assume there are 10 wrestlers in the battlefield and there is a fight between every two of them, only win or lose, not tie. There are 45 fights in total. How can we decide which one the best most likely is after all the fights done? A reasonable rule is called *voting process*, that is, a wrestler will earn one point if he wins, and his points will not be deducted if he loses and the points can be accumulated; finally, the wrestler who accumulates the most points will be the best.

After introducing those two definition, we are ready to design the process of One v.s One, which is sketched as below

Step 1: Classify the sampling units by their labels into 10 subsets;

Step 2: Randomly select 520 sample units from each subset as 10 new subsets;

Step 3: Pick two of them from 10 new subsets, note Φ_i and Φ_j . Here i and j are the original labels of two subsets. Reset the label of Φ_i as “+1” and Φ_j ’s as “-1”;

Step 4: Run linear SVMs algorithm using Φ_i and Φ_j , get the parameters \mathbf{w}_{ij} and b_{ij} ;

Step 5: Repeat the last two step for any two of subsets without the duplication.

Step 6: Obtain 45 classifiers if the testing unit is \mathbf{z} , then

$$y_{ij} = \mathbf{w}_{ij}\mathbf{z} + b_{ij} \quad i < j \in \{0, 1, \dots, 9\} \quad (3.37)$$

Step 7: We will get one output from every classifier and record them;

Step 8: Run the voting process for the testing sample unit by counting “+1” as the subset Φ_i win and “-1” as the subset Φ_i lose;

Step 9: Counting the score for each subset, then the unit will belong to the subset who has the biggest score;

Step 10: Change linear SVMs into nonlinear SVMs with three different kernel functions and run the same algorithm three times;

Step 11: Summarize the accuracy of recognizing of each digit from each algorithm.

The above steps complete the One v.s. One process of SVMs. Even though it has shorter steps than the process of One v.s. All, it has much more computation than the latter. During the manipulation, finding the classifier spends most of time. Obviously, this process need $\binom{10}{2}$ classifiers but the last one only need 10, and this difference will become more distinguishable if there are more classes. Moreover, the voting processing takes longer than finding the maximum. Hopefully, this process can give us the better result to make our extra work paid off.

4. EXPERIMENTAL RESULT

This chapter verifies our theory and the experimental design. The results are obtained from a laptop computer with Window 10 64-bit operating system, 8.00GB RAM with the processor Inter(R) Core(TM) i7-6500U CPU @2.50GHz 2.6GHz, x64-based. We will do comparisons between linear and nonlinear models, with One v.s. One and One v.s. All strategies.

4.1 Accuracy

Table 1 gives the accuracy result from 8 different experimental designs. It shows the results from the two strategies that were stated in the last chapter, One vs One and One vs All. Each course separated as two sections, linear and nonlinear. Particularly, nonlinear section has three cases stand for its own kernel functions, polynomial, Gaussian and sigmoid.

Table 1: The Result of Accuracy from 8 Experiments

Digit	1 vs. All				1 vs. 1			
	linear	Poly	nonlinear Gaussian	Sigmoid	linear	Poly	nonlinear Gaussian	Sigmoid
0	0.891	0.944	0.983	0.939	0.969	0.978	0.983	0.964
1	0.913	0.977	0.962	0.955	0.973	0.981	0.966	0.977
2	0.753	0.874	0.884	0.747	0.864	0.616	0.904	0.864
3	0.880	0.873	0.892	0.855	0.880	0.940	0.867	0.855
4	0.790	0.915	0.945	0.915	0.910	0.845	0.945	0.920
5	0.763	0.906	0.919	0.825	0.831	0.756	0.931	0.888
6	0.912	0.959	0.929	0.929	0.953	0.988	0.947	0.947
7	0.878	0.973	0.918	0.891	0.891	0.912	0.925	0.925
8	0.801	0.723	0.916	0.837	0.873	0.892	0.934	0.867
9	0.966	0.927	0.960	0.949	0.966	0.944	0.960	0.960

The above Table represents the percentage of algorithm recognized the correct digit from the testing sample. The first column are from 0 to 9, which are the ten classes in our experiments, e.g., “0.891” means that the algorithm has the accu-

racy of 89.1% when it is recognizing the digits 0 in *One vs All* design with the normal *linear kernel*. Hence, a bigger value implies better classification performance.

