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Implementing Instructional Scaffolding to Support Secondary Students' Abilities to Write Mathematical Explanations

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IMPLEMENTING INSTRUCTIONAL SCAFFOLDING TO SUPPORT SECONDARY STUDENTS' ABILITIES TO WRITE MATHEMATICAL EXPLANATIONS

A Master's Thesis

Presented to

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In Partial Fulfillment

Of the Requirements for the Degree

Master of Science in Education, Secondary Education

By

Camry J. Cowan

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ABSTRACT

This study examined the implementation of an instructional scaffolding teaching strategy in the secondary mathematics classroom. An iterative process was used to implement an initial design of instructional scaffolding, reflect on its efficacy, and adjust the design as needed. The goal was to assist students in learning to write responses of high epistemic complexity, which is an indicator of the degree of conceptual understanding. A total of 94 responses written by 35 students in two high school Algebra 2 courses were analyzed for epistemic complexity. Across three iterations of the implementation of instructional scaffolding, students wrote at the highest levels of epistemic complexity in 81.91% of the responses that were collected. These results provided evidence that instructional scaffolding is a viable teaching strategy that mathematics educators can employ to help students learn to write epistemically complex explanations of mathematics. Furthermore, two types of challenges are discussed that were encountered while implementing instructional scaffolding: a) challenges related to the instructional scaffolding process; and b) challenges related to coding responses while applying a coding framework designed to measure epistemic complexity.

KEYWORDS: instructional scaffolding, epistemic complexity, writing in secondary mathematics, conceptual understanding of mathematics, secondary mathematics education

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"I know what to do. I just don't know how to explain it." This quote is far from unique. In fact, students in my mathematics courses have voiced this lament time and time again. During my experience teaching multiple courses – Algebra I Support, Algebra I, Geometry, Algebra II, Algebra II Honors, Discrete Mathematics, College Algebra, Precalculus, and AP Calculus AB – my students have made this statement without fail, regardless of the perceived level of difficulty of the course or level of ability of the students. Students' inability to explain the *why* behind their mathematical solutions has frustrated me since I became a mathematics teacher. The challenge of helping them develop the ability to provide those explanations has been especially frustrating.

When I ask my students to explain their thinking when solving problems, I expect their explanations to convince me they understand the concepts as well as the procedures. As such, I wish their explanations would align with the eight Standards for Mathematical Practice established by the Common Core State Standards for Mathematics (CCSS-M). These eight standards identify a multitude of dispositions that mathematics students should develop during their mathematics education (Common Core State Standards Initiative [CCSSI], 2020b). The third Standard for Mathematical Practice (SMP3) states that students proficient in mathematics should be able to "construct viable arguments and critique the reasoning of others" (CCSSI, 2020b, CCSS.MATH.PRACTICE.MP3 section). Students not only should justify their conclusions, but they also should communicate those conclusions to others and respond to feedback they receive (CCSSI, 2020b). Closely related to SMP3 is the sixth Standard for Mathematical Practice (SMP6), which establishes that mathematically proficient students should "attend to precision" in their own reasoning and in their communicating with others (CCSSI, 2020b, CCSS.MATH.PRACTICE.MP6 section). After observing and participating in countless conversations in which students used imprecise or completely incorrect language, accepted vague or wrong answers from others unquestioningly, or justified their answers' being correct by saying, "Well, that's just how it works, because math," it became obvious that my students' abilities to explain their thinking were not aligned with the intentions described in the Standards for Mathematical Practice.

Rationale for the Study

Reflecting on my teaching practices helped me identify areas in need of improvement. First, I was rarely asking students questions that required them to explain their thinking to demonstrate conceptual understanding. Lessons featured explanations of how I understood concepts, but I hardly asked students to explain their understandings to me. How could I expect my students to be able to explain their thinking if they were never being asked to do so? Second, even when I did ask students to explain their thinking, their explanations were not demonstrative of conceptual understanding. For example, if I were to ask my students to explain why the quadratic trinomial $x^2 + 5x + 6$ is equivalent to the factored expression $(x + 2)(x+3)$, I may want their response to be similar in nature to the following:

> To start, I know the expressions $x^2 + 5x + 6$ and $(x + 2)(x+3)$ are equivalent because multiplying the factors $(x + 2)$ and $(x+3)$ produces the trinomial $x^2 + 5x + 6$ by applying the Distributive Property of Multiplication over Addition. This is similar to how I know the expressions 6 and $2(3)$ are equivalent because multiplying the factors 2 and 3 produces 6.

An explanation like that – one that illustrated the relationship between factoring and distribution and connected factoring polynomials to factoring whole numbers – would convince me a student understood the concept of factoring quadratic trinomials. Instead, my students were offering explanations like, “I got the numbers from the Magic X,” referencing a procedure I had shown them to help them complete their assigned problems about factoring more quickly.

It is not surprising my students would provide such surface-level explanations devoid of conceptual understanding. Although I did explain in my lecture how to factor quadratic trinomials using the reasoning in the exemplar given above, I did not effectively assess whether my students understood the concept. One strategy I could have employed would have been to ask students to decide if factorizations were correct or incorrect and to justify their reasoning. For example, I could have presented students with the factorizations $x^2 + 5x + 4 = (x + 1)(x + 4)$ and $x^2 + 8x + 9 = (x + 1)(x + 9)$ and then asked them to justify which factorization was correct and which one was incorrect. They could have explained or shown how distribution would reveal the first factorization was correct but the second was not, as multiplying $(x + 1)$ and $(x + 9)$ would actually produce $x^2 + 10x + 9$. Reading my students’ written responses to this question would have helped me assess their conceptual understandings and decide how to proceed with the lesson.

What did I do instead? I immediately showed students a procedure to help them factor quadratic trinomials faster, I guided them through four or five more examples to demonstrate how quickly the procedure could go, and I assigned them a twelve-problem worksheet with the instructions “Factor each polynomial” at the top of the page. No conceptual understanding, no justification, no thinking – I merely trained them to mimic yet another procedure they learned in their mathematics class. It is not inspiring for learners of mathematics to merely memorize formulas, follow algorithms, and employ tricks. Students should move beyond that status quo of learning mathematics. Part of the beauty of mathematics is making sense of ideas and concepts to truly understand if they work – or do not work – and *why*. When students leave my classroom, I hope they have internalized their understandings of mathematical concepts. I also hope they have learned how to convince others they have developed those understandings. To help students achieve these two goals, I decided to incorporate writing as a part of my curriculum.

Past research has shown that having students explain their understandings of concepts in writing helps teachers assess their level of understanding (Baxter, Woodward, & Olson, 2005; Bicer, Perihan, & Lee, 2018; Casler-Failing, 2013; Evans, 2017; Kostos & Shin, 2010; Martin & Polly, 2013, 2016; Martin, Polly, & Kissel, 2017; McIntosh, 1991; Miller & England, 1989; Miller, 1991, 1992a, 1992b; Nachowitz, 2018). Nachowitz (2018) analyzed the epistemic complexity of nearly 1,500 samples of student writing in mathematics classes in grades 6-12. Epistemic complexity represents the degree of conceptual understanding demonstrated in a written response (Nachowitz, 2018). A greater degree of epistemic complexity indicates a higher level of conceptual understanding. In Nachowitz's study, not one sample of student writing was classified as a *well-organized explanation*, which was the highest level on the coding framework that was used to measure epistemic complexity (Appendix A). In other words, no students were writing explanations of their thinking that demonstrated conceptual understandings of mathematics. Examples were provided that represented the nature of the majority of students' explanations. At best, these samples from Nachowitz's (2018) study simply provided a summary of steps taken within a procedure, failing to demonstrate any understanding of the underlying mathematical concepts that were employed:

- My estimate was correct.
- 1) You have to read all the problem carefully so you won't miss something important. 2) You get both of the numbers 8 foot and 10 foot. 3) Then you times 8 foot times 2. 4) After you do that you times 10 times 2. 5) Then you add both of these numbers and you should get 36 feet. 6) OK you got the answer for the fencing one, so now [it's] time to do the netting one. 7) You times 8 foot times 2. 8) You times 10 foot times h. 9) Then you should get 36 feet and [you're] done.

- I started 4 subtraction problems with the country to compare which one has the highest temperature and the lowest temperature. And I found out that Washington has the range of 166° and Kentucky has the range of 151°. As you move west the temperature gets lower and lower as you move east the temperature is colder. (p. 7)

These writing samples came from schools in different states across the United States that were locally renowned for excellence at incorporating writing in their teaching practices (Nachowitz, 2018). Even though these schools were well-regarded for their implementation of writing, no students were writing at a level consistent with the definition of a student proficient in mathematics according the Standards for Mathematical Practice (CCSSI, 2020b; Nachowitz, 2018). These findings illuminated the need for research that focuses on more than just the inclusion of writing strategies in the mathematics classroom. Research was needed that focuses on *how* to help students learn to write in a way that explains their thinking and demonstrates their understandings of concepts; that is the rationale for this study.

Nachowitz (2018) explained that if it is the goal of mathematics educators for students to be able to use written language to make conjectures, justify conclusions, effectively communicate those conclusions to others, and argue whether or not proposed solutions to problems are valid, then teachers must afford them the opportunity to directly learn to think, reason, and engage in discourse norms specific to the mathematics classroom. It was noted this goal could be best achieved through the use of instructional scaffolding, which has been established as an effective practice in the field of literacy. Nachowitz recommended studying the viability of instructional scaffolding as a practice to be used in the mathematics classroom. Nachowitz (2018) described instructional scaffolding as requiring teachers to "model their writing processes using think-aloud protocols, followed by collaborative practice, feedback with guided instruction, and individual student practice until mastery is achieved" (p. 11). As noted previously, Nachowitz found no evidence of student writing that demonstrated conceptual understanding of mathematics. It was suggested that employing instructional scaffolding to help students learn how to show their reasoning rather than just show their work may help students' writings become better aligned with the intentions outlined in the Standards for Mathematical Practice.

