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POSING PURPOSEFUL QUESTIONS IN A MATHEMATICS TUTORING SETTING

A Master's Thesis

Presented to

The Graduate College of

Missouri State University

In Partial Fulfillment

Of the Requirements for the Degree

Master of Science in Education, Secondary Education

By

Sara Elaine Jones

May 2019

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POSING PURPOSEFUL QUESTIONS IN A MATHEMATICS TUTORING SETTING

Mathematics

Missouri State University, May 2019

Master of Science in Education

Sara Elaine Jones

ABSTRACT

One of the eight Effective Mathematics Teaching Practices published by the National Council of Teachers of Mathematics (NCTM) is the posing of purposeful questions. Many studies have been conducted that support the need, importance, and effectiveness of purposeful questioning in conceptual mathematics teaching. The purpose of this action research study was to gain insight into the successes and challenges of a tutor when implementing purposeful questions in a mathematics tutoring setting. The experiences of the tutor were analyzed through the collection of qualitative data using video and audio recordings, journal entries of the tutor, and an observational protocol. Data analysis revealed three successes associated with purposeful questioning: a) encouraged the standards of mathematical practice b) enhanced students' cognitive mathematical engagement and c) promoted personal and professional growth for the tutor. Data analysis also revealed three challenges to asking purposeful questions: a) nature of the homework problems b) students' lack of conceptual mathematical experience and c) lack of mathematical confidence by both the tutor and the student. The findings of this study can be used to encourage and improve the questioning by current and future mathematics educators and tutors.

KEYWORDS: Common Core mathematical practices, effective teaching practices, mathematics questioning, tutoring, secondary mathematics education

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Approved:

Kurt Killion, Ed.D., Thesis Committee Chair

Gay Ragan, Ph.D., Committee Member

Sarah Nixon, Ph.D., Committee Member

Julie Masterson, Ph.D., Dean of the Graduate College

In the interest of academic freedom and the principle of free speech, approval of this thesis indicates the format is acceptable and meets the academic criteria for the discipline as determined by the faculty that constitute the thesis committee. The content and views expressed in this thesis are those of the student-scholar and are not endorsed by Missouri State University, its Graduate College, or its employees.

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CHAPTER I: OVERVIEW OF THE STUDY

Many times, when a person hears the word “math” it is followed by some negative response, whether it be a sarcastic laugh or the casual eye-roll. Unfortunately, in my opinion, the mathematics many individuals have been exposed to throughout their mathematics experiences is deserving of this type of negative response. In many mathematics classrooms across the United States, students are conditioned through teacher-driven instruction to see mathematics as a disconnected set of procedures and rules which they are required to memorize and repeat on exams. Most teachers who deliver procedural mathematics are unware they are spreading a misrepresentation of mathematics. They are simply teaching the algorithm-driven mathematics for which they were taught (Evans, 2017). Thus, a cycle of depthless mathematics is recycled.

Based on my own experience as a high school student, this restricted view of mathematics neglects the natural beauty, order, and mystery that exists within the subject. Although students have completed numerous mathematics classes, does mindlessly repeating procedures suggest a true and rich understanding of mathematics has been gained? Paul Lockhart, author of *A Mathematician’s Lament*, argued it does not. Lockhart (2009) suggested mathematics is an art form that should encourage creativity and inquiry, and because these actions are absent in many classrooms, meaningful, conceptual mathematics is not being taught. The role of the teacher is to ignite this curiosity by providing opportunities for students to discover the embedded layers that mathematics has to offer (Way, 2008).

One way a teacher can begin to foster true mathematical understanding is by the use of purposeful questioning (Stump, 2010). Posing purposeful questions is one of the Effective Mathematics Teaching Practices (See Appendix A) suggested by the National Council of

Teachers of Mathematics (NTCM). The kind of knowledge students construct and communicate within mathematics classrooms is positively correlated to teacher's questioning (McCarthy, Sithole, McCarthy, Cho, & Gyan, 2016). Thus, the questions posed by a teacher directly influence student learning. Purposeful questioning has been known to be a fundamental tool in effective teaching, but the practice of purposeful questioning is not being executed (Stump, 2010). Educators believe in the importance of purposeful questioning but lack the necessary skills to pose such purposeful questions (Stump, 2010). This action research study seeks to examine the successes and challenges of a tutor when implementing purposeful questions as it pertains to developing deeper mathematical understanding.

This chapter will identify the purpose for the study, provide the research questions, communicate the research design, and support the significance of the study. Assumptions, limitations, and definitions within this study are also described within this study overview.

Rationale for the Study

My personal mathematics journey was one rationale for this study. Throughout high school, I excelled in mathematics. I paid close attention to the teacher's examples, studied the review packet inside-and-out, and checked every answer on the test. These actions led to me having success within the subject, and therefore, sparked a liking for mathematics. This liking encouraged the desire to become a secondary mathematics educator. As an undergraduate student, I was hit with the realization that what I thought was *good at math* actually meant *good at memorizing procedures*. Due to the purposeful questions posed by my professors, I quickly recognized the mathematics I thought I knew and loved had many conceptual gaps. I realized that my mathematical education in high school lacked connections, deeper understandings, and conceptual meaning. This realization was heartbreaking to me, as one who loved and felt confident doing mathematics. Determined to not strip my students of the same deeper understandings, I decided to study how the implementation of purposeful questions could aid the learning of conceptual mathematics.

Questioning is a major tool for effective teaching (Stump, 2010). Because "differences in students' thinking and reasoning could be attributed to the type of questions teachers ask" (Wood, 2002, para. 1), the art of purposeful questioning must be studied. As a tutor at the collegiate level, I have seen the frustration and struggle of students who have only been exposed to a procedural-focused mathematics curriculum. In my opinion, this lack of exposure to a deeper, more conceptual understanding of mathematics has put them at a disadvantage. They were cheated of the opportunity to discover all the connections within mathematics. As a tutor, I was disappointed by the mathematical knowledge students were bringing to college. This motivated me to study how I could implement purposeful questions to aid their understanding. As I worked with students, I observed the need to improve mathematical teaching in such a way that would encourage conceptual student thinking and introduce students to the connections embedded within mathematics.

Most research on the topic of mathematical questioning included the use of observational and interview designs that shed light on the importance of questioning but neglected the personal challenges and successes of the questioning process (Babu & Mim, 2017; Robitaille & Maldonado, 2015; Stump, 2010). Research provided evidence of the positive impact purposeful questioning can have on student learning, but insights about the implementation of this purposeful questioning were neglected. The need for research that shifted from a theoretical argument *for* questioning to investigating the actual practice *of* purposeful questioning motivated me to take a personal approach in this study.

Purpose of the Study

The purpose of this action research study was to gain insight into the successes and challenges of a tutor when implementing purposeful questions in a mathematics tutoring setting. Another purpose for this study was to reflect on and improve my questioning techniques in order to better serve students' conceptual understandings of mathematics. I hope to provide pre-service and in-service educators with another source of research to support effective teaching practices.

Research Questions

The following research questions guided this study:

1. What successes did the tutor experience while posing purposeful questions in a mathematics tutoring setting?
2. What challenges did the tutor experience while posing purposeful questions in a mathematics tutoring setting?

Research Design

This action research design study examined the experiences of a tutor as I implemented purposeful questioning in one-on-one tutoring settings. Based on the similarities to past action research studies (Evans, 2017; McAninch, 2015), this study was best suited for an action research design to answer the research questions under investigation. The research design provided the opportunity to improve my teaching practices through in-depth personal narratives and experiences.

As a tutor, I had the opportunity to work one-on-one with students. During the months of November and December in 2018, four collegiate student-athletes, enrolled in College Algebra, Contemporary Mathematics, or Intermediate Mathematics, each met with me for five 30-minute one-on-one tutoring sessions. This, a total of 20 tutoring sessions were investigated. Throughout these tutoring sessions, the student and I used the student's homework as the course of action. He or she would open up the online assignment, and we would work through the problems one at a time. Rather than funneling students to answers, I focused on asking purposeful questions that promoted conceptual understanding and recognition of mathematical connections within the problem. After each tutoring session, I wrote in a self-guided journal to note interactions of successes and areas of challenges that took place within each session. Each tutoring session was videotaped and audio recorded using Swivel technology for later examination by both me and an outside observer. I analyzed the recordings using an observational protocol to draw conclusions about my questioning as the tutor and the students' responses. Three video recordings were also reviewed by an outside observer using the same observational protocol. These recordings provided accurate and precise interactions to use for qualitative data collection. After the fifth and final tutoring session with each participant, I asked the student to reflect on the effectiveness of my questioning techniques. These student reflections provided another data source into the success and challenges of the tutoring sessions.

Significance of the Study

First, this study examined the successes and challenges of posing purposeful questions within a mathematics tutoring session. In doing so, I have provided authentic insights that will encourage educators to focus on purposeful questioning, while also avoiding or persevering through the potential challenges for which I experienced. Although collected in a tutoring setting, many of the findings within this study can be generalized to a classroom setting. The findings of the research provided another resource for pre-service teachers and post-secondary teacher education programs by equipping them with personal insights about how to educate pre-service teachers about the successes of purposeful questioning and prepare them for the challenges of posing purposeful questions.

Secondly, this study was significant because I was able to improve my teaching practices, specifically my questioning abilities. Through deep reflection, I was able to learn more about myself as an educator in terms of areas of strength and areas in need of growth related to purposeful questioning. Exposing these areas allowed me to improve as a tutor, but perhaps more importantly, it provided critical insights for me to improve as a future teacher.

Lastly, this study allowed me to see the benefits that posing purposeful questions can have on students' mathematical understanding. This was significant because student learning is the goal of education. This research study will encourage other educators to use purposeful questioning in order for their students to discover the meaningful mathematical connections that exist beyond the procedures.

Assumptions

The following is a list of assumptions I made during the study:

1. I assumed students would have no significant prior experience with purposeful questions. Instead I expected them to have only been asked questions that are procedure-based.

2. I assumed students would want to complete homework every tutoring session with high accuracy.

Limitations

The following is a list of limitations I encountered during the study:

1. The 30-minutes allotted for each tutoring session could not accommodate the maximum number of purposeful questions and completion of the homework assignment. Thus, the number of purposeful questions was restricted due to limited time.
2. Questioning that promoted student discourse was limited due to the tutoring sessions being one-on-one. I had minimal opportunities to ask students to repeat a classmate's rationale or critique others reasoning since students were in sessions by themselves.
3. Since this study was conducted using one-on-one sessions, there were some limitations when generalizing findings to a whole class environment (large class size, classroom management, etc.).

Definition of Terms

The following was a list of definitions during the study:

1. Challenge: A challenge, for the purpose of this study, takes on two forms. First, any factor that resists or prevents the use of a purposeful question is described as a challenge. Second, a challenge may also be a barrier faced *after* a purposeful question is asked. This includes student deficiencies to answer the purposeful questions and the inexperience with purposeful questioning by both the tutor and student.