In general, the algorithm has a relatively good average accuracy on most of experiments as what the following Figure 10 shows. Most of them are around 90%, which is decent. Since this is handwritten digits not a printed body, even the human will get confused on certain curves, it is acceptable. However, the polynomial kernel gave a weak result on *One vs One* design.

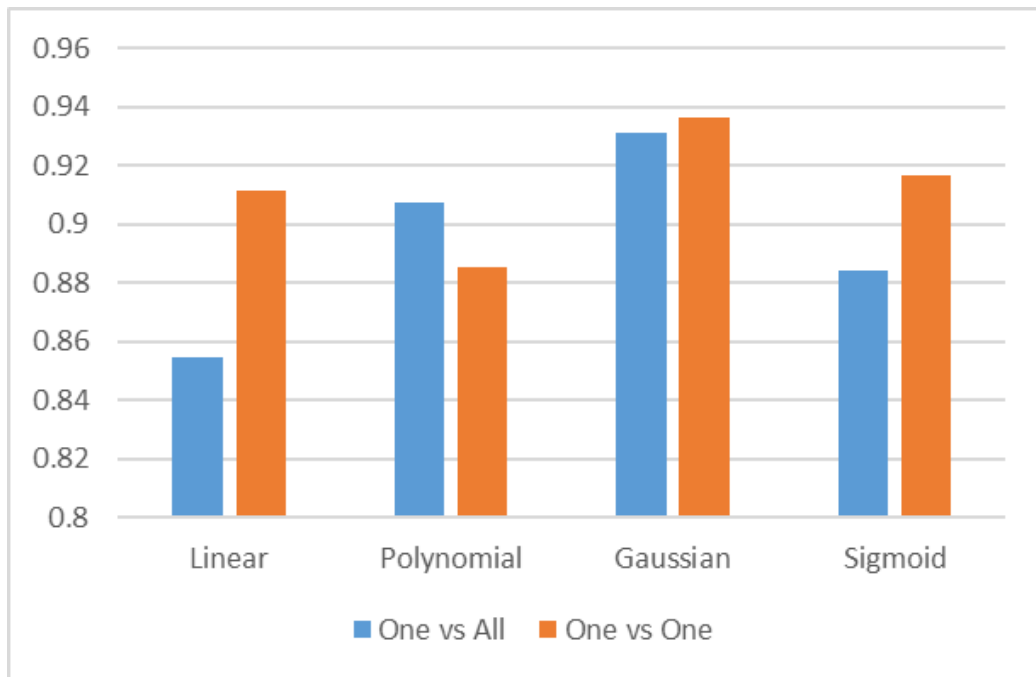


Figure 10: The Average Accuracy on Each Experiment

The above result are based on some specific parameters, which play a crucial role in this experiment. On certain ones, some accuracy are even 0. Table 2 shows the difference base on *Gaussian kernel* in *One vs All* design.

From Table 2, we notice that there are huge difference in accuracy between case 1 and 3 even though they just have a slight different in σ . In case 2 and 3, there is a little difference on accuracy since the different ϵ is minor. Hence, the parameters did effect the results a lot, but the variation may more or less. Therefore, this experiment does require the some experience to adjust the parameters.

Table 2: The Effect of The Parameters

The Effect of the Parameters												
	Parameters		Accuracy									
	ϵ	σ	0	1	2	3	4	5	6	7	8	9
case 1	1.0E-07	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
case 2	1.0E-05	10	0.99	0.97	0.87	0.87	0.94	0.91	0.95	0.92	0.90	0.95
case 3	1.0E-07	10	0.98	0.96	0.88	0.89	0.95	0.92	0.93	0.92	0.92	0.96

4.2 Confusion Table

In this section, we are going to look inside to find what mistake the algorithm made and if they are reasonable. By given Table 3 to 10, which are the confusion matrices of each algorithm, it will be much more easier to investigate what decision the algorithm had made in detail.