Purpose of the Study

This study analyzed the implementation of an instructional scaffolding teaching strategy in the secondary mathematics classroom. The first purpose of the study was to examine if engaging students in instructional scaffolding would help them learn to write epistemically complex written explanations of mathematics. The second purpose was to identify challenges experienced by the teacher while implementing instructional scaffolding. Because this was action research, the researcher hoped to improve his teaching practices based on the conclusions drawn from the collection and analysis of students' written responses and the challenges encountered during implementation. A third purpose was to provide other mathematics teachers with an in-depth look at how they could incorporate instructional scaffolding in their classrooms to improve their students' abilities to demonstrate conceptual understanding of mathematics in writing.

Research Questions

This study focused on answering two research questions:

1. Does engaging secondary mathematics students in an instructional scaffolding teaching strategy support their abilities to write explanations with high degrees of epistemic complexity?
2. What challenges were encountered while implementing instructional scaffolding to support students' abilities to write explanations with high degrees of epistemic complexity?

Research Design

The design of this qualitative action research study centered on the implementation of an instructional scaffolding teaching strategy in two high school Algebra 2 courses. An iterative process was used that involved: a) implementing an initial design of instructional scaffolding; b) reflecting on its efficacy; and c) adjusting the design as needed based on challenges encountered by the researcher. The initial design of the instructional scaffolding process that was recommended by Nachowitz (personal communication, March 3, 2020) is provided in the following steps:

1. *The teacher models writing processes using a think-aloud protocol.* This means the teacher talks through thought processes while composing a written answer to a prompt, allowing learners to see exactly what the teacher thinks while writing in mathematics and what is expected of their written responses.

2. *Collaborative practice.* Students participate in pair-shares to try writing their own responses to a similar prompt. One student explains their thought process about how to answer the prompt, and the other student writes it down. Volunteers then use a think-aloud protocol in front of the class to explain what they wrote and why.

3. *Debriefing process.* The teacher provides guided feedback to let the volunteers know what they did right and to offer suggestions for improvement.

4. *Independent practice.* Students work independently to try writing their own responses to a similar prompt.

5. Repeat steps (3) and (4) as needed.

This design was adjusted throughout the iterative process because of the reflections made by the researcher. The rationale for the adjustments that were made is discussed in Chapter V. The final design of instructional scaffolding implemented in this study is described in the below steps:

1. *Modeling with a think-aloud protocol.* The teacher talks through thought processes while composing a written answer to a prompt, allowing learners to see exactly what the teacher thinks while writing in mathematics and what is expected for their written responses to demonstrate conceptual understanding.

2. *Collaborative practice with debriefing.* Students participate in pair-shares to try writing their own responses to a similar prompt. One student explains their thought process about how to answer the prompt, and the other student writes it down. While pairs of students are working, the teacher monitors their conversations and provides guided feedback to let the students know what they did right and to offer suggestions for improvement.

3. *Score sample responses.* Working with a partner, students use a rubric to score sample responses to prompts. Students in one class score samples from a separate class. After all pairs have finished scoring the responses, the class engages in a whole-class discussion to share their scores and their reasoning for awarding those scores. The teacher facilitates the discussion and allows students to convince others why they are right, only intervening when it seems necessary.

4. *Final assessment*. Students take a final assessment featuring prompts similar to those they answered and scored during steps (2) and (3). Their responses are scored by the teacher using the coding framework for epistemic complexity.

The coding framework for epistemic complexity used in this study (Appendix B) was a modified version of the coding framework Nachowitz (2018) used (Appendix A). A comparison of the two coding frameworks is provided in Chapter III along with the rationale for the modifications that were made. The final, adjusted design of the instructional scaffolding teaching strategy was implemented to teach three units of study: a) functions; b) solutions to equations; and c) factoring polynomials.

Significance of the Study

The findings of this study provided evidence that an instructional scaffolding teaching strategy can be employed by teachers to help students learn to write epistemically complex explanations of mathematics. The design of the instructional scaffolding process is explained thoroughly so that other mathematics teachers can replicate the process in their own classrooms. The analysis and discussion of the challenges encountered in this study also provided insight into what teachers should expect when implementing instructional scaffolding in their classrooms, such as the importance of choosing concepts to scaffold strategically and making no inferences about the meaning of students' writings. The explanations of how the researcher responded to challenges will help other teachers know what to expect when implementing instructional scaffolding in their own classrooms.

Assumptions

The following is a list of assumptions made by the researcher while conducting the study:

1. Students had not been taught through an instructional scaffolding process in their past experiences in the mathematics classroom.

2. Students did not have prior experience learning to write epistemically complex explanations of mathematical concepts.

3. Students did not have a background in mathematics courses that incorporated writing into the curriculum in a way that was aligned with the intentions outlined by the CCSS-M.

4. Students did not have a background in mathematics courses that assessed their ability to construct viable arguments and critique the reasoning of others (SMP3) in writing.

5. Students did not have a background in mathematics courses that assessed their attending to precision (SMP6) in written justifications of solutions to problems.

Limitations

The following is a list of limitations of the study:

1. This study took place in two high school Algebra 2 courses. Most of the student-participants were juniors in high school, and a few were sophomores or seniors. The final design of the instructional scaffolding process recommended for practice by the researcher may not be able to be replicated exactly with students of other age groups or enrolled in other courses. Further research is needed on its efficacy with other grade-levels and courses.

2. This study was conducted during the COVID-19 pandemic. Because of this, student absences due to sickness or quarantine were an issue that interrupted the instructional scaffolding process. Some student participants missed vital steps of

the process that could not be recreated in a virtual, distance-learning setting. For example, in each iteration, some student participants took final assessments even though they were not present during the *collaborative practice with debriefing* or *score sample responses* steps of the instructional scaffolding process. Ideally, student participants would have been present for all steps of the instructional scaffolding process before taking the final assessment in each iteration.

3. The duration of the study was a limitation, as data were collected in three course units spanning two months. Had the study been conducted throughout the entire school year, more data could have been collected and analyzed.

4. Data were not collected on how accurately students scored sample responses before a rubric was provided compared to how accurately they scored sample responses after a rubric was provided. The researcher believes there was an improvement in the accuracy of students' awarded scores based on his experiences implementing that step of the instructional scaffolding process, but there is no formal evidence to support that claim.

5. No baseline data were collected to compare the degree of epistemic complexity of students' written explanations before engaging in the instructional scaffolding process with the degree of epistemic complexity of their written explanations after engaging in the instructional scaffolding process. Anecdotally, the epistemic complexity of students' written explanations improved after they engaged in the *collaborative practice with debriefing* and *score sample responses* steps of the instructional scaffolding process, but no formal evidence was collected to measure the degree of that improvement.

Definition of Terms

The following is a list of definitions of terms referenced in the study:

1. Instructional Scaffolding: The teacher models their writing processes using think-aloud protocols, followed by collaborative practice, feedback with guided instruction, and individual student practice (Nachowitz, 2018).

2. Epistemic Complexity: The degree to which a student explained or justified conceptual understanding of mathematics. A greater degree of epistemic complexity indicates a higher level of conceptual understanding. This will be measured using a coding framework (Appendix B) that is a modified version of one used by Nachowitz (2018) (Appendix A). Students' written responses will be coded as *Level 1 – Needs Help*, *Level 2 – Not Quite*, *Level 3 – Almost There*, or *Level 4 – Nailed It*. These codes will be described in greater detail and examples will be provided in Chapters III and IV.

3. Teacher Challenge: For the purpose of this study, a teacher challenge was defined in three ways:

   a. First, a teacher challenge can be faced *before* the implementation of instructional scaffolding. This includes the teacher's decisions about what he expects out of students' written responses to convince him they have developed conceptual understanding of a specific mathematical idea.

   b. Second, a teacher challenge can be faced *during* the implementation of instructional scaffolding. This includes students' difficulties writing responses to prompts that prove they understand a concept and how the teacher provides appropriate guided feedback to those students.

   c. Third, a teacher challenge can be faced *after* the implementation of instructional scaffolding. This includes the teacher's accurate and consistent application of the coding framework to assess students' written responses.

4. CCSS-M: an abbreviation for Common Core State Standards for Mathematics (CCSSI, 2020a). These are national standards that define what students should understand and be able to do in their study of mathematics.

5. SMP3: an abbreviation for Standard for Mathematical Practice 3, referencing the third Standard for Mathematical Practice as listed in the CCSS-M. SMP3 holds that students proficient in mathematics should be able to "construct viable arguments and critique the reasoning of others" (CCSSI, 2020b, CCSS.MATH.PRACTICE.MP3 section).

6. SMP6: an abbreviation for Standard for Mathematical Practice 6, referencing the sixth Standard for Mathematical Practice as listed in the CCSS-M. SMP6 states that mathematically proficient students should "attend to precision" in their own reasoning and in their communicating with others (CCSSI, 2020b, CCSS.MATH.PRACTICE.MP6 section).