2. Mathematical connections: Mathematical connections refer to a deeper, longer lasting, more meaningful learning of mathematics. Mathematical connections reveal relationships between mathematics concepts, interplay of concepts within other subjects, and relation to personal experiences. Mathematical connections fosters holistic learning that allows students to relate current mathematics concepts to future mathematics concepts.
3. Purposeful questions: Purposeful questions reveal student understanding and extend student thinking in order to help students make sense of problems and expose mathematical connections. Implementation of purposeful questioning was identified by having at least one of the following indicators:
    a. Question built-on and/or extended student thinking in order to make sense of the solution
    b. Question exposed mathematical connections that made the mathematics more conceptual and meaningful to students
    c. Question encouraged students to clearly communicate and elaborate their thinking
4. Success: A success, for the purpose of this study, is a positive outcome as a result of purposeful questions posed by the tutor. A positive outcome includes improvement in students' mathematical learning and/or personal growth as an educator.

Summary

Posing purposeful questions is one of the eight Effective Mathematics Teaching Practices published by the National Council of Teachers of Mathematics (NCTM) (2014). Researchers have studied the benefits of posing purposeful questions, but few have studied the role of the teacher as they implement such questions. Due to my own procedural-based mathematics education, I was motivated to take a closer look at the implementation of posing purposeful questions to aid students in more conceptual mathematical understanding. The purpose of this study was to gain insight into the successes and challenges of a tutor when implementing purposeful questions in a mathematics tutoring setting.

CHAPTER II: REVIEW OF RELATED LITERATURE

The NCTM (2014) listed posing purposeful questions as one of the eight practices that guide effective mathematics teaching. Questioning is a dynamic teacher instructional intervention (Tienkin, Goldberg, & Di Rocco, 2009). The roots of questioning were grounded over 2,000 year ago with Socrates, as he used questioning to engage others in inquiry-based discourse that encouraged critical thinking and problem solving (Robitaille & Maldonado, 2015). Questioning is a route of communication that takes many forms and has many purposes. Many studies have explored the effectiveness and implementation of questioning inside mathematics classrooms. This chapter will provide a summary of literature that describes the characteristics of purposeful questions and exposes the absence of these purposeful questioning in mathematic classrooms. These works will be followed by literature that highlights the importance of purposeful questioning in pursuit of conceptual understanding of mathematical concepts. The final pieces of literature describe the role of the teacher in this effective questioning process.

Investigating the Meaning of Purposeful Questioning

To implement purposeful questioning, one must first come to terms about what it means for a question to be purposeful. The NCTM (2014) described purposeful questions as ways "to access and advance students' reasoning and making sense about important mathematical ideas and relationships" (p. 3).

Teodoro, Donders, Kemp-Davidson, Robertson, and Schuyler (2011) examined the features that define a good question. Two subgroups, deeper and surface, were used to categorize the questions posed by teachers. Deeper questions were characterized by their open-ended and divergent nature, while surface questions promoted recall and procedure (Teodoro et al., 2011). Criteria for these subgroups were also determined through a whole-class discussion. Students characterized surface-level questioning as "not giving you much information about what was done" (Teodoro et al., 2011, p. 21) and deeper-level questioning as "helping others explain their own thinking more clearly" (Teodoro et al., 2011, p. 21). Although surface-level questions have a place in the classroom, the characteristics of deeper questions serve a greater purpose.

Classroom questions are not easy to classify because the context and purpose of the questions are not taken into account (Myhill & Dunkin, 2005). Myhill and Dunkin, professors of education in the United Kingdom, studied the types of questions asked within a Year 2 mathematics classroom (aged 6 to 7) and a Year 6 (aged 10 to 11) mathematics classroom, but rather than grouping questions into two groups as Teodoro et al. (2011) did, Myhill and Dunkin's study established four types of questions: *factual, speculative, process,* and *procedural.* Factual questions invite predetermined answers. Speculative questions, on the other hand, request opinions, hypotheses, and other nonpredetermined answers. An example of a speculative question is, *If I made the slope higher, what do you predict might happen to our graph?* Speculative questions value student thought and greatly influence a teacher's course of action. Process questions promote articulation of students' thinking and understanding, thus, checking students' prior knowledge (Myhill & Dunkin, 2005). Process questions generally promote classroom discourse and stimulate deep mathematical thought. *Why?* is a popular question underneath the process category. Both speculative and process questions stimulate higher cognitive levels. The final group, procedural questions, contain the unavoidable questions that involve managing and organizing a classroom. Examples include, *Can you please use a pencil instead of a pen?* or *Do you need to be excused?*

Myhill and Dunkin (2005) collected data from 54 teaching episodes, teacher reflections, and student interviews. During each third of the semester, the researchers examined the questions asked in a classroom and characterized them by type. During the first third, the data revealed that eight percent of the questions posed were procedural, 64 percent were factual, 16 percent were speculative, and 12 percent were process (Myhill & Dunkin, 2005). By the last third, the percentage of process questions increased, but the number of factual questions did not lower (Myhill & Dunkin, 2005). While a teacher should aim for process or speculative questions that invite students to share their thinking and have no predetermined answer, this study showed the most common type of question, by a great margin, was factual. Thus, the most frequently asked questions within these teaching observations were not purposeful.

NCTM (2014) defines purposeful questions as, "questions that assess and advance students' reasoning and sense making about important mathematical ideas and relationships" (p. 35). Their book, *Principles to Actions: Ensuring Mathematical Success for All*, uses four groups to categorize questions: gathering information, probing thinking, making the mathematics visible, and encouraging reflection of justification. Questions that gather information require students to recall facts, definitions, or procedures. Probing questions require students to explain or clarify their thinking. Questions that make mathematics visible discuss structures and connections that exist within mathematics. Finally, questions that encourage reflection and justification reveal deeper student understanding and reasoning. NCTM's *Principles to Action* (2014) outlines these four types of questions, but then shifts the attention from types of questions to patterns of questions.

Herbel-Eisenmann and Breyfogle (2005) described two patterns of questioning: funneling and focusing. Funneling questions involve a teacher asking a series of questions to guide students through a procedure in order to reach a predetermined end. These types of questions do not advance student learning but rather help a student follow a procedure which the teacher knows. Focus questions, on the other hand, require teachers to listen to student responses and guide them through their *own* thinking. Focus questions encourages teachers to meet students where they are and encourages students to clearly articulate their ideas. A teacher interacting through focus questions values student thinking and promotes student engagement (Herbel-Eisenmann & Breyfogle, 2005). Hence, focus questions contain the characteristics of a purposeful question within this study.

Absence of Purposeful Questioning

Although questioning skills have been adopted within the subjects of literacy and social studies, Way (2008), a researcher of effective pedagogies for mathematics education, argued questioning skills have not transferred to mathematics classrooms. Reformers of mathematics teaching have promoted the need of an instructional shift from teacher-centered instructional approaches to a more student-centered methodology. According to Stump (2010), amidst this theoretical shift, the actual practices taking place within classrooms have not shifted. Although questions are being posed, there are few questions that encourage students to use higher-order thinking and inquiry skills (Way, 2008). There is a gap between teachers recognizing the need for purposeful questioning and teachers actually asking purposeful questions (Robitaille & Maldonado, 2015).

Stump (2010) studied the teaching of 11 pre-service teachers as it pertained to student learning. He concluded, through the written comments of the teachers, that they acknowledged the importance of asking purposeful questions. The problem was, in actual teaching situations, that questions tended to focus more on procedures than concepts. Rather than supporting students' development of mathematical power, a majority of questions asked simply guided students to an arrival of an answer. Based on his findings, Stump (2010) suggested pre-service teachers examine their questioning in order to promote awareness of the mathematical understanding that questions are promoting.

The absence of purposeful questioning was also evident through the work of Babu and Mim (2017) in their study of 10 mathematics teachers in Bangladesh. With the use of video recording, 300 questions from each observed lesson were categorized by type and learning domain. These observations revealed three types of questions asked within classrooms: Open, closed, and yes-no answers. Only five percent of the questions witnessed were open questions, whereas 55 percent were closed, and 40 percent were yes-no questions (Babu & Mim, 2017). Although questions were being asked, *purposeful* questions were still nonexistent.

Myhill and Dunkin (2005) described the process of questioning in classrooms as a tool to align student thinking with that of the teacher. This process disregards student thinking and encourages the following of a script (Myhill & Dunkin, 2005). Teachers and tutors believe they need to funnel students to an answer rather than focus on the understandings of the student. By funneling student's understanding to that of the teacher, the students' natural curiosity to ask provoking, abstract, and conceptual questions is being stripped away (Lockhart, 2009). Thinking, questioning, and discussing are inherent processes that should be promoted within a classroom (Robitaille & Maldonado, 2015). Unfortunately, funneling students to a correct answer is denying those inherent processes. Lockhart (2009) wrote about the beauty of mathematics being camouflaged by procedures to the point that educators and students no longer view it as the art for which it is.

Purposeful Questioning Supports Conceptual Understanding

The absence of purposeful questioning is evident. Educators agree on the importance of teaching for understanding but have varying meanings of the word understanding (Simon, 2006). The depth of student understanding and the ability to communicate this understanding is determined by the types of questions asked within a classroom (McCarthy et al., 2016). If a teacher is unable to ask questions across the various cognitive domains of analysis, synthesis, and evaluation, then student thinking is being restricted. McCarthy et al. (2016) argued quality questioning shifts limited thinking to the exploration of new solutions and repetition of procedures to the investigation of reasoning and connections.

Teodoro et al. (2011) recognized a lack of depth of student mathematical understanding which provided the rationale for their study. One goal of these researchers was to deepen student learning and reveal the connections that exist within mathematics. In pursuit of this goal, it was revealed one can accomplish richer and more conceptual understandings of mathematical concepts through the use of purposeful questioning (Teodoro et al., 2011).

In this study, four teachers used sorting activities in order to improve purposeful question identification and implementation. These sorting activities involved both teachers and students sorting questions into “surface level” or “deep level” categories based on collaboratively established criteria. As students were exposed to characteristics of meaningful questions, they gained the skills necessary to pose purposeful questions of their own. Teachers are a primary source for students’ mathematical questioning because students can adopt the skills and strategies modeled by their teacher as they gain proficiency in their own skills (Stolk, 2013). This improvement of questioning allowed students to engage in conceptual mathematical consolidation which further enhanced their learning (Teodoro et al., 2011).

The relationship between the type of question asked and the response in which a student provides is substantial (Myhill & Dunkin, 2005). Thus, if teachers provide students with weak questions, then student responses will lack depth and conceptual meaning. Teachers should ask questions that support students verbalizing their own mathematical ideas and questions that support students confirming their understandings.