Table 3: Confusion Table of Linear Kernel Based on One vs All

		Output Digits									
		0	1	2	3	4	5	6	7	8	9
Actual Digits	0	320	0	3	7	4	2	13	0	7	3
	1	0	241	1	2	8	0	4	1	2	5
	2	2	0	149	10	14	1	3	2	14	3
	3	1	0	3	146	0	6	1	2	3	4
	4	0	1	3	0	158	0	5	2	4	27
	5	2	0	1	14	4	122	3	1	7	6
	6	2	0	5	0	4	2	155	0	2	0
	7	0	0	0	2	8	0	0	129	0	8
	8	2	0	4	5	5	7	4	2	133	4
	9	0	0	0	0	3	0	0	2	1	171

Table 4: Confusion Table of Linear Kernel Based on One vs One

		Output Digits									
		0	1	2	3	4	5	6	7	8	9
Actual Digits	0	348	0	1	0	3	0	5	1	1	0
	1	0	257	1	2	1	0	3	0	0	0
	2	4	1	171	6	4	2	3	3	4	0
	3	4	0	2	146	0	8	0	1	4	1
	4	1	3	5	0	182	1	2	1	1	4
	5	4	0	1	12	2	133	2	0	2	4
	6	0	0	2	0	2	2	162	0	2	0
	7	1	0	1	0	6	0	0	131	0	8
	8	4	0	2	3	4	7	0	1	145	0
	9	0	0	0	0	3	1	0	2	0	171

Table 5: Confusion Table of Polynomial Kernel Based on One vs All

		Output Digits									
		0	1	2	3	4	5	6	7	8	9
Actual Digits	0	339	3	1	0	3	1	5	6	0	1
	1	0	258	0	1	2	0	2	1	0	0
	2	0	2	173	5	3	0	1	13	1	0
	3	0	0	1	145	0	12	0	6	0	2
	4	0	4	2	0	183	1	1	5	0	4
	5	1	1	0	3	1	145	0	5	0	4
	6	0	1	1	0	4	1	163	0	0	0
	7	0	0	0	0	4	0	0	143	0	0
	8	2	5	3	8	3	5	3	17	120	0
	9	0	2	0	0	1	0	0	10	0	164

Table 6: Confusion Table of Polynomial Kernel based on One vs One

		Output Digits									
		0	1	2	3	4	5	6	7	8	9
Actual Digits	0	342	12	1	0	0	0	2	2	0	0
	1	0	261	0	0	3	0	0	0	0	0
	2	4	107	75	2	2	0	1	5	2	0
	3	2	25	0	137	0	1	0	0	0	1
	4	0	40	0	0	157	0	1	1	0	1
	5	4	79	0	19	1	53	0	3	0	1
	6	1	35	1	0	2	0	131	0	0	0
	7	0	12	1	0	3	0	0	131	0	0
	8	3	35	0	6	2	2	0	10	108	0
	9	0	12	0	0	2	1	0	17	0	145

Table 7: Confusion Table of Guassian Kernel based on One vs All

		Output Digits									
		0	1	2	3	4	5	6	7	8	9
Actual Digits	0	353	0	2	0	3	0	0	0	0	1
	1	0	254	0	2	3	0	3	1	1	0
	2	3	0	175	3	8	1	1	1	6	0
	3	2	0	4	148	0	7	0	0	4	1
	4	0	2	3	0	189	0	1	1	0	4
	5	3	0	0	4	1	147	0	0	1	4
	6	3	0	3	0	3	2	158	0	1	0
	7	0	0	2	0	6	0	0	135	1	3
	8	2	0	2	5	1	3	0	0	152	1
	9	0	0	0	0	3	1	0	1	2	170