Summary

The eight Standards for Mathematical Practice established by the CCSS-M include SMP3 and SMP6. SMP3 states that students proficient in mathematics should "construct viable arguments and critique the reasoning of others (CCSSI, 2020b, CCSS.MATH.PRACTICE.MP3 section), and SMP6 adds they should "attend to precision" in their own reasoning and communicating with others (CCSSI, 2020b, CCSS.MATH.PRACTICE.MP6 section). Prior research has supplied evidence that including writing as a feature of the core mathematics curriculum helps both students and teachers assess students' understanding of mathematics concepts (Baxter et al., 2005; Bicer et al., 2018; Casler-Failing, 2013; Evans, 2017; Kostos & Shin, 2010; Martin & Polly, 2013, 2016; Martin et al., 2017; McIntosh, 1991; Miller & England, 1989; Miller, 1991, 1992a, 1992b; Nachowitz, 2018). However, samples of student writing that were collected from mathematics classrooms featured responses that were not epistemically complex (Baxter et al., 2005; Martin & Polly, 2013, 2016; Martin et al., 2017; Nachowitz, 2018). In other words, student writing samples did not demonstrate conceptual understandings of mathematics. Furthermore, it has been recommended that research investigate what actions teachers can take to improve students' abilities to explain conceptual understandings of mathematics in writing (Baxter et al., 2005; Martin & Polly, 2013, 2016; Martin et al., 2017; Nachowitz, 2018).

This study had two primary goals: a) to examine if engaging students in instructional scaffolding would help them learn to write mathematical explanations with high degrees of epistemic complexity; and b) to identify challenges experienced by the teacher while implementing instructional scaffolding. An iterative process was used to implement an initial design of instructional scaffolding, reflect on its efficacy, and adjust the design as needed based on challenges encountered by the researcher. At the end of each iteration, students' written responses were analyzed using a coding framework for epistemic complexity (Appendix B) that was a modified version of one Nachowitz (2018) used in a related study (Appendix A). The findings of this study established that instructional scaffolding is a viable teaching strategy that secondary mathematics teachers can employ to help their students learn to write epistemically complex explanations of mathematics. The analysis and discussion of the challenges encountered in this study also provided insight about how teachers should focus their attention when implementing instructional scaffolding in their classrooms. The explanations of how the researcher responded to challenges will help other teachers know what to expect when implementing instructional scaffolding in their own classrooms.

CHAPTER II: REVIEW OF RELATED LITERATURE

The Common Core State Standards for Mathematics (CCSS-M) establish many understandings students should develop throughout the course of their mathematics education (CCSSI, 2020a). Experts have argued it is important for students to balance their procedural abilities and conceptual understandings of mathematics (CCSSI, 2020a; Hiebert et al., 2005). In the United States, though, the balance is tilted heavily in favor of procedural fluency over conceptual fluency (Hiebert et al., 2005; National Council of Teachers of Mathematics [NCTM], 2018). Evidence spanning decades has shown that including writing as a core part of the mathematics curriculum offers many benefits. First, writing helps both teachers and students assess students' conceptual understandings (Baxter et al., 2005; Bicer et al., 2018; Casler-Failing, 2013; Evans, 2017; Kostos & Shin, 2010; Martin & Polly, 2013, 2016; Martin et al., 2017; McIntosh, 1991; Miller & England, 1989; Miller, 1991, 1992a, 1992b; Nachowitz, 2018). Second, writing engages learners in mathematical thinking (Baxter et al., 2005; Evans, 2017; McIntosh, 1991; Miller, 1992a, 1992b). Third, writing encourages precision of mathematics vocabulary (Kostos & Shin, 2010; Miller, 1992b, 1993).

However, research has also determined the enactment of writing in the mathematics classroom has fallen short of the expectations outlined in the CCSS-M (Nachowitz, 2018), and research has been recommended on strategies teachers can implement to improve the quality of students' written explanations of concepts (Baxter et al., 2005; Martin & Polly, 2013, 2016; Martin et al., 2017; Nachowitz, 2018). Nachowitz (2018) recommended investigating if instructional scaffolding is a viable strategy in the mathematics classroom to support students' abilities to write epistemically complex explanations of mathematics. In order to address the research that has been conducted related to writing in the mathematics classroom, it was important to address four aspects that will be discussed in this chapter: a) nature of instructional focus in mathematics classrooms; b) benefits of writing in mathematics; c) nature of students' written explanations; and d) gaps in existing research on writing in mathematics.

Nature of Instructional Focus in Mathematics Classrooms

Nachowitz's (2018) study, among others, established that writing can be used as a tool to assess students' conceptual understandings of mathematics. His study also revealed students in the United States, aside from a few exceptions, were unable to justify their thinking and provide evidence of conceptual understanding. Nachowitz's findings showed students' written explanations did not align with the expectations and recommendations of mathematics education experts and the Standards for Mathematical Practice. Many experts in mathematics education have argued for the need to balance students' procedural skills and conceptual understandings (CCSSI, 2020a; Hiebert et al., 2005), yet mathematics education in the United States has tended to focus efforts mostly on developing students' procedural skills (Hiebert et al., 2005; NCTM, 2018). This was unearthed by a Video Study conducted in 1999 as part of the Third International Mathematics and Science Study (TIMSS) (Hiebert et al., 2005). The TIMSS 1999 Video Study selected samples of eighth-grade mathematics classrooms that were nationally representative of the nations of Australia, the Czech Republic, Hong Kong SAR, Japan, the Netherlands, Switzerland, and the United States. Each classroom was filmed for one lesson, and teachers were specifically asked not to adjust their intended instruction for the recordings. Lessons were then analyzed and coded by Hiebert et al. (2005) according to the following categories: "(a) structure and organization of daily lessons, (b) nature of the mathematics presented, and (c) way in which the mathematics was worked on during the lesson" (p. 115). The analyses of the recorded lessons were then compared with the average scores achieved by each nation's students on the TIMSS 1995 and 1999 Eighth-Grade Mathematics Assessments.

One key finding by Hiebert et al. (2005) was "teachers in the higher-achieving countries attended more to the conceptual development of mathematics than teachers in the United States" (p. 120). United States teachers were found to teach students to solve problems almost exclusively by using procedures or providing answers only. Even educational systems like Hong Kong SAR that mainly presented problems procedurally were found to explicitly examine the conceptual reasoning behind the procedures used to solve those problems. This attention to conceptual underpinnings of procedures was virtually nonexistent in the mathematics lessons in the United States; instead, the United States lessons placed a uniquely heavy emphasis on procedural skill in the types of problems they presented to students. Hiebert et al.'s findings suggested the most successful teaching systems were those that featured a balance of attending to procedural skill and to developing conceptual understanding of *why* specific procedures worked.

The results of international assessment results in 2015 comparing the United States to these nations and others led to similar conclusions about the lack of conceptual understanding in mathematics education in the United States (NCTM, 2018; Organisation for Economic Co-operation and Development [OECD], 2016; Provasnik et al., 2016). Provasnik et al. (2016) found that, among other educational systems, Hong Kong and Japan were outperforming United States eighth-grade students based on the TIMSS 2015 international benchmarks. Only 10% of United States eighth-grade students reached the Advanced benchmark in 2015 compared to 34% in Japan and 37% in Hong Kong (Provasnik et al., 2016). This benchmark was reserved for students who demonstrated the ability to make generalizations, justify conclusions, and reason in a variety of problem situations (Provasnik et al., 2016). The United States also lagged far behind the world's top-scoring countries in mathematical literacy based on the results of the Programme for International Student Assessment (PISA) in 2015 (OECD, 2016). Within PISA 2015, mathematical literacy was defined as "students' capacity to formulate, employ and interpret mathematics in a variety of contexts. … [including] reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena" (OECD, 2016, p. 28). Continuing with the trend established in the TIMSS 1995, TIMSS 1999, and TIMSS 2015 assessments, both Hong Kong and Japan ranked among the best countries in the world on the PISA 2015 (OECD, 2016). The educational systems that balanced procedural skill and conceptual understanding fostered greater student abilities to demonstrate conceptual understanding (OECD, 2016), an observation similar to those made by Hiebert et al. (2005).

Typical United States classrooms have focused most of their attention in mathematics education on students' abilities to execute low-level skills without nearly enough emphasis on developing their conceptual understanding as well, despite the expectations established by the Standards for Mathematical Practice, the CCSS-M, and the NCTM (Hiebert et al., 2005; NCTM, 2018). Even lessons that appeared to feature problems asking students to make connections and explain concepts were transformed by teachers' showing students how to solve those problems through the use of procedures (Hiebert et al., 2005). *Catalyzing Change in High School Mathematics* (NCTM, 2018) stated teachers must move past this fixation on procedures and look beyond whether their students' answers are simply right or wrong. Rather, it was argued teachers must elicit and assess evidence of students' mathematical thinking to establish the frame of mind that being correct is not more valuable than engaging in mathematical thinking (NCTM, 2018). By requiring students to justify their reasoning, teachers can engage students in mathematical thinking, elicit evidence of their thinking, and respond to that evidence to develop conceptual understanding (NCTM, 2018).

Benefits of Writing in Mathematics

Several benefits of including writing in the mathematics classroom have been well-established in the research literature. First, various studies have established writing reveals the degree of students' understanding of mathematics, thus helping teachers gauge if their students possess conceptual understanding or just procedural ability (Baxter et al., 2005; Bicer et al., 2018; Casler-Failing, 2013; Evans, 2017; Kostos & Shin, 2010; Martin & Polly, 2013, 2016; Martin et al., 2017; McIntosh, 1991; Miller & England, 1989; Miller, 1991, 1992a, 1992b; Nachowitz, 2018). Second, writing has been shown to be effective in engaging students in mathematical thinking (Baxter et al., 2005; McIntosh, 1991; Miller, 1992a, 1992b), even more so than other strategies like class discussions (Baxter et al., 2005; Miller, 1992b). Third, having students explain their thinking in writing encourages both students and teachers to be precise with their language and to use technical mathematics vocabulary accurately (Kostos & Shin, 2010; Miller, 1992b, 1993).