Role of the Teacher

Researcher suggestions were consistent in arguing the role of the teacher is to facilitate conceptual learning through the use of purposeful questions. A teacher should use purposeful questioning to shift the mathematical authority to his or her students (Evans, 2017). As this authority is passed, students begin to take ownership of their learning and responsibility for asking good questions (Teodoro et al., 2011). Rather than being a speaker of information, teachers should switch to a listener role as they let students verbalize reason and mathematical understanding (McCarthy et al., 2016). By posing questions that promote discussion, teachers shift from direct lectures to facilitators of student learning (McAninch, 2015). Teachers are to design questions that encourage learners to think, create viable arguments, challenge their assumptions, and engage in provoking thought (Babu & Mim, 2017).

As a teacher, there is a linguistic dominance that provides the teacher, or tutor, with the authority to control and manipulate classroom discourse to achieve the desired educational purpose (Myhill & Dunkin, 2005). The foundation of this dominance is the way teachers use questions. Effective use of this power involves teachers taking on a managerial role in which they ask questions that take the classroom discussions to deeper levels (Myhill & Dunkin, 2005).

Purposeful questioning requires a great deal of preparation by the teacher, or tutor, in order to ensure its effectiveness. Thus, teachers have a vital role as a planner. It is critical for a teacher to think about questions to ask and misconceptions students may have before a lesson begins in order to create effective questions ahead of time (Teodoro et al., 2011). Because generating deep questions in the heat of the moment can be difficult, preparation allows teachers to prepare questions which they are certain will foster productive thinking (Tienken, Goldberg, & Di Rocco, 2009). Teodoro et al. (2011) describes questioning as a “real art” (p. 27).

Although preparation can minimize some of the difficulty of asking purposeful questions, the real challenge lies in the student responses for which a teacher cannot prepare. While many teachers can ask initial questions to simulate mathematical thinking, they struggle to use questions to challenge or extend student ideas (Franke, Webb, Chan, Ing, Freund, & Battey, 2009). To effectively question based on a student’s response requires a teacher’s own deep mathematical understanding of concepts and how students think mathematically (McCarthy et al., 2016). This ability to engage in purposeful questioning at a student’s present level of understanding is not an easy skill. McCarthy et al. (2016) argued that even experienced teachers with their collection of strategies have a hard time interpreting and responding to unforeseen answers from students.

A purposeful question begins with strong preparation and finishes with deep reflection. Robitaille and Maldonado (2015) recorded teachers speaking about the importance of reflection in terms of improving questioning and discussion techniques. Without reflection, the effectiveness of a purposeful question cannot be determined. A peer evaluator in this study specifically addressed the need for teachers to reflect on who is participating in the classroom’s discussions: students or the teacher? Reflection speaks truth to the areas in need of improvement and the areas of great student learning.

Summary

There are many types of questions posed within a mathematical classroom but only a few have the goal of enhancing student learning. Understanding the characteristics of these purposeful questions is critical to their implementation. It is evident that this implementation of purposeful questions is not taking place in all classrooms. Teachers are posing questions that serve no greater purpose than a meaningless answer. The questions asked by teachers should place the mathematical authority on students so they can begin to discover the relationships of mathematics that exists beyond the surface (Lockhart, 2009). Not only is purposeful questioning critical, it is also difficult. Through preparation and reflection, teachers can begin to improve the questioning strategies they bring to their classrooms. The depth of student mathematical understanding is dependent on the purpose and intentionality of posed questions, hence the need to investigate the implementation of purposeful questioning further.

CHAPTER III: METHODOLOGY

Researches have suggested the types of questions teachers should be asking in order to promote conceptual mathematical understanding. The goal of this study was to investigate, through a personal lens, the successes and challenges of posing purposeful questions in a mathematics tutoring session. As a tutor, I posed purposeful questions during one-on-one tutoring sessions to promote a more conceptual learning of mathematics. Decisions involving the methodology of this study, including recording sessions and keeping a free-response journal, were made in order to provide authentic insights to the successes and challenges that transpire during tutoring sessions as purposeful questions were posed. This chapter will explain the research design, describe the site and participants of the study, disclose ethical considerations, and clarify the data collection and analysis procedures.

Research Design

This study was an action research design. This type of design is piloted by a teacher in their classroom with the goal to improve their teaching practices. Because action research "values the interpretations that teachers make based on data collected with their students" (Hendricks, 2013, p.12), it was an ideal design for this particular study. During an action research design, the researcher is the primary consumer of the conclusions found within the study. Thus, the goal of this action research was to collect data from my students to gain understanding and promote improvement of my own teaching practices.

When engaging in a new teaching practice, such as posing purposeful questions, many researchers feel the need to collect quantitative data in order to provide numerical findings that will speak to the effectiveness of the practice (Myhill & Dunkin, 2005; McAninch, 2015; Sharma, 2013). Since there was already research available to support the effectiveness of good questioning, quantitative data would not speak to the research questions I am investigating. The personal complexities for which I was looking to discover were more likely to be exposed through the collection of qualitative data. Qualitative research provided depth and detail while analyzing open-ended questions (Sharma, 2013), making it more suitable for this research study.

The four participants in this study were students whom I had already been tutoring throughout the semester. I usually met with these students on a needs-only basis to help with a homework problem or prepare for a test. For this study, rather than working quickly to simply arrive at a correct answer, I focused on promoting conceptual mathematics understanding through the use of purposeful questioning. I required each participant to meet with me five times between November and December of 2018. Each session was one-on-one.

In each session, the plan of action was determined by the assigned coursework in the participant's mathematics course. This course work included reviewing classroom notes, completing homework problems on an online program, or working through study guides. I had no prior knowledge or planning of the material in order to ensure an authentic tutoring situation. My role in the tutoring sessions was to advance the student's understanding of the material through the use of deep questioning.

Site of the Study

This study took place on a university campus located in the mid-west region of the United States. With an enrollment of approximately 24,000 students, the university is the second largest post-secondary institution in its state. The university offers 180 undergraduate degree programs, 100 graduate degree options, and 17 NCAA Division I sports. With an 85-percent acceptance rate and average admission requirements, the university is made up of a diverse range of individuals.

Specifically, each tutoring session took place in a study room of the Academic Achievement Center. This facility is a place where the university's student-athletes can gain access to support their academic responsibilities. Verbal permission was granted from my supervisor to conduct my study in this location and written consent from the participants before beginning the study.

Participants

Based on the nature of an action research design, I participated in my own study as a tutor-researcher. My participation in the study included posing purposeful questions within tutoring sessions, writing reflections about the events that take place within the tutoring sessions, and analyzing the audio and video recordings of the tutoring sessions.

As for the student participants in my tutoring sessions, they were selected using a convenient sample technique. The participants were selected from the group of students whom I was currently tutoring in the Fall 18 semester. They were between the ages of 18 – 20 years-old. Three of them were male, and there was one female. The four chosen students were college student-athletes enrolled in one of the following mathematics courses: College Algebra, Contemporary Mathematics, or Intermediate Mathematics. Participants were individuals who have self-enrolled in mathematics tutoring because of encouragement by an advisor or by personal acknowledgement of needed assistance. Since these students desired help in mathematics, this study worked to engage each of them in deep conceptual mathematical learning through the use of purposeful questioning.

Ethical Considerations

Prior to conducting this study, I gained approval for study number IRB-FY2019-46 by the university's Institutional Review Board (IRB) on November 16, 2018 (See Appendix B). I also gained verbal permission from my Graduate Assistantship supervisor. This study only included willing participants who signed the informed consent form (See Appendix C). Participants were fully aware of the benefits and risks of the study before taking part. The first responsibility of my role as the mathematics tutor was to provide the highest quality mathematics tutoring possible to each student. Throughout the study, I tutored more students than just my research participants, but no matter their role in my study, I ensured quality tutoring practices to each student.

Data Collection Procedures

From November to December of 2018, each of the four participants met with me for five 30-minute tutoring sessions. These sessions were not consecutive due to the schedules of the participants and me. The following were topics covered within the tutoring sessions:

1. Probability
2. Solving Expressions
3. Unit Conversations
4. Factoring Polynomials
5. Parabolas

The qualitative data generated in this study were collected in multiple ways. After each tutoring session, I completed a self-guided free-response journal to record the initial feelings I had following the session. These included emotions of excitement, as well as incidents that left me disappointed, frustrated, or embarrassed. The journal allowed me to record genuine reactions that aided the research questions.

Secondly, each of the five sessions were audio and video recorded using Swivel technology. The recordings allowed for in-depth and accurate reflection about the actions that transpired in each session. The analysis of the recordings was guided by an observational protocol (See Appendix D). This protocol assisted in looking for purposeful questions that aligned with the study's definitions. Based on my observational protocol, purposeful questions were identified by the following characteristics:

a. Question built-on and/or extended student thinking in order to make sense of the solution

b. Question exposed mathematical connections that made the mathematics more conceptual and meaningful to students

c. Question encouraged students to clearly communicate and elaborate their thinking

Three of the recordings were also analyzed by a trusted, veteran mathematics education professor. He examined the recordings using the same observational protocol to ensure consistency of the study's definition of purposeful questions. Using collected student work, personal reflections, and video analysis from multiple individuals, I qualitatively uncovered the successes and challenges of posing purposeful questions in a mathematics tutoring session.

Instrumentation. One tool for measurement I used in my study was an observational protocol (See Appendix D) which I created to ensure consistency in defining the characteristics of purposeful questions. Although my journals were free-response, I included a list of successes and difficulties encountered in the session. I defined a success as any instance in which I engaged a student with a purposeful question and a challenge as any factor that restricted the use of a purposeful question.

Role of the Researcher. First and foremost, my role as the researcher was to tutor my students in mathematics. Second, my role was to collect, analyze, and share the findings that the data revealed. My role in the study was unique. Having just completed my final season as a softball player at Missouri State University, I brought a very relatable approach to my tutoring sessions with current athletes. This background aided rapport as I was able to create close relationships with my participants due to our common interests and my first-hand experience with being a student-athlete at the same university. The ability to create strong relationships, allowed students to be more comfortable engaging in dialogue and acting as co-participants in this research project.

Because this study had a personal viewpoint, I was aware of the opportunity for bias. As measures to reduce or eliminate the bias, I asked a professor to act as an outside observer. This encouraged me to be honest and realistic with myself in terms of my own weaknesses and strengths and successes and challenges. His observations protected the integrity of the study and promoted unbiased findings.

Data Analysis

Qualitative data analysis is the process of taking collected qualitative data and using it to form an explanation or understanding of the situation we are investigating (Bernard & Gery, 2010). To identify both expected and unexpected themes that emerged from the data collection, I analyzed data concurrently with implementing tutoring sessions (Merriam, 1998). Within qualitative research, triangulation, the use of more than one method to answering a research question, is very important (Forbes & Heale, 2013). By utilizing two or more independent measures, confidence in the findings is increased and a more comprehensive picture of the results is produced. In this study, the measures included video and audio recording of tutoring sessions, self-guided personal journal entries, and completion of an observational protocol.