Table 8: Confusion Table of Guassian Kernel based on One vs One

		Output Digits									
		0	1	2	3	4	5	6	7	8	9
Actual Digits	0	353	0	2	0	2	0	0	0	2	0
	1	0	255	0	1	3	0	4	0	0	1
	2	2	0	179	3	5	2	0	1	6	0
	3	1	0	4	144	0	11	0	0	5	1
	4	0	1	3	0	189	1	1	1	1	3
	5	3	0	0	3	1	149	0	0	1	3
	6	0	0	3	0	2	3	161	0	1	0
	7	0	0	1	0	6	1	0	136	1	2
	8	2	0	0	1	0	4	1	1	155	2
	9	0	0	0	1	4	0	0	0	2	170

Table 9: Confusion Table of Sigmoid Kernel Based on One vs All

		Output Digits									
		0	1	2	3	4	5	6	7	8	9
Actual Digits	0	337	0	3	2	3	1	7	1	3	2
	1	0	252	1	2	3	0	3	0	1	2
	2	3	0	148	5	20	2	3	2	14	1
	3	1	0	3	142	1	12	0	2	1	4
	4	0	1	1	0	183	0	5	1	1	8
	5	5	0	0	10	5	132	1	0	2	5
	6	1	0	2	1	4	3	158	0	1	0
	7	0	0	1	0	6	0	0	131	1	8
	8	3	0	1	5	4	4	2	4	139	4
	9	0	0	0	0	4	1	0	2	2	168

Table 10: Confusion Table of Sigmoid Kernel Based on One vs One

		Output Digits									
		0	1	2	3	4	5	6	7	8	9
Actual Digits	0	346	0	1	0	3	0	6	1	2	0
	1	0	258	0	1	2	0	3	0	0	0
	2	5	0	171	6	6	1	3	1	4	1
	3	2	0	2	142	1	14	0	1	3	1
	4	0	3	3	0	184	0	3	1	1	5
	5	4	0	0	8	2	142	0	0	0	4
	6	0	0	3	0	3	3	161	0	0	0
	7	0	1	0	1	6	0	0	136	1	2
	8	4	0	0	3	4	7	0	2	144	2
	9	0	0	0	1	4	0	0	2	0	170

The above 8 Tables demonstrate the actually recognized result from each experiment. Every one has 10 rows and 10 columns. Since the testing sample units have its own label, we separate them digit by digit. For example, there are

$$320 + 0 + 3 + 7 + 4 + 2 + 13 + 0 + 7 + 3 = 359 \quad (4.38)$$

testing units with label “0” in our test sample and the algorithm did correctly recognize 320 of them, and recognized 3 of them as digit “2”, 7 of them as digit “3”, etc. So, the accuracy that the algorithm with Linear Kernel based on One vs All design is

$$\frac{320}{359} = 0.891, \quad (4.39)$$

which is consistent with the first element upper right on table 1.

Similarly, each digit from Table 3 to Table 10 has the same meaning. These tables are very important for improving the accuracy, they suggest the possible trend of error for each algorithm. Then, we can develop the suitable modification individually by these reference in order to increase the performance of the algorithms.

4.3 Time

As we all know, the sufficiency of one experiment is not only about the outcome, but also the cost. The main issue is time on our design and it can be considered as two parts, training and recognizing. Since the training part we can do ahead, it takes up a smaller weight. Here are the details in the following Figure 11.