Writing Reveals Understanding. Miller and England (1989) found that writing in the mathematics classroom can reveal whether students actually understand the concepts they are learning or are just proficient at manipulating symbols to replicate memorized algorithms. Misconceptions that may have been hiding behind correct answers that students obtained procedurally were revealed in written explanations. Miller and England argued that identifying these misconceptions affords teachers time to adjust instruction to help students improve their understandings. They pointed out that summative assessment is often the first instance when a teacher gains insight about the nature of students' understandings. By that time, it is likely too late for the students who never understood concepts during a unit of study. Miller and England contended that uncovering students' understandings, or lack thereof, before a test would help both the teacher and students better prepare for summative assessments. Miller (1991) then found that teachers could uncover and actually respond to the misconceptions they identified by commenting on students' writings and returning them to the students, talking in private with students about what they wrote, or initiating a whole-class review session based on misconceptions displayed in several students' writings.

Miller (1992b) further studied writing in mathematics through teachers' use of impromptu writing prompts in Algebra classes. Students were given five minutes to write a response to a prompt an average of four out of every five instructional days; their responses were expected to provide a clear demonstration of their conceptual understanding of a given topic. Miller found that the students in two Algebra 1 classes and one Algebra 2 class were unable to explain concepts of problems they were able to solve accurately by manipulating symbols and numbers or by recalling memorized facts. For example, no students' answers demonstrated conceptual understanding when they were asked to explain why $0/5 = 0$, why $5/0$ was undefined, and why $0/0$ was indeterminate, even though the vast majority of students accurately filled in blanks for those expressions just days before. These students could recall memorized facts tied to the location of the zero in relation to the fraction bar, but they could not explain the reasoning that supports those facts. Miller found the use of these impromptu writing prompts allowed the teachers in the study to assess the degree of their students' understanding, or rather the absence of their understanding. By recognizing their students' deficiencies, teachers were able to alter their instructional practices by reteaching concepts, delaying exams and designing reviews based on what they learned from the writings, and conferencing individually with students to address misconceptions.

In a related study, Kostos and Shin (2010) focused on how mathematical writing journals affected students' abilities to communicate their mathematical thinking. Data were collected and analyzed from sixteen second-grade students who wrote in their journals an average of three times per week. Prompts focused on lessons students had learned previously in addition to mathematical skills they had already developed. One of the main findings of Kostos and Shin's study was that the journals functioned as an effective assessment tool. Kostos and Shin noted the "journals provided insights into the students' thought process and understanding of mathematical concepts, rather than simply checking the right answers" (p. 229). Students' responses allowed the teacher to see when students had procedural ability but lacked conceptual understanding, as some students obtained correct solutions but provided no explanation to justify why the solutions were correct. The journal entries also enabled the teacher to determine if concepts needed to be retaught to the entire class, small groups of students, or a few individuals.

Martin and Polly (2013) also studied how students' writing in mathematics journals influenced their understanding of mathematics. Over the course of two months, seven teachers assigned a minimum of two writing prompts each week to a combined 120 students in third-grade and fourth-grade. Students were given between three and ten minutes per prompt to write responses in their journals. At the end of the two-month period, five student journals were selected for deeper analysis. Martin and Polly's analysis supported the results of similar studies asserting that students' written responses to prompts allowed the reader to see the thinking taking place while the student was engaged in mathematical activities. These findings were consistent with views expressed by McIntosh (1991):

> Although half the benefit of keeping a journal is students' enhanced learning, the other half is teachers' enhanced awareness of students' learning, knowledge of what students are thinking related to the class (and sometimes not related to the class), and information indicating where students are in need of assistance. (p. 430)

The results of these studies provided evidence that students' conceptual understandings of mathematics – or lack of understandings – are revealed when they are asked to write.

Writing Engages Learners in Mathematical Thinking. In her study involving impromptu writing prompts in Algebra classes, Miller (1992b) found another benefit resulting from students' writing in class: students were much more willing to engage in mathematical thinking in their writings than they were willing to demonstrate in whole-class discussions. Throughout Miller's study, teachers made attempts to facilitate whole-class discussions focusing on the same types of questions they were asking students in the writing prompts. When students were asked to explain their understanding of concepts aloud in front of the entire class, "teachers indicated that students were either reluctant or unable to respond, they verbally pleaded ignorance, or the same few students wanted to dominate the discussion" (Miller, 1992b, p. 339). However, students did not display this same reluctance when they were given time to gather their thoughts privately and write out their mathematical thinking. In fact, many students wrote they believed writing helped them express their own thoughts and gave them opportunities to ask questions they would not normally ask. Writing caused them to feel their input mattered, not just the input of students who were comfortable speaking out. One Algebra 2 student wrote the following about his attitude toward writing in class:

> I think it (writing) lets us clear our minds and express our opinions. It means you [the teacher] care enough about what we think to allow us time to write and then read what we write. Most of the time I don't want to ask questions in class because everybody will think I'm dumb. I can ask you questions in my writings that you answer without everybody else knowing I was the one who asked. I hope we keep writing. (Miller, 1992b, p. 337)

Similar findings surfaced when Baxter et al. (2005) analyzed seventh-grade mathematics students' journal entries to determine what writing would reveal about their understandings. There were four target students, all of whom qualified for special education services, whose journals were the focus of the study. Over the course of the school year, the classroom teacher gradually transitioned students from writing primarily about their attitudes toward mathematics to writing about how they would justify their solutions to problems. Explanations were coded at four different levels – *Recording (Level 1)*, *Summarizing (Level 2)*, *Generalizing (Level 3)*, and *Relating (Level 4)* – with *Relating* being the highest level that demonstrated evidence of students' "conceptual understanding, strategic competence, and adaptive reasoning" (Baxter et al., 2005, p. 123). The target students' journal entries revealed whether or not students possessed understanding of the concepts behind the problem-solving strategies they learned. The teacher used these written responses to guide her instructional planning as it allowed her to see into students' minds and assess if they were truly comprehending material. These benefits coincided with those established by the previously mentioned studies conducted by Kostos and Shin (2010), Martin and Polly (2013), McIntosh (1991), Miller and England (1989), and Miller (1991, 1992b).

What was most notable in the findings of Baxter et al. (2005), though, was how their assumptions about the target students based on classroom observations were contrasted by their analysis of the target students' journals. In the classroom observations, it appeared all four target students were passive participants in class because they did not engage in attempting to explain their mathematical thinking and understanding during class discussions. However, analyses of their journals revealed that three of the target students actually attempted to explain their mathematical reasoning and feelings in their journals regularly. Two of them were writing at the

Summarizing and *Generalizing* levels in over half of their responses, and the third was writing at those levels in almost 40% of her responses. Baxter et al. remarked these students rarely, if ever, contributed to class discussions, yet they frequently and consistently demonstrated their engagement in mathematical thinking in their journals. This medium allowed the teacher and the researchers to witness students' mathematical thinking they otherwise were unable to observe.

Writing Encourages Precision of Mathematics Vocabulary. In their study on using mathematics journals with second-grade students, Kostos and Shin (2010) found that, in addition to the journals' helping teachers assess students' mathematical thinking, the journals improved the students' abilities to use proper mathematics vocabulary. Over the course of the study, students' explanations featured correct mathematics terminology more frequently. Students reported they believed writing encouraged them to use more "math words" because they had to think more carefully when explaining their thinking in writing (Kostos & Shin, 2010, p, 229). Not only did students accurately name processes they were employing to solve problems, but they also were able to utilize their understandings of vocabulary to prove why their answers were correct. One student sample even demonstrated a conceptual understanding of even numbers by explaining she could split 18 into two equal groups with no leftovers, a rare example according to Nachowitz's (2018) findings.

The correct, precise use of mathematics vocabulary has been shown to be important for teachers, too, not just for students (Miller, 1992b, 1993). Miller (1992b, 1993) suggested teachers must help students connect the language of mathematics to their understanding of mathematical concepts. Written responses help students demonstrate their understandings and learn from any misconceptions their teachers may notice and address; this finding has been established by numerous researchers previously cited (Baxter et al., 2005; Bicer et al., 2018;

Casler-Failing, 2013; Evans, 2017; Kostos & Shin, 2010; Martin & Polly, 2013, 2016; Martin et al., 2017; McIntosh, 1991; Miller & England, 1989; Miller, 1991, 1992a, 1992b; Nachowitz, 2018). Additionally, the language students use in their writings can also help teachers recognize if they need to be more precise with their own language (Miller, 1992b, 1993). This self-recognition has been a challenge for teachers to realize, as teachers often reflect more on the language students are using than they reflect on their own use of classroom language. For example, Miller (1993) referenced a study that found only seven percent of a group of over six hundred eighth-grade students could define the term *quotient*. Upon reflection, all teachers involved in the study reported they never modeled the use of the word *quotient* in their teaching (Miller, 1993). As a result, Miller (1993) raised the following question: “If mathematics teachers do not model appropriate vocabulary within a meaningful context, how can students be expected to use the language of mathematics?” (p. 314).