Analyze of my data involved watching the recorded tutoring sessions and noting interactions that illustrated the successes of research question one or the challenges of research question two. Qualitative researchers build their patterns and themes from collecting details and building up to abstract units of information (Sharma, 2013). With each video recording, similar interactions would take place across multiple tutoring sessions, resulting in overarching themes to surface.

Once overarching themes were established, I began to seek supporting patterns embedded within the personal journal entries and observational protocols. What began as a large collection of unrelated data, revealed the many reoccurring successes and challenges that take place when posing purposeful questions. Examination of my journal entries and observational protocols, revealed futher patterns of successes and difficulties that supported the triangulation of an established theme.

Summary

This action research study took place during November and December of 2018. There were four participants enrolled in an entry-level math course at the University. Each student participated in five one-on-one tutor sessions. Each student brought his or her homework to the tutoring session in order to have material to work through. As the student worked to solve the problem, I focused on asking purposeful questions about the main ideas contained within the problem. Videotaping, audio recording, and personal journaling were used as instruments to collect data for analysis. The data collected help compile themes of both successes and challenges of posing purposeful questions in an individualized tutoring session.

CHAPTER IV: FINDINGS

This chapter describes the detailed findings of the analyzed data in regard to the successes and challenges of posing purposeful questions in a mathematics tutoring setting. These findings come from analysis of personal journals by the tutor, observation protocols, and analysis of audio and video recordings. Research question one is addressed first and exposes three themes of success encountered while engaging students with purposeful questioning. These successes include student engagement in the standards of mathematical practice, enhanced cognitive mathematical thinking by the students, and personal and professional growth for the tutor-researcher. The challenges of posing purposeful questions were the focus of research question two, which are outlined using three main themes. They include the nature of the homework problems, students' lack of conceptual mathematical experience, and a lack of mathematical confidence by both the tutor and the student.

Research Question One: Successes

The first research question focused on the successes of posing purposeful questions in a mathematics tutoring setting. By the study's definition, a posed question was purposeful if it fit within at least one of the following categories:

a. Question built-on and/or extended student thinking in order to make sense of the solution

b. Question exposed mathematical connections that made the mathematics more conceptual and meaningful to students

c. Question encouraged students to clearly communicate and elaborate their thinking

Based on the findings, purposeful questions encouraged the standards of mathematical practice, enhanced students' cognitive engagement, and promoted personal growth for the tutor-researcher.

Engaged Standards of Mathematical Practice. The Eight Standards for Mathematical Practice (See Appendix E), a list created by Common Core, outlines proficiencies of longstanding importance that mathematics educators should seek to develop in their students (National Governor's Association Center for Best Practice, 2010). This list varies from the Effective Mathematics Teaching Practices because the standards focus on what the *students* are doing, while the teaching practices focus on the actions of the *tutor* or teacher. After analyzing the data, the first theme to emerge due to the posing of purposeful questions was the promotion of the mathematical practices. By fostering the mathematical practices, successes sprouted as students were engaged in making sense of problems, constructing viable arguments, modeling mathematics, and looking for structure.

<u>Mathematical Practice One (MP1)</u>. The first mathematical practice states, "Make sense of problems and persevere in solving them" (National Governor's Association Center for Best Practice, Council of Chief State School Officers, 2010). Data analysis revealed students were hesitant to respond and think on their own, and they reacted negatively to incorrect answers. Therefore, it was evident students were accustomed to an environment absent of MP1. The absence of MP1 in previous mathematical experiences is a result of teachers being the mathematical authority, while students act as passive recipients of procedures that magically produce correct answers. In order for students to have an opportunity to make sense of problems, they must have the mathematical authority, not the teacher (Evans, 2017). By posing purposeful questions, I handed the mathematical authority over to my students which encouraged them to

accept more of the mathematical responsibility for their learning. They began using previous knowledge to make sense of problems, make new connections, and discover the *why* behind the steps they were once mindlessly using to solve problems.

Analysis revealed evidence of this type of thinking took place during a tutoring session over quadratics. The student was given the formula $h(t) = -16t^2 + 40t + 1.5$ to describe a ball being shot from a cannon. Rather than jumping right to the problems that followed, I transferred the mathematical authority to the student by purposing the question "What does this equation represent?"

Student: "The U-shape".

Tutor: "Do you know what we call the U-shape?"

No response. The student gave a head shake signaling he had no answer.

Tutor: "We call this a parabola. Let's take a look at its shape on Desmos."

Tutor used Desmos, an online graphing calculator, to display the parabola.

Tutor: "How does the parabola describe the relationship between height and time?"

Student: "As time is passing, the height of the ball is changing."

Tutor: "What do you mean?"

Student: "The ball gets higher, as time passes."

Tutor: "So the ball keeps going up?"

Student: "Wait, no. The ball rises and eventually comes back down."

Tutor: "Okay. That would make sense to a real-life situation. What do you think the 1.5 means in this equation? Where is that value represented on our graph?"

Student: "It is where our parabola starts on the axis."

Tutor: "Good. It is our y-intercept. What is the time at the point?"

Student: “Zero because no time as passed yet.

Tutor: “Awesome! Let’s look at question one.”

Time was given for the student to read question one, “How long does it take for the ball to reach the ground?” A couple seconds passed as the student thought about what to do before writing something down.

Student writes the equation with zero substituted for $h(t)$.

Tutor: “Why did you set the equation equal to zero?”

Student: “Because if the ball was at on the ground, then the height would be zero. *H* stands for the height, which is where the zero goes.”

Tutor: “Okay! Does this equation look familiar? “

Student: “Hmmm, I am looking for time, which means I am solving for *t*. I just forgot how to do that.”

Eventually within the interaction the student makes sense of the values of *t* obtained from factoring, identifies and describes the vertex, and clarifies the relationship between time and height within the function. It was tempting to just explain to the student the process for which to solve this problem but doing so would have stripped him of the opportunity to think. Instead, through the use of purposeful questions, the student was required to make *his own* sense of the problem and persevere through the reflection required to clarify understanding. Thus, purposeful questions had students engaging in Mathematical Practice One (MP1).

Evidence of MP1 also stood out within word problems as students were asked to simply provide characteristics of a reasonable answer before he or she even started the problem. Rather than going straight to a procedure after reading the problem, it was important for students to take a second to make sense of the problem and determine reasonable answers. For instance, when a student was asked about time or length, he or she knew the answer had to be positive. When solving for the measure of an acute angle, he or she knew the answer had to be less the 90 degrees. When asked about probability, he or she knew it had to be greater than zero but less than one. The sense-making step provided students with the ability to detect mistakes more easily as they knew what constituted as an unreasonable answer from the start. This aided the perseverance half of MP1.

Data analysis also revealed that purposeful questions encouraged sense making at the end of problems. For example, a student was confronted with a table for which he was to find the probability for selecting different colored marbles from a bag that contained eight red marbles, four blue marbles, three white marbles, and one yellow marble. Using his current knowledge, he filled in the chart with probabilities based on the idea that fractions are part over whole. He calculated a probability of .50 for the red marbles. If he typed this in, he would be notified his answer was correct, but his conceptual understanding of the problem would be questionable. I asked, “Is .50 a reasonable answer for choosing a red marble?”, “What does the .50 mean?”, “What would I do to increase the probability of choosing a red marble?”, and “Is that the only way I can increase the probability of choosing a red marble?” By questioning beyond the low-level given question, students were engaged in making sense of probability at a theoretical level.

The use of purposeful questions focused on refining the students’ understanding rather than funneling students to an answer using the tutors’ understanding. The goal became to clarify or build-on existing knowledge rather than worrying about answering the low cognitive-level question on the computer screen.

Mathematical Practice Three (MP3). The third mathematical practice states, “Construct viable arguments and critique the reasoning of others” (National Governor’s Association Center for Best Practice, Council of Chief State School Officers, 2010). In a regular classroom with 15+ students, this standard could be identified by students verbalizing and justifying mathematics amongst one another through classroom discussion or small group conversations. Within this research study, the tutoring sessions were individualized, so verbal communication between peers as identification was not a possibility. Instead, guidelines described by Evans (2017) were used to determine the surfacing of MP3. Evans (2017) described engagement in MP3 using eleven characteristics. Three of the characteristics included students making conjectures, building a logical progression of statements to explore the truth of their conjectures, and justifying their conclusions. Because these three characteristics appeared within this research study, it was evident that students were engaging in Mathematical Practice 3.

Data analysis provided an interaction that supports the engagement in MP3 within a tutoring session involving linear equation word problems. The student was given the following question: Sandy buys a book for $14.70, which is a 30% discount off the regular price. What is the regular price of the book?

After some wait time, the student writes: $14.70 = .70x$

Tutor: "Why did you write that equation?"

Student: "Because if the item is 30% off then I am only paying the remaining 70%."

Tutor: "Okay. So, what do you get for an answer?

Student: "21"

Tutor: "What does your answer mean?"

Student: "The regular price of the book is $21."

Tutor: "Is that a reasonable answer?"

Student: “Yes, I know the answer has to be higher than \$14.70 because the book cost \$14.70 *after* a discount.”

Within this interaction, the student started with her own conjecture and justified her reasoning with truth logical. Her viable arguments were evidence of engagement in MP3.

As seen in the interaction above, wait time attributed to the impact of the many of the purposeful questions posed within this study. Wait time played a big role in the engagement of MP3. In order for students to create viable arguments, they were provided with the amount of time required to think and form these viable arguments. If an explanation would have been given directly after the question was presented, the student would have been denied the opportunity to engage in MP3. As students faced homework problems, wait time allowed them to think deeply in order to process what the question is asking, determine a strategy to solve the question, and ultimately form the viable arguments MP3 suggests.

In the discount example above, the student happened to start with a correct conjecture, but in other situations when students were given the opportunity to verbalize their justification, the power of *incorrect* answers became impactful. When wrong conjectures were verbalized, or false justification was used, the use of purposeful questions exposed misconceptions and improved mathematical understanding.

For example, one student was able to solve $x^2 - x + 12$ with ease. After he typed in the correct answer, I asked him why he set both $(x - 4)$ and $(x + 3)$ equal to zero. His response was like most students, “It is how my teacher told me to do it.” There was no justification or understanding of the zero-factor property for which permits this procedure to work. By following a student’s conjecture with a purposeful question “Why?” the student was required to explore and justify his reasoning. Rather than critiquing the reasoning of others, these purposeful questions encouraged the student to generate and critique his *own* reasoning. Thus, the student was engaged in MP3 as a result of purposeful questioning. If the student was not asked to verbalize his answer, I would have had no way to determine the truth of his reasoning or identify the need for clarification.