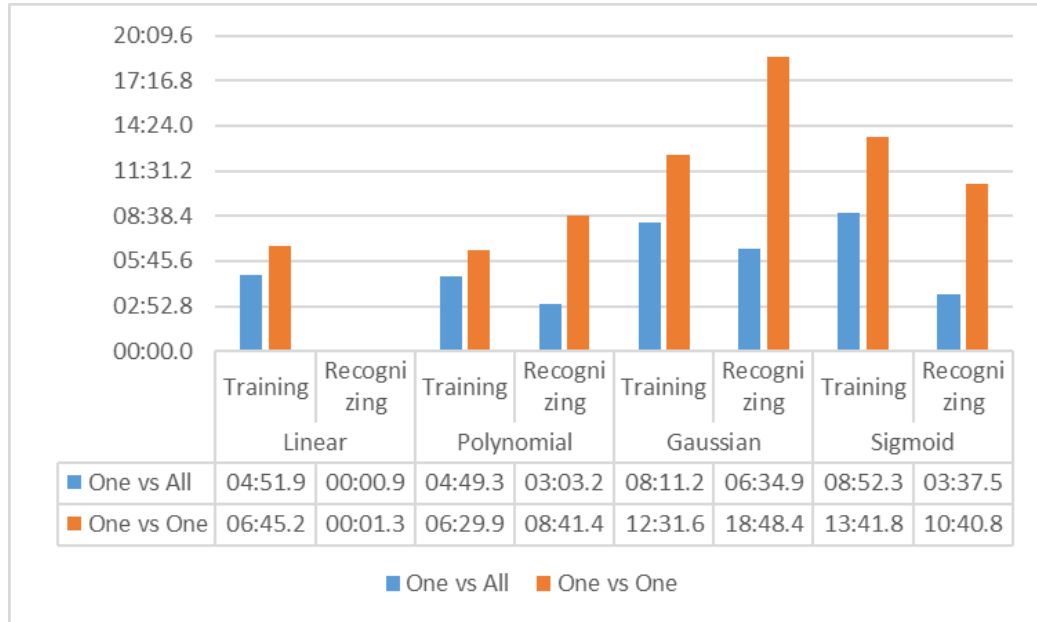


Figure 11: The Time Spend on Training and Recognizing

From this figure, we can clearly see that the time spend on One vs. All design is uniformly less than One vs One design, which makes sense because the One vs One has a lot more classifiers than One vs All design does. Furthermore, the process of recognizing needs to bring every unit of the training sample into the classifiers in the second design. In fact, we would rather the algorithm spend time as less as possible even though we have the super power computer today. No one is willing to wait the outcome in a few hour after give some input. Thus, we prefer to use the linear kernel than other kernels, One vs All design than One vs One design from the aspect of time consuming.

5. DISCUSSION

In the last chapter, we have given the main result of our experiments, which proved our theory is durable and effective in real scenario. However, it is worth to discuss more in detail, in particular, what advantages and disadvantages of SVM, compared to other classification algorithms. if there is any disadvantage, what improvement should we do to increase the performance of experiment in the future.

5.1 Conclusion

From Figure 10, we can clearly see that One vs One design performs better than One vs All in accuracy as what we expect except the polynomial kernel case, this is because the second design has much more classifiers involved and the voting process is more convincing than one ticket call.

The algorithm with Gaussian Kernel has the best result, no matter in which design. These two designs deliver slightly different results and both are greater than 93%. As for the algorithm with linear kernel, it doesn't have the best result on accuracy but it is unbeatable at the speed of recognition. Hence, it is really difficult to conclude which one is the best overall.

However, we can make an evaluation system based on their ranks to help us out. The weight of each index will be specified ahead by the preference of experiment. If we don't care time consuming but the accuracy, give the rank of accuracy heavier weight, vice versa. In our case, we set the weight of accuracy as 70%, time as 30%. Then we have the following Table 11.

Based on this evaluation, we would like the algorithm with Gaussian Kernel in One vs All design. In the future, this evaluation system still can be used, just change the weight of indexes.

Table 11: The Rank of Algorithm

	Rank		Performance	
	Average Accuracy	Time Consuming	Score	Rank
Linear_1vsAll	8	1	5.9	6
Ploynomial_1vsAll	5	3	4.4	5
Gaussian_1vsAll	2	5	2.9	1
Sigmoid_1vsall	7	4	6.1	8
Linear_1vs1	4	2	3.4	3
Ploynomial_1vs1	6	6	6	7
Gaussian_1vs1	1	8	3.1	2
Sigmoid_1vs1	3	7	4.2	4
Weight	70%	30%		

5.2 Advantage and Disadvantage

In general, the Support Vector Machine classifier has better result on accuracy compared to Regression Analysis. Since the regression model is more based on the continuous variables but classification problem only has two values originally, it is a discrete case.