Miller (1992b) reported that observing specific misconceptions and misuses of vocabulary in secondary mathematics students’ written responses helped teachers reflect on the language they were using in their teaching. For example, some students viewed factoring as a subtractive process. After reflecting on their practices, teachers believed this may have been a result of their use of the phrase *take out* when teaching students how to identify common factors of binomial terms, language remarkably similar to the subtractive *take away* phrase. Miller (1993) provided additional examples of teachers’ expectations of students to use mathematics vocabulary and language in their written explanations that the teachers did not even model with their own language. One teacher was frustrated that none of his students used the language of numerator and denominator when writing about fractions. Yet, an observer noticed the teacher did not use either of those words during a teacher-led review of fractions a few days later, even though the teacher was convinced he had. By reading students' written responses, teachers can identify when students are misusing or not using proper vocabulary and then adjust their instruction to focus on modeling the correct, intentional use of vocabulary (Miller, 1992b, 1993).

Nature of Students' Written Explanations

Nachowitz (2018) focused his study on two topics: a) the enactment of writing in the mathematics classroom; and b) the extent to which that writing was aligned with the intentions outlined by the CCSS-M and the NCTM. The study analyzed the writings of 138 students from around the United States who were in grades 6, 8, 10, and 12 and enrolled in courses such as Pre-Algebra, Algebra, Geometry, AP Statistics, and Calculus. Various types of writing samples were collected, such as tests, quizzes, class notes, and journal entries, and the researchers examined more closely writings they classified as short-answer exercises. Nachowitz coded these writing samples according to a measurement of their epistemic complexity – the degree to which a student explained or justified conceptual understanding of mathematics. The goal was to evaluate the quality of students' explanations compared to the intentions of the CCSS-M and the NCTM. Nachowitz (2018) employed the below framework (Appendix A) for coding the samples of writing – with Level 5 being aligned with the CCSS-M and NCTM's intentions:

1. *Separated pieces of facts*. A statement consisting of a list or table of facts with hardly any integration or connections/or explanation.

2. *Partially organized facts*. A statement consisting of facts that were loosely organized together. The facts were stated without relating them to each other by means of causal or some other connections. Only a minimal amount of inference seemed to be involved.

3. *Well-organized facts*. A statement consisting of rather well organized factual or descriptive information. Although the ideas did not explicitly provide an explanation, it was meaningfully organized and had a potential for facilitating understanding of the issue in question.

4. *Partial explanation*. A statement represents an explicit attempt to construct an explanation and to provide new information, but the explanation was only partially articulated. It was only an explanatory sketch that was not further elaborated.

5. *Well-organized explanation*. A statement containing postulations of common causes, reasons and other explanatory relations, or theoretical entities. (p. 7)

After coding the selected samples, Nachowitz (2018) found that "not one included a statement of causes, reasons, or other explanatory relationships [Level 5]. At best, students were listing partially organized facts when asked to write about their understanding of mathematical concepts" (p. 8). Almost three-fourths of all written samples were coded at Level 1 or Level 2. One example of student writing coded at Level 1 was "My estimate was correct" (Nachowitz, 2018, p. 7). While this is certainly a written statement, at best, it simply stated what the student may have learned when he or she checked the answer. It did nothing to demonstrate the student's thinking when solving the problem. The following is another example of student writing from Nachowitz's (2018) study:

> 1) You have to read all the problem carefully so you won't miss something important. 2) You get both of the numbers 8 foot and 10 foot. 3) Then you times 8 foot times 2. 4) After you do that you times 10 times 2. 5) Then you add both of these numbers and you should get 36 feet. 6) OK you got the answer for the fencing one, so now [it's] time to do the netting one. 7) You times 8 foot times 2. 8) You times 10 foot times h. 9) Then you should get 36 feet and [you're] done. (p.7)

Like the example coded at Level 1, this response, which was coded at Level 2, did not demonstrate the thinking of the student. Rather, it was a summary of steps the student took to reach the solution, a common trend among students' writing samples analyzed by Nachowitz.

One pattern Nachowitz (2018) observed was teachers were not asking students to justify their answers nearly as much as they were directing students to *explain how* they obtained their answers. This resulted in students' tending to list what steps they took to find solutions instead of explaining their thinking and conceptual understandings. Even when prompts were worded as *explain your answer*, students still provided surface-level justifications that did not demonstrate conceptual understanding. Students' responses were mostly algorithmic, and the assigned prompts did not elicit *why* students employed particular problem-solving strategies or how they verified their answers were accurate based on their understanding of the concept being assessed. The data indicated students may not have been able to discriminate the difference between what showing one's thinking rather than just showing one's work, as posited by Nachowitz (2018):

> Perhaps, after years answering prompts to "do the math" and "show work," students have conflated *show your work* with the algorithmic steps rather than *show your thinking*, which the standards and principles recommend. The data presented here demonstrate a pattern of listing steps using written language rather than explaining, describing, or justifying strategies … Language conveys a great deal, and "work" is not the same as "thinking." *Work* implies what you do, not how you think or why you think it. If thinking is what the standards and practices call for, then the only way to address this disconnect is to develop student understanding of what the prompt, "explain your thinking" really means. (p. 10)

Analysis of samples of student writing provided by Baxter et al. (2005), Martin and Polly (2013, 2016), and Martin et al. (2017) leads to similar conclusions. Each of these studies employed student writing in various ways, such as through the use of journals or by having students reflect in writing about what they learned during a lesson. Inspecting student writing samples cited in their research reveals the same trends Nachowitz (2018) identified. Below is a list of several examples of student writing analyzed within these studies. These written responses commonly listed steps, summarized procedures, and explained *what* they did to solve problems. The responses did not elucidate students' thought processes or explain why they understood their solutions were valid, which is what the Standards for Mathematical Practice, the CCSS-M, and the NCTM call for students to do (Nachowitz, 2018; NCTM, 2018).

1. Well, I really don't know how to do this but I'll try it anyway. 16/32 I think of 1/2 because 16 is half of 32. (Baxter at al., 2005, p. 128)

2. A range is when you take two numbers and subtract them. The numbers you subtract should be the biggest and the smallest number in a set of data. (Baxter at al., 2005, p. 128)

3. 5 x 5 = 25 + 4 + 6 = 35 She ran the miles she wanted to run, she ran extra 5 miles. I multiplied 5 times 5 and the answer was 25. I added 4 miles equals 29 miles. I added 6 miles equals 35 miles [because] I added. Yes, she ran 35 but she only needed to run 30 miles. She ran 5 miles more. I can do 35 subtract 30 = 5 miles. He (Bert) should have added 5 extra but before that he should have 25 + 4 + 29 and 29 + 6 = 35. (Martin & Polly, 2013, p. 257)

4. I would tell him to do 5 x 5 because she ran 5 miles for 5 days. Then add 4 + 6 = 10 because she ran 4 miles and 6 miles on the last 2 days. Then add it together 25 + 10 = 35 and so Marley is 5 miles over. (Martin & Polly, 2016, p. 70)

5. Division is easy it's like adding, 8 + 8 + 8 = 24, 24 ÷ 8 = 3. (Martin et al., 2017, p. 551)

Hiebert et al. (2005) stated that educators often focus more on the inclusion of a feature in instruction rather than how that feature can be implemented effectively to achieve learning goals. This appears to be the case of how writing has been enacted in the mathematics classroom. Teachers may have been including writing in their classes, but they may not have been assigning prompts that evoke student thinking and conceptual understanding (Nachowitz, 2018). Sample responses from multiple studies revealed students were not able to employ written language to make conjectures, justify conclusions, communicate those conclusions to others, and argue whether or not proposed solutions to problems are valid (Baxter et al., 2005; Martin & Polly, 2013, 2016; Martin et al., 2017; Nachowitz, 2018). Instead, students were listing steps, summarizing processes, and *showing work* instead of *showing thinking*. Nachowitz (2018) remarked that teachers' implementation of writing in mathematics is not aligned "with the standards and principles calling for students to explain and justify their conceptual understanding and application of problem-solving processes" (p. 8). Thus, writing in the mathematics classroom has lacked conceptual understanding, falling short of achieving the goals established by the Standards for Mathematical Practice, the CCSS-M, and the NCTM (Nachowitz, 2018).

Gaps in Existing Research on Writing in Mathematics

Numerous studies have asserted teachers can use students' writings to assess their understanding and adjust instruction to improve their understanding (Baxter et al., 2005; Kostos & Shin, 2010; Martin & Polly, 2013; McIntosh, 1991; Miller & England, 1989; Miller, 1991, 1992b). However, there appears to be a gap in the literature on what actions a teacher can take to improve students' abilities to write epistemically complex explanations of mathematics. It was reported that teachers in the Miller and England (1989) and Miller (1991, 1992b) studies altered their instructional practices by reteaching concepts, delaying exams and designing reviews based on what they learned from student writings, and conferencing individually with students to address misconceptions, but the research lacks an explanation of what these alterations looked like when teachers put them into practice. McIntosh (1991) claimed half the benefit of journals was students' enhanced learning but offered little evidence of how their learning was enhanced or what teachers can do to accomplish this. The teacher in the Baxter et al. (2005) study reported her students' writings were a valuable assessment tool that guided her instructional planning as it allowed her to see into her students' minds and assess if they were truly comprehending material. However, Baxter et al. did not elaborate on any specific adjustments the teacher made when she determined students were not demonstrating comprehension of concepts at the level she intended; no evidence was provided regarding the nature of prompting or scaffolding done by the teacher to improve the quality of students' writing. Kostos and Shin (2010) and Martin and Polly (2013) also did not provide this kind of evidence.