Mathematical Practice Four (MP4). The fourth mathematical practice states, "model with mathematics" (National Governor's Association Center for Best Practice, Council of Chief State School Officers, 2010). There were three tutoring sessions for which the student did not have any homework problems to complete. I decided to take the opportunity to discuss the topic of factoring. I tried to question students deeply about the mathematical meaning of factoring with the goal to expose the connection between algebraic functions and their corresponding graphs.

Each factoring lesson varied based on student response, but they all three started off with the tutor asking them to factor and solve the following quadratic: $x^2 + 3x - 4 = 0$.

One student immediately solved the problem to be: $x = -4$ and $x = 1$.

Tutor: "What does the solution $x = -4$ mean?

Student: "Negative four is a solution."

Tutor: "Go to Desmos and type in the quadratic."

Tutor: "What do you notice?"

Student: "The graph is a parabola."

Tutor: "That is true. What is the relationship between your solutions and the graph?"

**Students clicks different points on the graph."

Student: "The graph crosses the x-axis at the two solutions."

Tutor: "What do we call points that cross an axis?"

Students: "Intercepts."

Tutor: “Based on the graph, can you write your solutions as ordered pairs?”

Student writes $(-4,0)$ and $(1,0)$

Tutor: “Can you describe these two solutions in terms of inputs and outputs?”

Silence

Tutor: “What happens when I plug in $x = -4$ into the given equation?”

Student: “We get zero.”

Tutor: “Or an output, or y-value, or zero. Now let’s look at the graph. If I plug in $x = -4$, my y-value or solution is zero.” (*points finger at the point (-4,0))* What if I plug in $x = 0$? What is my solution?

Student points finger on $x = 0$ and slides down to the point (0,-4)

Student: “Negative four.”

Tutor: “Now plug in 0 for x into our function and see what happens.”

Eventually, within this interaction, the student was able to discover that the ordered pairs on the graph were solutions to the function. She was able to use the geometric model of the function to make connections to its algebraic solutions. In doing so, she engaged in MP4 by identifying important qualities and mapping their relationship on such tools as a graph (National Governor’s Association Center for Best Practice, 2010). Based on the verbal and non-verbal responses of amusement, she had never made this connection before.

Mathematical Practice Six (MP6). The sixth mathematical practice states, “attend to precision” (National Governor’s Association Center for Best Practice, Council of Chief State School Officers, 2010). As within many mathematics settings, the precision of notation and symbols is critical. It was important that a student understands that $\leq$ has a different meaning than $<$, that a negative sign can change an entire answer, and that squaring a value is not the same thing as square rooting a value. MP6 was easily identified in the manner of notation and symbols, but it was also identified in terms of precision to vocabulary. Many of the purposeful questions centered around the vocabulary students were using to communicate their reasonings. Although it was frustrating at times for students, it was important that they were equipped with precise vocabulary to communicate effectively and understood the meaning of the words they were using to justify. This strictness on vocabulary and notation engaged students in MP6.

Precision of vocabulary forced me to listen carefully to how students were communicating their mathematics and allowed me to informally assess students' levels of understanding for both vocabulary words and bigger mathematical concepts. There were times in sessions when students would justify their answers to me, but they would run into gaps with vocabulary that blocked them from clearly communicating their reasonings.

Data analysis revealed students eventually used the vocabulary they had available to provide a convincing argument that they had reached understanding. At times, it was tempting to stop here and be excited at the glimpse of wisdom, but I would have done my students a disservice by not holding them accountable or equipping them with precise vocabulary. A particular example of MP6 was when a student was solving for a greatest common factor within the polynomial, $25x^3 + 15x^2 - 5x$. The student took out the factor $5x$. When asked how he got that answer, he said, "That is the greatest factor I can take out of each number." This interaction introduced the vocabulary word *term*.

Data analysis suggested replacing "little number" with exponent, "number in front" with coefficient, and "the bottom of the fraction" with denominator were all instances of engagement in MP6. The differences in the words *solve* verses *factor* and *equal* verses *equivalent* also required precision. By holding students and myself accountable for the words we were using, I was fostering deeper learning and filling in gaps of understanding. The power of the words used throughout the tutoring sessions demanded precision.

Mathematical Practice Seven (MP7). The seventh mathematical practice states, “look for and make sense of structure” (National Governor’s Association Center for Best Practice, Council of Chief State School Officers, 2010). Based on the structure of the procedural, repetitive homework problems, the purposeful questioning enabled students to look for structure in previous problems to help them solve new, similar problems. A common question for engagement in MP7 was, “What do you notice is similar or different from the previous problem?” This question directed students to find patterns and look for connections that could lead to similar thinking and strategies within a new problem.

The following conversation provides an example of engagement in MP7:

> ***Student is presented with scientific notation problem 5.42×10^{-5} in which he is asked to convert into expanded form. The tutor writes the previous problem and current problem side by side on the board. ***
>
> Tutor: “What do you notice about these two problems? How are they different? In what ways are they the same?”
>
> Student: “The exponent is the second problem is negative.”
>
> Tutor: “Since both problems are asking the same question, how do you think you will solve this?”
>
> Student: “I think the answer is going to be negative.”
>
> Tutor: “Why?”
>
> Student: “Because a negative and a positive make a negative.”
>
> Tutor: “Good thought. What did the exponent in the first problem have us do?”

Student: "Oh, we are going to move the decimal left this time because we went right last time when the exponent was positive."

Using questions to engage students in the strategy of identifying structure provides a holistic view of concepts rather the departmentalized, unrelated questions. I realized early on in the study that equipping students with the ability to manipulate a new problem into a familiar problem was critical to conceptual learning.

Data analysis revealed the use of concrete examples to help transfer structure to more abstract concepts was another way which purposeful questioning promoted MP7. When students were confronted with an abstract idea, rather than giving away answers, I worked to represent the abstract concept in a more familiar and concrete example. This allowed students to find structure within a more accessible problem then make use of that structure within an unfamiliar problem.

For example, one student was evaluating absolute value equations. When confronted with a multi-step problem he became hesitant and struggled to determine what operations would get the absolute value bars by themselves. Knowing he was very comfortable with solving single variable equations, I gave him the same problem but replaced the absolute value bars with an "x" and asked him to solve. Effortlessly, he did. After seeing the connection between the two problems, he was able to see the absolute value problem in a more familiar way, and thus utilize the structure on a new problem.

Another example of this transition was encountered in a word problem. The student was asked to write an equation involving the phase "10 less than x." He proceeded to write "$10 - x$." I followed this action by asking a more concrete question. "What is 10 less than 25?" The student snickered and replied with the correct answer. I then asked, "How did you get that answer?" A little puzzled, the student responded with, "I did 25 minus 10." Taking it full circle, I asked, "Based on how you solved that problem, how should we write this equation?" The student was able to make the connection and correct himself. The use of procedural or funneling questions throughout an abstract problem would have guided student to a correct answer, but purposeful questions gave students the cognitive control and engaged them in the important problem-solving strategy of finding structure.

Enhanced Students' Cognitive Mathematical Engagement. The second theme of success to emerge from data analysis was the ability to help students make sense of problems, discover connections, and gain conceptual understanding of mathematical ideas. Rather than worrying exclusively about getting a correct answer, the purposeful questions required students to think deeply about concepts embedded in the answer. Prediction, connectivity, and verbalization are all patterns found through data analysis that enhanced students' cognitive mathematical engagement.

<u>Power of Prediction.</u> Rather than students immediately hitting the paper with a straight shot procedure to lead them to an answer, purposeful questions were posed that allowed the power of prediction to spark a deeper cognitive demand. By asking students to predict reasonable answers or describe characteristics of a reasonable answer, I was able to quickly gage the level of understanding brought into each question. For example, when a student was confronted with a problem about finding a discounted sale price, before a pencil hit the paper, I asked, "What amount do you predict the correct answer to be?" I was not looking for her to give me a prediction down to the cent value. Instead, I was assessing her number sense and looking for her to know that if it was discounted then the new sales price should be a lower number. This technique got students thinking from the start without even having to write anything down.

Within one session, a student was being asked to manipulate unit conversions. His logic to solve the problems was to make sure matching units were on opposite sides of the fraction. Not convinced he had the understanding I desired, I asked him to predict an answer before solving. This allowed me to listen to him reason through the relationships of different units in terms of what units of measurements were larger or smaller.

Connected Concepts. Analysis would suggest the most desirable mathematics learning is holistic and connected. Through purposeful questioning students were able to discover relationships between mathematical concepts. An example of holistic cognitive engagement found within the data analysis occurred when working with equations of lines. The student was given the following equation for a line: $2y - 4 = 4x$. The homework question asked him to write the equation in slope-intercept form. In a regular setting, if the student knew that slope-intercept form was $y = mx + b$, then he or she would be able to get the correct answer quickly, but without verifying he or she actually had understanding of the problem. For this reason, after the student used algebra to correctly manipulate the equation to $y = 2x + 2$, purposeful questioning were used to evaluate and expand understanding.

First, I graphed $2y - 4 = 4x$ in Desmos, an online graphing calculator. I then asked, "What do you predict the graph of your new question to look like?" The student responded with, "I don't know. The same?" Based on the hesitation, I pushed further by asking, "What would be the relationship of the equations if their two graphs were the same?" The student's inability to answer this question sparked, "What information does a graph represent in relation to its equation?" Still reluctant to answer, I then asked the student to pick out a point on the graph. "What do you think will happen when you plug that point into the equation that was given to you?" After seeing that the point was a solution to the given equation, we tried the same point in his new equation. After seeing the connection, but not being fully convinced, he selected another point on the graph to verify his findings once more. Thus, this realization allowed him to see that the graphs were displaying two equivalent solution sets, which verified his answer. Although the two equations looked different, by showing the geometric representations the student was able to see that the solutions sets were the same for both, making the equations equivalent. This purposeful interaction exposed students' conceptual understanding of algebraic manipulations conserving equality and the concepts of graphs displaying solutions sets for an equation.

Verbalize Mathematics. A lot of the increased cognitive mathematical engagement came from getting the students talking aloud. As students spoke their reasoning, it solidified the understandings they had or disclosed the understandings they lack. The less I talked, the more student were required to think critically and form their own conclusions, whether right and wrong. In the past, I was guilty of trying to take mathematical ideas to a deeper level through my own explanation. I thought that if I said it to the student, then he or she would "get" it. Unfortunately, I learned that that is not true. Instead, by focusing on asking better questions based on what the students were verbalizing, they ended up making their own discoveries and connections based on their own previous knowledge. "Young children lack knowledge, but they do have the abilities to reason with the knowledge they understand" (Bransford, Brown, & Cocking, 2000). Purposeful questioning passed the mathematical authority to the students and provided them with opportunities to reason.