Also, SVMs doesn't require any prior knowledge about the object we focused on, the algorithm will learn the features from the training data set. For this reason, the SVM model can be generalized very easily. For example, it could do face detection, image classification, bioinformatics, etc, as long as we have the correspond training sample. The new classifiers can be found in the same pattern, and what we need to do is just changing a few parameters.

However, there are shortcoming as well. SVMs is a typical supervised training algorithm, we need adjust the parameters to get a better output as what we showed in Gaussian kernel on One vs All design, which means that the result we have may not be the best since the parameter is not the best fit.

In addition, SVMs cannot memorize the new coming data units. For example, there is a testing unit coming and it gave an incorrect classification. There is

nothing the experimenter can do to help it improve. In another word, it will still give the same wrong answer if the same unit comes to classify. Therefore, it cannot be self improved.

5.3 Future Work

Two main assignments are increasing the accuracy and decreasing the time consuming in the future. Firstly, recalling the experimental design, the "Not" training set is randomly selected to balance the number of elements in two subsets. Since it is random, there exist some variations on the selected training set which will make a different classifier. Thus, the accuracy will change somehow.

From this perspective, there is a maximum in the varied accuracy. So, we can run the algorithm as many time as possible and store the parameter every time. Then compare with the outcome, chose the selected set as the final training set which gives the best result. In this way, I believe we can increase the accuracy.

Secondly, from Figure 9 we notice there is no sufficient value located in four corners. So we can get rid of the gray values in the four corners, to reduce the dimension of the vector and simply the manipulation and save the time eventually.

REFERENCES

- [1] B. Schölkopf and A. J. Smola, *Learning with kernels: support vector machines, regularization, optimization, and beyond*. MIT press, 2002.
- [2] C. L. Devasena, T. Sumathi, V. Gomathi, and M. Hemalatha, “Effectiveness evaluation of rule based classifiers for the classification of iris data set,” *Bonfring International Journal of Man Machine Interface*, vol. 1, p. 5, 2011.
- [3] N. Zahir and H. Mahdi, “Snow depth estimation using time series passive microwave imagery via genetically support vector regression (case study urmia lake basin),” *The International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences*, vol. 40, no. 1, p. 555, 2015.
- [4] Z. Ye, M. Wang, C. Wang, and H. Xu, “P2p traffic identification using support vector machine and cuckoo search algorithm combined with particle swarm optimization algorithm,” in *Internet Conference of China*. Springer, 2014, pp. 118–132.
- [5] S. Suralkar, A. Karode, P. W. Pawade *et al.*, “Texture image classification using support vector machine,” *International Journal of Computer Applications in Technology*, vol. 3, no. 1, pp. 71–75, 2012.
- [6] Z. Songfeng, L. Xiaofeng, Z. Nanning, and X. Weipu, “Unsupervised clustering based reduced support vector machines,” in *Acoustics, Speech, and Signal Processing, 2003. Proceedings.(ICASSP’03). 2003 IEEE International Conference on*, vol. 2. IEEE, 2003, pp. II–821.
- [7] R. Berwick, “An idiot’s guide to support vector machines (svms),” *Retrieved on October*, vol. 21, p. 2011, 2003.

- [8] C.-C. Chang, C.-W. Hsu, and C.-J. Lin, “The analysis of decomposition methods for support vector machines,” *IEEE Transactions on Neural Networks*, vol. 11, no. 4, pp. 1003–1008, 2000.
- [9] B. Schölkopf, R. Herbrich, and A. J. Smola, “A generalized representer theorem,” in *International conference on computational learning theory*. Springer, 2001, pp. 416–426.
- [10] Y. K. Jain and S. K. Bhandare, “Min max normalization based data perturbation method for privacy protection,” *International Journal of Computer & Communication Technology*, vol. 2, no. 8, pp. 45–50, 2011.