The literature seems to leave many questions unanswered or even unaddressed that could benefit teachers who want to improve their students' abilities to explain concepts in writing. Miller (1991) did remark teachers would provide written commentary on student responses and return those to students, but how did they assess if the written feedback was beneficial for the students? What specific actions did teachers take when they identified misconceptions hiding behind procedurally correct solutions? When teachers retaught concepts, did they present anything differently than they did during initial instruction? What were discussions like in the individual conferences with students? What evidence do teachers have to convince others their students' conceptual understandings were improved as a result of these adjustments? Was the simple inclusion of writing responsible for the improvements, or were there specific elements of feedback that were critical in making this happen? It seems likely the role the teacher played made an impact on any observed improvements, but the literature lacks evidence to support that conclusion. The literature certainly establishes that writing provides a window into students' thinking, but the literature does not provide a picture of teachers' reactions when students' thinking lacks conceptual understanding.

Summary

Even though experts in mathematics education have emphasized there should be a balance of students' procedural abilities and conceptual understandings of mathematics (CCSSI, 2020a; Hiebert et al., 2005), mathematics education in the United States has focused mainly on the development of students' procedural skills (Hiebert et al., 2005; NCTM, 2018). There is a wide body of existing research that has provided evidence that including writing in the mathematics classroom: a) helps teachers and students assess students' conceptual understandings (Baxter et al., 2005; Bicer et al., 2018; Casler-Failing, 2013; Evans, 2017; Kostos & Shin, 2010; Martin & Polly, 2013, 2016; Martin et al., 2017; McIntosh, 1991; Miller & England, 1989; Miller, 1991, 1992a, 1992b; Nachowitz, 2018); b) engages students in mathematical thinking (Baxter et al., 2005; Evans, 2017; McIntosh, 1991; Miller, 1992a, 1992b); and c) encourages precision of mathematics vocabulary (Kostos & Shin, 2010; Miller, 1992b, 1993). However, even though numerous studies have stated teachers can use written responses to assess student understanding and adjust instruction (Baxter et al., 2005; Kostos & Shin, 2010; Martin & Polly, 2013; McIntosh, 1991; Miller & England, 1989; Miller, 1991, 1992b), they have not described what those adjustments looked like or provided evidence that the adjustments resulted in students' learning to demonstrate conceptual understanding of mathematics in writing.

Catalyzing Change in High School Mathematics (NCTM, 2018) recommended teachers move beyond just eliciting evidence of student thinking; rather, its recommendation was teachers should utilize evidence of student thinking to build conceptual understanding. Hiebert et al. (2005) remarked that when educators attempt to incorporate specific features in classrooms, "teachers and administrators often focus on *what* to implement rather than *how* to implement it, focusing on the presence of a feature rather than its purpose and how it interacts with other features to achieve or block learning goals" (p. 128). The existing literature on writing in mathematics classrooms seems to have followed this pattern. Specific writing strategies have been described, such as journals and impromptu writing prompts. Benefits of these strategies have been discussed, especially how writing helps teachers assess students' understandings. It also has been noted frequently that teachers can use student writing to make adjustments to instruction. What the research appears to lack is *how* teachers respond to student writing and adjust instruction to better their students' conceptual understandings. Furthermore, past research has determined the nature of student writing in the mathematics classroom has fallen short of the expectations outlined in the CCSS-M (Nachowitz, 2018), and new research has been recommended on strategies teachers can implement to help students learn to demonstrate conceptual understanding of mathematics in writing (Baxter et al., 2005; Martin & Polly, 2013, 2016; Martin et al., 2017; Nachowitz, 2018). Furthermore, Nachowitz (2018) recommended research investigate the viability of instructional scaffolding as an effective practice to help students learn to write epistemically complex explanations of mathematics.

CHAPTER III: METHODOLOGY

Existing research on writing in the mathematics classroom has not appeared to focus on what actions teachers can take to teach their students how to write epistemically complex explanations of mathematics. The methodology employed within this qualitative action research study focused on answering two research questions:

1. Does engaging secondary mathematics students in an instructional scaffolding teaching strategy support their abilities to write explanations with high degrees of epistemic complexity?
2. What challenges were encountered while implementing instructional scaffolding to support students' abilities to write explanations with high degrees of epistemic complexity?

Further details and key considerations of the methodology employed in this study are discussed in the following sections: a) site of the study; b) participants; c) ethical considerations; and d) research design.

Site of the Study

This study was conducted in a suburban high school located in the Midwest region of the United States. During the 2020-2021 school year, there were 1,497 students enrolled in the high school. Of those students, 34.67% qualified for the Free and Reduced Price School Meals program, but the high school was not considered Title I. The high school had the following demographics at the time the study was conducted: 0.53% of all students were American Indian, 0.73% were Asian, 1.67% were Black, 5.14% were Hispanic, 3.94% were Multi-racial, 0.13% were Pacific Islander, 87.84% were White, and 0.53% were English Language Learners. All students in the high school were provided access to a school-issued Chromebook to use at school and at home. The high school also participated in a state scholarship program that provides scholarships to graduates who fulfill certain obligations and then attend participating public community colleges or vocational/technical schools.

Participants

The participants of this study were two classes of Algebra 2 students that were taught by the researcher. Because this study focused on the researcher's two Algebra 2 classes, convenience sampling was used to select the students who were participants in this study. Student participants and at least one of their parents or guardians signed informed consent agreements (Appendix C) to indicate they voluntarily participated in the study. A total of 35 out of 38 enrolled students gave consent to be participants. The participating Algebra 2 students consisted of 3 sophomores, 31 juniors, and 1 senior, all of whom ranged from 15 to 18 years of age. Of the 35 participating students, 22 were males and 13 were females. There were 31 students who were White, 3 students were multi-racial, and 1 student was Hispanic. One student received special education accommodations through an Individual Education Program, and one student received accommodations through a 504 Plan. When the data collection process concluded, 4 student participants were earning a course grade of an A, 18 had a B, 8 had a C, and 5 had a D. Of the student participants who had taken the Algebra 1 End of Course (EOC) Exam, 3 scored Advanced, 14 scored Proficient, 12 scored Basic, and 1 scored Below Basic. No score for the Algebra 1 EOC was available for 5 students. All participants' identities were kept confidential in this study.

Ethical Considerations

The Missouri State University Institutional Review Board approved this study on

September 22, 2020 (Appendix D). The school district where the study was conducted also gave approval on October 5, 2020 (Appendix E). Informed consent was obtained from both student participants and at least one parent or guardian of each student participant. The researcher intended to deliver the same instruction to every student participant, regardless of their agreement to participate in the study or not. As such, even if a student or a parent or guardian of a student did not provide consent to participate in this study, that student still received the same instructional experience in the classroom as those who did provide consent. The only difference was that data collected and analyzed for this study did not include data of students who did not provide consent or whose parents or guardians did not provide consent.

Research Design

This was a qualitative action research study centered on the implementation of an instructional scaffolding teaching strategy in two high school Algebra 2 courses. An iterative process was used that involved: a) implementing an initial design of instructional scaffolding; b) reflecting on its efficacy; and c) adjusting the design as needed based on challenges encountered by the researcher. The initial design of the instructional scaffolding process, which was recommended by Nachowitz (personal communication, March 3, 2020), is provided in the following steps:

1. *The teacher models writing processes using a think-aloud protocol.* This means the teacher talks through thought processes while composing a written answer to a prompt, allowing learners to see exactly what the teacher thinks while writing in mathematics and what is expected of their written responses.

2. *Collaborative practice.* Students participate in pair-shares to try writing their own responses to a similar prompt. One student explains their thought process about how to answer the prompt, and the other student writes it down. Volunteers then use a think-aloud protocol in front of the class to explain what they wrote and why.

3. *Debriefing process.* The teacher provides guided feedback to let the volunteers know what they did right and to offer suggestions for improvement.

4. *Independent practice.* Students work independently to try writing their own responses to a similar prompt.

5. Repeat steps (3) and (4) as needed.

This design was adjusted throughout the iterative process because of reflections made by the researcher. The rationale for the adjustments that were made is discussed in Chapter V. The final design of instructional scaffolding implemented in this study is described in the below steps:

1. *Modeling with a think-aloud protocol.* The teacher talks through thought processes while composing a written answer to a prompt, allowing learners to see exactly what the teacher thinks while writing in mathematics and what is expected for their written responses to demonstrate conceptual understanding.

2. *Collaborative practice with debriefing.* Students participate in pair-shares to try writing their own responses to a similar prompt. One student explains their thought process about how to answer the prompt, and the other student writes it down. While pairs of students are working, the teacher monitors their conversations and provides guided feedback to let the students know what they did right and to offer suggestions for improvement.

3. *Score sample responses.* Working with a partner, students use a rubric to score sample responses to prompts. Students in one class score samples from a separate class. After all pairs have finished scoring the responses, the class engages in a whole-class discussion to share their scores and their reasoning for awarding those scores. The teacher facilitates the discussion and allows students to convince others why they are right, only intervening when it seems necessary.

4. *Final assessment*. Students take a final assessment featuring prompts similar to those they answered and scored during steps (2) and (3). Their responses are scored by the teacher using the coding framework for epistemic complexity.

The coding framework for epistemic complexity used in this study (Appendix B) was a modified version of the coding framework Nachowitz (2018) used (Appendix A). The final, adjusted design of the instructional scaffolding teaching strategy was implemented to teach three units of study: a) functions; b) solutions to equations; and c) factoring polynomials.