Promoted Personal and Professional Growth. There is evidence to suggest that posing purposeful questions has many successes in terms of student learning. After further analysis of my research journey as a whole, a third theme of success emerged for me as the tutor-researcher.

Patterns within this theme include improved questioning abilities, richer mathematical understanding, better anticipation of student understanding, and an enhanced love for teaching.

Improved Questioning and Understanding. One major success of posing purposeful questions was simply the improvement of my own questioning abilities. This experience shifted my philosophy of asking questions from a quantitative focus of correct answers to more of a qualitative focus on conceptual understanding. With each tutoring session I conducted, I saw my ability to ask purposeful questions improve in terms of finding ways for students to engage in rich thinking. Rather than questions like, “What do I do next?” and “Do you agree?” I started creating habits of asking questions that furthered student thinking and exposed mathematical connections.

This improvement of questioning is directly correlated to the growth of my own conceptual understanding. By asking students purposeful questions, the responses were unpredictable. The nature of purposeful questions not only pushed students to think deeply about underlying connections but also pushed me. Many of the purposeful questions I asked were a result of my own curiosity and desire for richer understanding. As a rookie teacher, the only mathematic perspective I had going into this research was my own, but by asking questions that promoted students to vocalize their understanding, I was able to hear different perspectives about certain concepts. These perspectives improved my own mathematical understandings and equipped me with proficiencies to pose better questions as a tutor and a future teacher.

Anticipation of Student Understanding. Another area where I saw growth throughout this research study was in my anticipation of student understanding. By posing purposeful questions, I was able to see first-hand the learning that takes place when students possess the cognitive control. Because student thinking was guiding the sessions, I learned how to adjust questioning and remain poised when students brought me down unexpected paths. With each session, I gained new experiences that equipped me for future student interactions. I was able to anticipate areas that students would have misconceptions; and therefore, be confident and prepared to purposefully question to clarity. Based on my video recordings, it was clear that if I was helping three different students with similar homework concepts, by the third student I was much more equipped to question purposefully. The experience I have gained towards student reasoning helped me grow as a tutor within this research study, but it also promoted my questioning as a teacher in a future classroom.

Love for Teaching. Lastly, my research helped my love for teaching mathematics grow. The opportunity to work one-on-one with students and engage them in rich mathematical thinking brought me so much joy and excitement about being a mathematics educator. The expression of students when concepts finally clicked reminded me that students do have a desire to learn, and that I have such a special opportunity to help them do so. The process strengthened my confidence, passion, and hope as a future mathematics educator.

Research Question Two: Challenges

When implementing purposeful questions, the challenges that arose seemed to be much more prominent and easier for me to identify compared to the successes. Three themes attributing to challenges surfaced as a result of data analysis. The first challenge was the nature of the homework problems my students were trying to solve. The second challenge was the students' inexperience with conceptual mathematical thinking. Thus, students were unable to use appropriate vocabulary, had a lack of prior knowledge, and struggled to see the purpose for deeper questioning beyond “right answers.” Lastly, there was a great lack of mathematical confidence in both my students and me.

Nature of the Homework Problems. This study reflected a majority of tutoring settings in that students brought in their homework for the day with the goal to complete it with high accuracy. This plan presented some challenges to my purposeful questioning due to the nature of the homework problems. Types of questions asked on the homework and the number of problems assigned on each homework were both patterns discovered through data analysis. Both caused friction for purposeful questioning.

<u>Types of Homework Problems.</u> The first challenge to this approach was that the homework problems students were being asked to solve were concrete, low-level, right-or-wrong type of questions. Questions such as “Solve for x.” or “Evaluate $\sqrt[2]{-16}$” or “Find the slope.” can be answered correctly without actually assessing students understanding of underlying concepts. These questions were not tailored to try and get students thinking deeply about the meaning of the mathematics behind the answers; instead, they encouraged a mindless procedure that led to a “Great Job” or a “Nice Work” icon indicating that their answer was correct. The questions assigned were what Myhill and Dunkin (2005) would have categorized as factual due to their predetermined answers and methods.

These types of homework questions made it a challenge for students to see the purpose in engaging in deeper questioning and understanding. To students, if they were able to get a correct answer, then it was validation that they knew the material. However, when questioned to verbalize a strategy or provide justification for their procedure, students were unable. Students were getting the correct answer but lacked the deeper mathematical understanding that education should value. The types of problems I was helping students solve encouraged me to ask more, “pretend” purposeful questions, such as “Do you agree?” or “What do we do next?” rather than fewer, more purposeful questions.

Amount of Homework Problems. Not only was the type of homework questions a challenge to purposeful questioning, but also the amount of homework problems. Within each 30-minute session, there was a conflict of interest. The student and I had opposing goals for the tutoring session. Students were trying to finish a batch of 25 questions in order to complete the homework, while I valued spending 10-minutes on one problem in order to ask the necessary purposeful questions to reach meaningful understanding. In one journal I wrote, “I felt frustration from the student because we only completed three problems.” In another I wrote, “I felt like I cheated the student of understanding on some of the problems in order to get through more of the homework problems.”

There were a couple of sessions where students brought in no assigned homework problems, so the conflict of interest was eliminated. Without the stress of completing a large amount of problems, I felt less pressure to rush and more freedom to explore and question purposefully. These lessons included displaying a single problem and questioning students purposefully to build rich understanding. There was one lesson in particular that the student and I spent the entire 30-minutes on one problem. Although we completed one problem, my journal for that day expressed that I felt like learning was achieved and the student left excited. The student gained conceptual understanding and mathematical connections were exposed, but the challenge was in the amount of time it required. It would have been much faster to just tell the student the necessary conclusions, but this would have weakened the learning. The number of questions asked created friction with the time restraint, thus, providing a challenge for purposeful questioning.

Inexperience with Conceptual Mathematics Thinking. At first, many of my students resisted the need and challenge of purposeful questioning. Up until the time we worked together, their mathematical careers were strictly procedural and survival. This deeper type of mathematical thinking was different, difficult, and threatening. This was evident through the nonverbals, gaps in understanding, and words of insecurity and frustration that were present within the tutoring sessions. The inexperience with conceptual mathematics was the second theme to emerge within the second research question. Inexperience presented challenges as students were not bought in to the importance of purposeful questions and conceptual thinking and lacked the necessary skills, such as verbalization of mathematics, to take the learning to a deeper level. My own inexperience with conceptual teaching also presented challenges as I was ill prepared for the mental demand required for purposeful questioning. I also asked questions without a specific learning goal and lacked the ability to identity potential learning opportunities.

Lack of Buy-In. Getting students to buy-in to the purposeful questioning was the first challenge of their inexperience. I had to work to convince students that I was not trying to make them feel inadequate or waste their time. Many times, when asked to provide justification, students automatically assumed I was asking because their answers were wrong. It was challenging to build an atmosphere where each session the student saw the questioning as purposeful when he or she had made it through his or her entire mathematical careers without needing any underlining concepts. I had sessions where students thought that if they said, "I don't know why," enough times or stayed silent for long enough that I would eventually give in and just tell them how to get the answer. Although tempting at times to take back the authority, I had to remind myself that the perseverance and patience required was aiding student learning.

Because students had never been required or encouraged to buy-in to conceptual understanding before, students came into the tutoring sessions expecting not to think. There were instances when students would glance at the problem and immediately announce, “I have no idea how to do this problem.” My first thought was that there was no way this student even read the question, but in reality, students were resisting the ability to have to think and problem-solve. Students were not just lacking a background in conceptual understanding; instead, they were lacking the strategies and desire to *think.* In one journal, I wrote that it felt like students thought they could get to an answer without having to think. These beliefs were rooted in their previous mathematics environments where they became accustomed to being asked low-level questions and using a meaningless procedure to produce a correct answer. When they were faced with a problem which a procedure was not obvious, they resisted. There were times when the resistance was a result of laziness, but other times it was as if they believed by having to think they were not intelligent. Students paired being smart with knowing procedures. Combatting these misbeliefs was a challenge.

The previous mathematical environments of students introduced behaviors that paralyzed them from taking chances, exploring possibilities, and utilizing recourses, which are all characteristics of problem solving. For example, a student was asked to manipulate a line in slope-intercept form to standard form. After working the algebra, I asked him if there was a way we could verify the new equation was correct. He looked at me for a while, then I suggested using an online graphing calculator. He replied, “I was going to say that, but I thought that would be cheating.” This is just one instance in which I was presented with the challenge to push back the unproductive norms of previous mathematical experiences. In the tutoring sessions, students were required to not only think to get an answer, but also to think in order to justify and communicate their answers.

Lack of Vocabulary. Because students lacked experience with conceptual learning, they were not equipped with the vocabulary or strategies required to verbalize mathematics at a conceptual level. Students lacked the necessary vocabulary to access the purposeful questions I was trying to ask, which was a challenge. This required me to have to make adjustments in my questioning in order to find what understandings each student had and build connections from their various starting points. For example, the goal of one of my tutoring sessions was to help students understand the reason for factoring. I knew the question was going to be inaccessible to the student at the start, so I had the challenge of building on vocabulary words like terms, factors, and intersection in order to provide the necessary tools for accessing a question like “why do we factor?”

The lack of vocabulary was accompanied by gaps in understanding. The challenge was for me to make decisions about what was beneficial for students to struggle through and what I was going to have to provide. There were many times when I would ask a question and a student would struggle to provide an explanation. They would use incorrect vocabulary or say, “I know what I did, but I don’t know how to explain it.” These instances made purposeful dialogue difficult and tempted me to funnel students to the desired answers I wanted rather than keep them as the mathematical authority. The gaps presented challenges of helplessness and frustration as I worked to question purposefully but felt like I was getting nowhere. With a past of struggling through mathematics fueled with an inexperience of ever engaging in conceptual mathematics, many of them did not believe they would be able to understand these deeper concepts.

Tutor Inexperience. My own inexperience with concpeutal teaching exposed some challenges. The first being the mental and emotional demand required to promote conceptual understanding through purposeful questioning. Encouraging students to think in new ways, thinking critically about purposeful questions to ask, and persevering through student resistance left me mentally exhausted at the end of every tutor session. Although the students had the mathematical authority, this type of questioning required me to think deeply about what students were saying and quickly determine the necessary questions that would promote and/or clarify their understanding. I was required to make on the spot decisions that had purpose and promoted learning. This was a challenge.

Another area of challenge based on my inexperience with conceptual teaching was the lack of a learning goal for the purposeful questions I asked. Based on the comments from the outside observer, I found that many of my "purposeful" questions were not advancing students to a particular learning goal. I was asking questions without actually having a clear goal of what understandings I wanted students to develop. In a tutoring setting, I do not have the opportunity to set a learning goal. Without a clear learning target, I asked questions for which I was unable to justify the understandings I was trying to help students develop. Thus, it is challenging to ask purposeful questions to begin with, but an even greater challenge to ask purposeful questions that are always aligned with a clear learning objective.