Data Collection. From October to December of 2020, three types of data were collected for this study: a) students' written responses to prompts asking them to justify their answers; b) free-response journal entries made by the researcher; and c) video recordings of classes during which instructional scaffolding was implemented. A total of 94 student written responses were collected – 31 at the end of the first iteration, 31 at the end of the second iteration, and 32 at the end of the third iteration. These written responses were collected and analyzed to respond to both research questions of the study. The researcher wrote a total of 7 reflections in the free-response journal. These reflections were written at various steps of each iteration of implementation of instructional scaffolding as the researcher reflected on the challenges encountered before, during, and after each iteration. A total of 18 class periods were video recorded. The video recordings captured various steps of each iteration of implementation of instructional scaffolding so the researcher could reflect on the efficacy of the process and the challenges encountered during implementation. The free-response journal entries and the video recordings were collected and analyzed to respond to the second research question of the study.

Data Analysis. Students' written responses to prompts asking them to justify their answers were analyzed using a coding framework to measure epistemic complexity (Appendix B). Four different levels of codes were applied: *Level 1 – Needs Help*; *Level 2 – Not Quite*; *Level 3 – Almost There*; and *Level 4 – Nailed It.* Responses with the greatest degree of epistemic complexity earned the score of *Level 4 – Nailed It*. This coding framework was a modified version of one used by Nachowitz (2018) in a related study (Appendix A). The coding framework used by Nachowitz was modified for use in this study for two reasons.

First, when the researcher read students' responses during the first iteration of instructional scaffolding, the responses were not able to be appropriately sorted into the five levels featured in the coding framework used by Nachowitz (2018). Instead, the responses were more appropriately able to be sorted into four distinct levels: 1) responses that were entirely incorrect; 2) responses that recited something about the definition of the concept being assessed but did not meaningfully attempt to apply its definition; 3) responses that knew the definition of the concept being assessed but failed to properly apply the definition; and 4) responses that knew the definition of the concept being assessed and applied it properly and effectively.

The second reason for modifying the coding framework used by Nachowitz (2018) was the researcher wanted it to be transferable to a rubric form so students could use it during the *score sample responses* step. Because of this, the framework needed to feature language that was accessible to students. The terminology featured in the coding framework used by Nachowitz – such as *separated pieces of facts* or *partially organized facts* or *partial explanation* – may not have been appropriately accessible for student use. Thus, the coding framework used in this study was modified to include language seemingly more accessible to students – such as *Needs Help*, *Not Quite*, *Almost There*, and *Nailed It* – that defined four different levels of codes. These codes were used to measure the epistemic complexity of students' written explanations. A side-by-side comparison of the two coding frameworks is given in Table 1.

Interrater reliability was established by calculating the value of Cohen's Kappa and the value of Cohen's Weighted Kappa for codes that were applied to a sample of 15 responses collected during the first iteration of instructional scaffolding. The calculated value of Cohen's Kappa was 0.6296 with a standard error of 0.1586; a 95% confidence interval had a lower bound of 0.3188 and an upper bound of 0.9404. The calculated value of Cohen's Weighted Kappa was 0.7727 with a standard error of 0.0901; a 95% confidence interval had a lower bound of 0.5961 and an upper bound of 0.9493. Both the unweighted and weighted Kappa values indicated

Table 1. Comparison of Coding Frameworks for Epistemic Complexity.

Level and Definition – Nachowitz	Level and Definition – Cowan
1. *Separated pieces of facts*. A statement consisting of a list or table of facts with hardly any integration or connections/or explanation.	1. *Needs help.* An explanation that demonstrates little, if any, awareness of the concept being assessed. This could be a response that is entirely incorrect, or it could be a response that is incorrect due to a misunderstanding of the foundational ideas behind a concept.
2. *Partially organized facts*. A statement consisting of facts that were loosely organized together. The facts were stated without relating them to each other by means of causal or some other connections. Only a minimal amount of inference seemed to be involved. 3. *Well-organized facts*. A statement consisting of rather well organized factual or descriptive information. Although the ideas did not explicitly provide an explanation, it was meaningfully organized and had a potential for facilitating understanding of the issue in question.	2. *Not quite.* An explanation that demonstrates awareness of a concept's definition but does not illustrate how a particular problem or situation is connected to that definition.
4. *Partial explanation*. A statement represents an explicit attempt to construct an explanation and to provide new information, but the explanation was only partially articulated. It was only an explanatory sketch that was not further elaborated.	3. *Almost there.* An explanation that demonstrates knowledge of a concept's definition and attempts to make a connection between that definition and a given problem or prompt. It may describe an example that almost connects to the definition but falls short by making an incomplete or slightly incorrect statement.
5. *Well-organized explanation*. A statement containing postulations of common causes, reasons and other explanatory relations, or theoretical entities.	4. *Nailed it.* An explanation that demonstrates understanding of a concept's definition and its application. It makes a direct connection between that definition and a given problem or prompt by providing a clear and accurate description, justification, or example of how the concept is being applied.

substantial agreement among raters (Landis & Koch, 1977). Further interrater reliability tasks were not performed in the second and third iterations. Distributions of scores were created after the *final assessment* step of each iteration of implementation of instructional scaffolding was finished, and an overall distribution of scores of all written responses collected in the study was created. Challenges related to coding responses were also identified in the findings of the study.

The free-response journal entries and the video recordings were analyzed by looking for challenges that were present throughout the three iterations of implementation. The free-response journal entries featured the researcher's reflections about various steps of each iteration, such as challenges encountered during the process or while coding responses. The researcher also made notes of what aspects of the design of instructional scaffolding seemed to be working and should remain included in the design. Furthermore, the researcher reflected on the details and efficacy of the design of instructional scaffolding by watching the video recordings. These recordings provided a clearer picture of how students were interacting with each other and with the teacher during each iteration, and they showed how the researcher responded to challenges encountered.

Instrumentation. The central measurement tool of this study was a coding framework designed to measure the epistemic complexity of students' written explanations (Appendix B). Swivl technology also was used to capture video recordings of classes that featured instructional scaffolding. The Swivl device focused what it recorded on the participants who had the tracker. This tracking feature allowed the researcher to listen to conversations between students as they worked during various steps of each iteration. It also allowed the researcher to reflect on conversations he had with students when he was providing them feedback on their written responses. This helped the researcher recall challenges encountered while responding to student difficulties.

Role of the Researcher. The researcher had two primary roles while this study was conducted. First, the researcher implemented the instructional scaffolding teaching strategy in his two Algebra 2 classes. This implementation included: a) choosing concepts to scaffold; b) creating materials to be used in each step of the process, such as prompts to assign to students and summative assessments; c) responding to student difficulties; and d) scoring students' written responses using the coding framework for epistemic complexity. Second, the researcher analyzed data that were collected to respond to the study's two research questions. This data included: a) students' written responses to prompts asking them to justify their answers; b) free-response journal entries written by the researcher; and c) video recordings of classes during which instructional scaffolding was implemented.

Summary

This qualitative action research study took place from September to December of 2020 in a suburban high school in the Midwest region of the United States. The researcher was a full participant as the classroom teacher, and there were 35 student participants in two Algebra 2 classes. An iterative process was used to implement an initial design of instructional scaffolding, reflect on its efficacy, and adjust the design as needed based on challenges encountered by the researcher. Three iterations of implementation of instructional scaffolding were employed to teach the mathematical concepts of functions, solutions to equations, and factoring polynomials. Three types of data were collected during the study: a) 94 written responses to prompts asking students to justify their answers; b) 7 free-response journal entries made by the researcher; and c) 18 video recordings of classes during which instructional scaffolding was implemented. A modified coding framework (Appendix B) was used to measure the epistemic complexity of the

94 written responses that were collected. Interrater reliability was established in the first iteration of implementation; there was substantial agreement among the two raters (Landis & Koch, 1977). No further interrater reliability tasks were completed in the second and third iterations of implementation. Challenges encountered while coding responses also were identified. Furthermore, the free-response journal entries and video recordings were analyzed to look for challenges that were present throughout the three iterations of implementation.

CHAPTER IV: FINDINGS

This study analyzed the implementation of instructional scaffolding in two high school Algebra 2 classes to help students learn to write epistemically complex explanations of mathematics. Data analysis focused on addressing two research questions:

1. Does engaging secondary mathematics students in an instructional scaffolding teaching strategy support their abilities to write explanations with high degrees of epistemic complexity?
2. What challenges were encountered while implementing instructional scaffolding to support students' abilities to write explanations with high degrees of epistemic complexity?

In this chapter, research question 1 is addressed through the analysis of 94 written responses collected during the study. A modified coding framework (Appendix B) was used to measure the epistemic complexity of these written responses. Research question 2 is addressed through the analysis of the following sources of data: a) 94 written responses; b) 7 free-response journal entries made by the researcher, and c) 18 video recordings of classes during which instructional scaffolding was implemented. These data were analyzed to identify common challenges that were encountered throughout the three iterations of implementation.

Research Question 1: Epistemic Complexity of Students' Written Explanations

At the end of each iteration of implementation of instructional scaffolding, students took final assessments in which they wrote responses to prompts asking them to justify their answers to problems. A total of 94 written responses were analyzed. The researcher applied the modified coding framework for epistemic complexity (Appendix B) to code these responses. A summary of the distribution of all written responses coded in the study is given in Figure 1.