Along with the absence of learning goals, the outside observer also exposed my lack of expectations for my student responses. My observer wrote, "What type of answer would you like to hear that would suggest to you that the student has the deeper conceptual understanding that you desire?" After reading this, I realized I did not have an answer. This uncovered that I was so focused on asking purposeful questions that the challenge of anticipating student responses and interpreting them in such a way that suggested strong understanding was neglected. Based on my need for clear learning goals and the inability to determine what constitutes as a correct answer, I found that the biggest challenge was not actually the posing of purposeful questions, but instead, the interactions after a purposeful question. The art of purposeful questioning brings challenges in many directions due to the unpredictable nature of student responses and inability to be prepared ahead of time in a tutoring setting. Staying true to using student responses as the path for instruction was demanding yet rewarding.

Missed Opportunities. Through analysis, I was able to identify that my own inexperience caused me to miss opportunities to press student learning. For example, after a student manipulated an equation, I asked him if the original equation and new equation were equivalent. He responded with the correct answer of yes, and I moved forward. The purposeful follow-up would have been, "Can you convince me that these two polynomials are equivalent?" Again, the unpredictable nature of how a lesson is going to go when you pass the mathematical authority to the student presents many challenges. My inexperience with the transfer of authority made me susceptible to missed opportunities for evaluating students' deeper understanding.

Lack of Mathematical Confidence. The final theme to emerge within the challenges of posing purposeful questions was the lack of mathematical confidence by both the students and me as the tutor-researcher. For students, patterns of low self-efficacy and a dependence on authority for affirmation prevented them from taking chances, enjoying the challenges, and believing that they were capable of gaining understanding. As for myself as the tutor-researcher, my absence of self-efficiency robbed students of potential discovery and my lack of confidence in my students caused me to condone negative practices of dependence.

Absence of Self-Efficacy. As I posed purposeful questions, before I could even worry about arriving at an answer, the first challenge was helping students believe they were even capable of gaining mathematical understanding. For many of my students, their mathematical pasts were filled with poor mathematical experience. They had convinced themselves that they were not "math people" and were never going to be good at it. The previous right-or-wrong, procedural approach completely hypnotized students into believing that mathematics was a subject that was inaccessible to them. This lack of confidence blocked even the first step of engaging in deeper mathematical thinking.

Not only was self-efficiency absent for students, it was also absent for me at times. My lack of experience and inability to prepare for tutoring sessions caused self-doubt which sometimes paralyzed my ability to ask purposeful questions. I was worried about students leading me to situations for which I did not know the answers. The unbelief in myself to be able to pose purposeful questions caused me to take back the mathematical authority and resort to my prideful mathematics abilities. To restore my credibility and mask my insecurities of asking purposeful questions, I would funnel students to a correct answer. This took away opportunities for students to make mistakes and learn from them. I journaled about how as a tutor I felt like I should never get incorrect answers to questions. I thought that if students missed a question with me that my creditability was weakened. My need for confidence took away from learning opportunities. I realized that my desire to "save students" was not only coming from a place of concern for them, but also a place of concern for me.

Dependence on the Authority. As students took the lead on working through a problem, their lack of confidence led them to rely heavily on me as the ultimate authority. After every decision made throughout the problem-solving process, students would look back at me for affirmation or ask, “Am I wrong?” These gestures suggested a lack of confidence as students did not hold conviction for the problem-solving strategies they were choosing to use. It also suggested the fear they had of being wrong. Wrong answers to many of my students meant failure rather than an opportunity to learn from their mistakes.

Although hesitant to admit, students’ dependence on authority also condoned by my actions as the tutor. There are numerous occasions where I verbally expressed belief in my students and worked to help build their own confidence, but my actions translated the contrary. As students worked on a problem, the minute I saw them start to make a mistake the urge to correct them would overtake me. That need to “take over” or “take control” was unintentionally communicating that I did not believe they were capable of discovering mistakes or solving the problem. By taking over, I was sending the message that they were not capable of reaching conclusions on their own; instead, they were reliant on me to get the right answer. Confidence required balance in a tutoring session in order for a me to both have the confidence to hand over the mathematical authority and be humble enough to allow students to make mistakes.

Summary

Many successes arose as a result of purposeful questioning. Students’ conceptual mathematical engagement was one of the biggest successes. Purposeful questioning invited Standards for Mathematical Practices one, three, four, six, and seven into the tutor sessions, which promoted quality mathematical strategies and professional growth. The main challenge of asking purposefully questions was equipping students with the proficiencies and confidence to take the first step in solving any type of problem: thinking. Ensuring students are doing the thinking was cognitively demanding on the tutor. Being intentional about asking purposeful questions required commitment and perseverance as I worked to change the culture of mathematics for my students. Although the nature of the homework problems did not foster purposeful questioning, students were able to gain conceptual mathematical understanding by the questions posed by the tutor about the homework problems.

CHAPTER V: DISCUSSION AND RECOMMENDATIONS

The Eight Effective Teaching Practices were created to provide educators with tools to help elevate their own teaching and in return foster rich student learning. This action research study identified the challenges and successes that accompanied the teaching practice of posing purposeful questions. As a tutor-researcher, I believe the successes and challenges of posing purposeful questions uncovered through my personal experience within this case study will help teachers gain the confidence to implement purposeful questioning in their own tutoring sessions, or in classrooms. When looking at the findings as a whole, it is interesting that many of the challenges involved myself, where many of the successes were gained on behalf of the students. The persistence through the challenges as a researcher-tutor was worth it for the result of meaningful student learning.

Discussion

The purpose of this action research study was to gain insight into the successes and challenges of a tutor when implementing purposeful questions in a mathematics tutoring setting. The tutoring sessions within this study included one student. To keep the setting as authentic as possible, the homework requirements that each student brought with them were used to guide the trajectory of the sessions. This required the questioning to be genuine and realistic. From the personal reflections and recorded sessions, I found many successes of posing purposeful questions. One success was that students were being encouraged to engage in mathematical practices such as creating viable arguments and modeling with mathematics. Engaging in these mathematical practices, then enhanced students' cognitive engagement. Rather than getting right answers with a procedure, they were making sense of problems and discovering mathematical connections about correct *and* incorrect answers. Lastly, alongside student learning improvement was my own improvement as a tutor. These improvements included gaining the ability to anticipate student misunderstanding, gaining the confidence to hand over the mathematical authority to students, and gaining passion for the excitement of rich mathematic learning.

It was not surprising that these successes were accompanied with some challenges. The nature of the homework problems and students' inexperience with conceptual thinking were two of the challenges encountered. My lack of confidence also created some personal challenges for myself. Purposeful questioning required me to think critically due to the amount of unpredictability within each session. It was unclear what understandings a student was bringing with him or her, and it was important that my questions focused on clarifying and strengthen their connections rather than sharing my own. There were times when I felt discouraged and doubted the worth of the questions I was asking. However, based on the feedback from students and the advances of student learning revealed in the data, purposeful questioning is necessary to an effective mathematics learning environment.

The start of this research reflected the studies of Myhill and Dunkin (2005), Teodoro et al., (2011), and Babu and Mim (2017) as they classified questions posed within a lesson. This strategy would reveal if I was indeed asking purposeful questions within a tutoring setting. By the end of the research, I realized that the posing of purposeful questions was *the easy part*. Instead, data analysis suggested that the successes and challenges of posing purposeful questions were not so much the asking the questions but instead were the surrounding factors of the posed questions.

Wood (2002) wrote about differences in students' thinking being attributed to the type of questions teachers ask. Data analysis supports this to be a valid proposition as purposeful questions influenced the thinking and reasoning of students within the tutoring sessions. Stump (2010) stated that educators believe purposeful questioning is important but lack the necessary skills to implement such questions. Again, data analysis supports this claim due to evidence of inexperience being a challenge to implementing purposeful questions within this study. Lastly, Stolk (2013) suggested the power of modeling questioning in order for students to adopt quality questioning habits. Based on data analysis, this claim was upheld as students learned the characteristics of a purposeful questions through repeated implementation by the tutor. By the end of the study, the role of asking purposeful questions became a joint responsibility.

Comparing these research results to other literature, there are many consistencies. Teachers often hold the perception that they must be the holder of the knowledge and are fully responsible for student learning, as opposed to the students being responsible for their own learning, which could discourage teachers from wanting to venture into areas where they may not feel fully knowledgeable (Robitaille & Maldonado, 2015). This pressure was felt often in my tutoring sessions, which restricted me from trusting my ability follow student's understanding.

Recommendations for Future Research

This action research study provided great insight for educators as it pertains to the implementation of posing purposeful questions. Due to the data uncovered by just one of the eight effective mathematical teaching practices, purposeful questioning, it is recommended more research studies be conducted about the implementation of the other effective mathematical practices. It is one thing for someone to conduct research and determine effective teaching practices, but another to study the transfer of these practices from theory to action. Research is needed about the implementation of the practices with real students in authentic teaching settings. Because this study was conducted in a tutoring setting, continuing the study of purposefully questioning at the classroom level as it might affect the transfer of authority and the mathematical discourse opportunities encouraged by purposeful questions.

Recommendations for Future Practice

Because this study took place in an individualized setting, there are some underlying differences between the environment of a tutoring setting and an environment of a classroom. Several findings from this study can transfer into classroom implementation.

The following is a list of recommendations for future practice:

1. Verbalize Mathematics. Classrooms and tutoring sessions should be filled with the voices of students speaking about mathematics. Rather than just watching students complete a procedure, have students verbally reason through a problem and justify the strategies they choose to use.
2. Demand Precision. Students should not only be verbalizing mathematics, but they should be verbalizing mathematics with precise vocabulary. Teachers must demand a verbal precision from their students and their selves. Modeling the use of precise verbalization and purposeful questioning helps construct a framework for which students can imitate.
3. Shift Culture. The effectiveness of purposeful questioning depends on the type of culture established where they were asked. Educators must make a culture shift within mathematics environments, including classrooms and tutoring sessions, that emphasizes conceptual understandings over correctly answering procedural questions.

In order for effective learning to take place and purposeful questions to have an impact, a teacher must establish a positive and supportive classroom culture where students are encouraged to think deeper. The effectiveness of a purposeful question is determined by the culture in which it is asked. This culture shift much seep vertically to all levels of education, including secondary, middle, and primary classrooms. This culture shift will deduce the challenges to posing purposeful questions and foster the successes of posing purposeful questions.

4. Build Relationships. The first step in a culture shift is to build relationships with students. Once students know you care about them and value their learning, they stop believing you are using purposeful questions to embarrass them and start trusting you are taking actions to further their learning. Within a strong relationship, if the teacher is committed, the student is likely to follow.
5. Prepare Effectively. As a tutor, I went into the sessions blindly, so I did not have the opportunity to prepare purposeful questions ahead time. This presented challenges. This study would recommend educators preparing in advance in order to conceptual understand the topic themselves and prepare purposeful questions ahead of time.
6. Inform Students. The study would suggest that teachers need to be transparent about their actions and the reasoning of their actions. In this study, the participants were aware I was investigating the use of purposeful questions. This awareness helped them buy-in to their implementation. As educators, transparency with students is critical to the actions we take.
7. Provide Wait Time. When purposeful questions are posed, students have to absorb the questioning being asked, make sense of the problem, decide of a strategy, and

determine how to verbalize their actions. All of these steps require time. Providing wait time always breaks the barrier that being fast at mathematics makes you better at mathematics.

8. Pose Purposeful Questions. Based on the findings, this study would recommend educators focus on asking better questions within their classrooms in order to hand the mathematical authority over to the students. I recommend teachers put great thought into the homework they assign and great consideration for the learning goals they prioritize.
9. Implement Effective Mathematical Teaching Practices. Because of the success gained from implementing one of the eight effective mathematical teaching practices, I would also recommend the implementation of the other practices. This may include facilitating meaningful mathematical discourse or establishing mathematics goals to focus learning.
10. Reflect Often. Whether new or experienced, quality teachers should strive to improve yearly, weekly, and even daily. This improvement can only take place through intentional and thoughtful reflection. Throughout the study, I was required to journal after each tutoring session in order to recognize challenges and acknowledge successes. I would recommend this reflection practice to all educators within any setting.

Notice that the study did *not* recommend asking *more* questions. Instead, this study would encourage the opposite. Educators can ask less questions, if the questions asked are more purposeful. A single purposeful question can lead to a very powerful, rich discussion. The need for classroom practices to shift from procedural to conceptual is imperative if we want students gaining conceptual mathematics understanding. Based on the study, one of the first and quickest steps for this shift is to place focus on asking purposeful questions.

Summary

The purpose of this action research study was to gain insight into the successes and challenges of a tutor when implementing purposeful questions in a mathematics tutoring setting. While students worked through their homework problems, purposeful questioning were implemented to bring out the rich conceptual meanings embedded within the problem rather than guiding students through a procedure that would procedure a correct answer. Because the homework problems did not aid a conceptual understanding, I was required to be diligent and strategic about the type of questions posed to help students form connections and make sense of problems. The goal of the purposeful questions was to give students the mathematical authority to make sense of problems for themselves and vocalize their understandings.

Although this study was conducted in a tutoring setting, many of the conclusions gained can be transferred for use in classroom practice. With student learning as the focus of the education system, the challenges of asking purposeful questions should not stop by from experiencing the successes.

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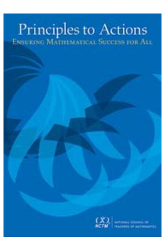
APPENDICES

Appendix A: Effective Mathematics Teaching Practices

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.



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Writing Team: Steve Leinwand, Daniel J. Brahier, DeAnn Huinker, Robert Q. Berry III, Frederick L. Dillon, Matthew R. Larson, Miriam A. Leiva, W. Gary Martin, and Margaret S. Smith.

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Appendix B: IRB Approval

Missouri State
UNIVERSITY

To:
Kurt Killion
Mathematics
Gay Ragan

RE: Notice of IRB Approval
Submission Type: Initial
Study #: IRB-FY2019-46
Study Title: Posing Purposeful Questions in Mathematics Tutoring Settings
Decision: Approved

Approval Date: November 16, 2018
Expiration Date: November 16, 2019

This submission has been approved by the Missouri State University Institutional Review Board (IRB) for the period indicated.

Federal regulations require that all research be reviewed at least annually. It is the Principal Investigator's responsibility to submit for renewal and obtain approval before the expiration date. You may not continue any research activity beyond the expiration date without IRB approval. Failure to receive approval for continuation before the expiration date will result in automatic termination of the approval for this study on the expiration date.

You are required to obtain IRB approval for any changes to any aspect of this study before they can be implemented. Should any adverse event or unanticipated problem involving risks to subjects or others occur it must be reported immediately to the IRB.

This study was reviewed in accordance with federal regulations governing human subjects research, including those found at 45 CFR 46 (Common Rule), 45 CFR 164 (HIPAA), 21 CFR 50 & 56 (FDA), and 40 CFR 26 (EPA), where applicable.

Researchers Associated with this Project:
PI: Kurt Killion
Co-PI: Gay Ragan
Primary Contact: Kurt Killion
Other Investigators: Sara Jones

Appendix C: Participant Informed Consent Form

INFORMED CONSENT AGREEMENT

TITLE OF STUDY
Posing Purposeful Questions in Mathematics Tutor Settings

PRINCIPAL INVESTIGATOR
Sara Jones
Dr. Mary Jo Wynn Academic Achievement Center
901 S. National Avenue, Springfield, MO 65897
573-645-6824
Jones2386@live.missouristate.edu

PURPOSE OF STUDY

You are being asked to take part in a research study. Before you decide to participate in this study, it is important that you understand why the research is being done and what it will involve. Please read the following information carefully. Please ask the researcher if there is anything that is not clear or if you need more information.

The purpose of this study is to examine a tutor's experience of posing purposeful mathematics questions within tutoring settings.

STUDY PROCEDURES

As a participant, you will report to an agreed upon tutor session time. During the tutor session, the participant will be encouraged to engage in mathematical discourse while the tutor poses purposeful questions about the current topic of study. The required assessments for the MSU mathematics course in which the participant is enrolled will guide our plan, discussions, and questioning for each tutoring session. Although a majority of the tutoring sessions will be one-on-one, you may be asked to participate in a pair tutoring session, involving you and another participant. The time required by participants in this study is five 30-minute tutoring sessions (150 total minutes). The tutoring sessions will take place between February and March of 2019.

The use of audio and video recorders will be used for data collection and reflection purposes. These recordings will only be seen by myself and one outside observer.

Note: The researcher may ask for course information in advance of the tutor session for planning purposes (unit topic, homework problems, etc.). Other than this, no actions will be requested of the participant beyond the tutor sessions.

RISKS

Students may feel uncomfortable or challenged when faced with different types of questions. This may lead to frustration and self-doubt. To reduce this risk, the tutor will explain the new strategy in advance in order to help students understand what will take place and the expectations required on both sides.

Note: Participating in this study will have no effect on your MTH 101/130/135 course grade.

You may decline and terminate your involvement at any time if you choose.

BENEFITS

First, this study will provide useful information to mathematic tutors and school teachers about the successes and challenges of posing purposeful questions. Second, mathematical engagement and conceptual understanding could improve as a result of the questioning strategies.

CONFIDENTIALITY

For the purposes of this research study, your collection work will be confidential. Every effort will be made by the researcher to preserve your confidentiality including the following:

- Pseudo names will be used for participants in all research notes and documentation
- Video recording and personal reflection journals will be stored on the personal laptop of the researcher that is secured by a password
- Collected student work will be locked in a file cabinet in the personal possession of the researcher

Participant data will be kept confidential except in cases where the researcher is legally obligated to report specific incidents. These incidents include, but may not be limited to, incidents of abuse and suicide risk.

CONTACT INFORMATION

If you have questions at any time about this study, you may contact the researcher whose contact information is provided on the first page. If you have questions regarding your rights as a research participant, or if problems arise which you do not feel you can discuss with the Primary Investigator, please contact the Dr. Kurt Killion at 417-836-6385.

VOLUNTARY PARTICIPATION

Your participation in this study is voluntary. It is up to you to decide whether or not to take part in this study. If you decide to take part in this study, you will be asked to sign a consent form. After you sign the consent form, you are still free to withdraw at any time and without giving a reason. Withdrawing from this study will not affect the relationship you have, if any, with the researcher. If you withdraw from the study before data collection is completed, your data will be returned to you or destroyed.

CONSENT

I have read and I understand the provided information and have had the opportunity to ask questions. I understand that my participation is voluntary and that I am free to withdraw at any time, without giving a reason and without cost. I understand that I will be given a copy of this consent form. I voluntarily agree to take part in this study.

Participant's signature ______________________________ Date __________

Investigator's signature _____________________________ Date __________

Appendix D: Observational Protocol

Purposeful Questioning Observation Protocol

Name of Observer: Date:

1. The tutor asks accessible questions that build-on and/or extend student thinking in order to make sense of the solution.

Look-For: The homework problem asks the student to factor a polynomial. The student works out a procedure to get an answer of x=0 and x=-2. Tutor: "What do those values mean?" Student: No response. Tutor: "What can you expect to happen if you plug x=-2 into your equation?" Student: No response. Tutor: Let's graph it. "What do you notice about the graph?" Student: "The graph crosses at x=0 and x=-2." Tutor: "How does that relate to our solutions?" Student: "Those are where the graph crossed the x-axis. Tutor: "What do we call points that cross an axis?" Student: "Intercepts." Tutor: "So, what does the graph tell us about factoring?" Student: "We factor to determine the x-intercepts." Tutor: "Does factoring always give us our x-intercepts?" .... Explore.

Evidence:

2. The tutor asks questions to expose mathematical connections that make the mathematics more conceptual and meaningful to students.

Look-For: A student is asked to write y = 5x +3 in Standard Form. Using algebraic manipulation, the student arrives at the solution of 5x – y = -3. The tutor suggests graphing the first equation. Tutor: "What would you predict the graph of your equation to look like?" Student: "It will be the same." Then graphs to confirm or deny his or her claim. Tutor: "What do the graphs tell us about the relationship of the equations?"

Evidence:

3. The tutor uses questions that encourage students to clearly communicate and elaborate their own thinking.

Look-For: The student is presented with two parallel lines cut by a transversal. Student: "Angles 3 and Angle 4 are alternate exterior angles." Tutor: "What does alternate exterior angles mean in relationship to the picture?" Student: "They are the angles alternative of the transversal and outside the parallel lines." Tutor: "What can you tell me about the relationship of alternate exterior angles?" Student: "They are the same." Tutor: "What do you mean they are the same?" Student: "They are equal." Tutor: "What does it mean for two angles to be equal?" Student: "They have the same degree measures." Tutor: "Knowing this, what would that suggest we need to do to solve this problem?"

Evidence:

Appendix E: Eight Standards for Mathematical Practice

CCSS.MATH.PRACTICE.MP1 Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP3 Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP4 Model with mathematics.

CCSS.MATH.PRACTICE.MP5 Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP6 Attend to precision.

CCSS.MATH.PRACTICE.MP7 Look for and make use of structure.

CCSS.MATH.PRACTICE.MP8 Look for and express regularity in repeated reasoning.

(National Governor's Association Center for Best Practice, Council of Chief State School Officers, 2010)