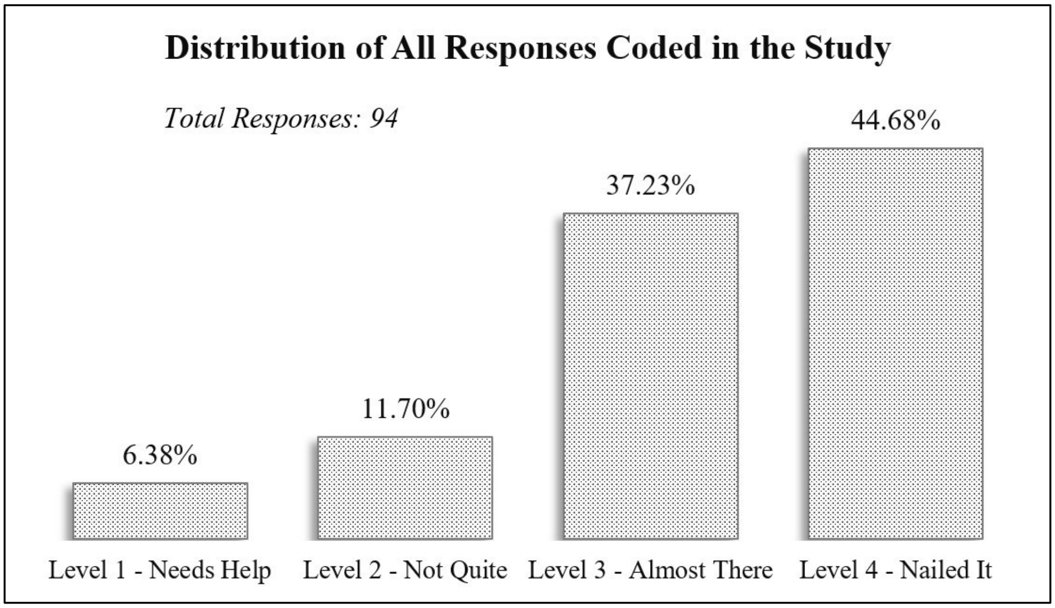


Figure 1. Distribution of all responses coded in the study.

Of the 94 total responses that were coded, 77 (81.91%) were coded at Level 3 or Level 4, indicating a greater degree of epistemic complexity was featured in these responses. An analysis of the codes awarded throughout the three iterations of implementation of instructional scaffolding follows, along with examples of responses that were coded at each level.

Written Responses Coded after First Iteration. The final assessment of the first iteration of instructional scaffolding was taken by 31 students. Students were given a three-question quiz on the concept of *functions* (Appendix F). Only the responses to the third prompt were chosen to be coded because the third prompt was representative of the prompts that students practiced answering the most during the *collaborative practice with debriefing* step and scored during the *score sample responses* step of instructional scaffolding. Of the 31 responses that were coded, 4 (12.90%) were coded as *Level 1 – Needs Help*, 2 (6.45%) were coded as *Level 2 – Not Quite*, 12 (38.71%) were coded as *Level 3 – Almost There*, and 13 (41.94%) were coded as *Level 4 – Nailed It*. In all, 25 (80.65%) of the 31 total responses were coded at Level 3 or Level 4, indicating a greater degree of epistemic complexity was featured in these responses.

The 4 responses that were coded as *Level 1 – Needs Help* provided no evidence they understood the concept of *functions*. Although they may have mentioned some of the terminology from the concept, such as *function* or *input* or *output*, the responses as a whole were entirely incorrect. Consider Ezra's response (Figure 2). His explanation shows that he did not understand the foundational idea of how to read and interpret the given graph that modeled input-output pairs of the given rule *f*. He did not connect that the outputs shown on the graph of *f* were what he should have used to think through the prompt. Specifically, he should have given examples like *f(1) = 0.8* or *f(1) = -0.8*. Had he recognized input-output pairs like those, he may have been able to better demonstrate his understanding of the concept of a *function*, but his response was entirely incorrect because he lacked that foundation. Instead, he gave examples that were lacking for two reasons: a) he did not include inputs that were actually part of the domain of *f*; and b) he incorrectly stated the output was always the notation *f(x)* regardless of the input. Ezra's response and the other 3 coded at Level 1 provided no evidence that they were aware of the concept being assessed. This suggested these students needed help in developing their understanding of *functions*.

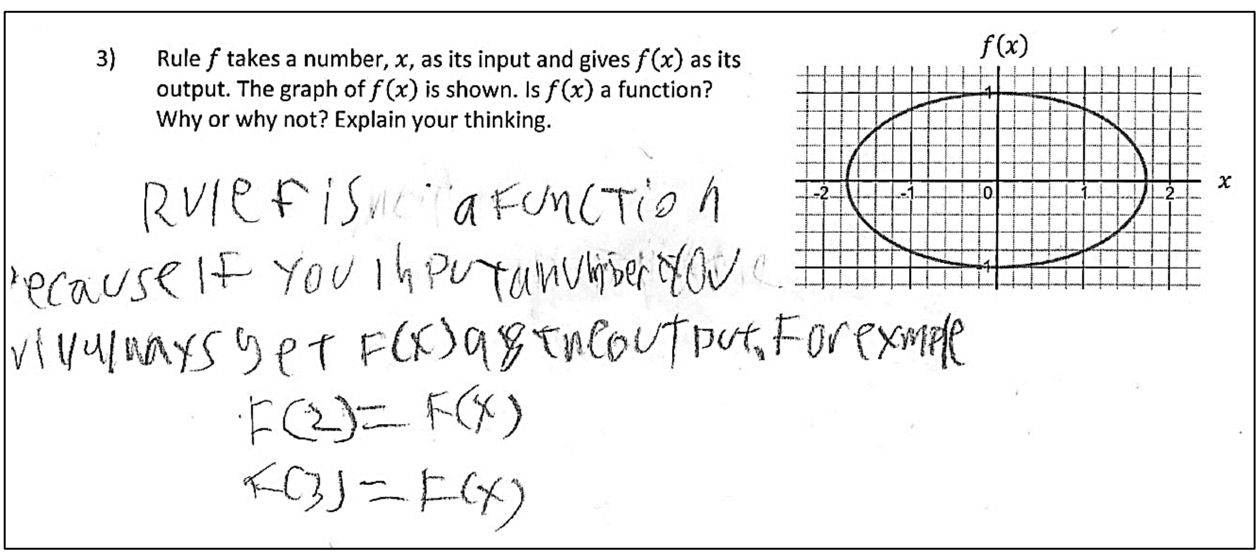


Figure 2. Ezra's Level 1 written response from final assessment in first iteration.

The 2 responses that were coded as *Level 2 – Not Quite* were able to demonstrate they had some awareness of the definition of a *function*, but they were unable to illustrate how the given prompt was connected to that definition. These responses attempted to address the idea that a given *rule* was not considered a *function* if there were multiple outputs possible for a given input. However, the language they used was incomplete or was not communicated clearly enough to prove to the reader they fully understood the concept. Finn's response (Figure 3) demonstrated he was aware of the definition of a *function* by stating *f* was not a function "because for every input you get multiple outputs." While this statement was not entirely true – as two inputs only had one possible output – it demonstrated Finn's awareness of the definition of a *function*. However, there was no further elaboration. Finn did not provide any examples from the graph of *f(x)* to support his written statement. As such, his statement did not connect the definition to the problem. It was unclear if Finn truly understood why that was a valid statement for this specific problem or if he had perhaps just memorized how to word an explanation for why a *rule* would not be considered a *function*. Finn's response showed that although he had some awareness of the definition of a *function*, he was not quite able to apply that definition and connect it to the given prompt.

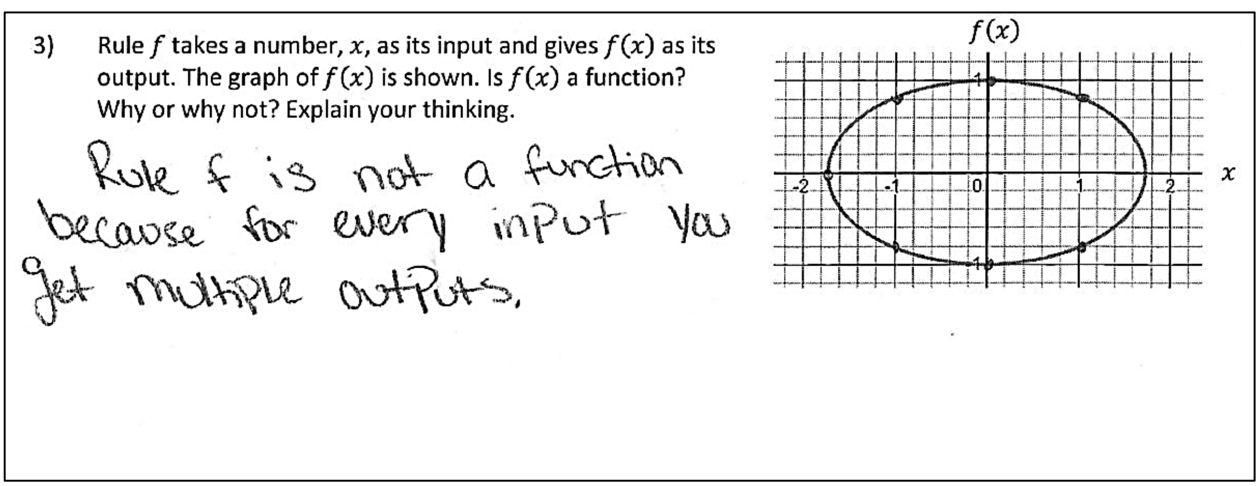


Figure 3. Finn's Level 2 written response from final assessment in first iteration.

The 12 responses that were coded as *Level 3 – Almost There* featured explanations that showed students knew a concept's definition. They attempted to apply that definition and connect it to the given prompt, but they fell short by making an incomplete or slightly incorrect statement. The incorrectness of these responses tended to result from a false absolute statement or an example that did not support what was stated. For example, in Jyn's response (Figure 4), she made the statement that "there can be multiple outputs for each input." Many responses coded at Level 3 made a similar statement, but the statement is untrue because of the word *each*. Rule *f*, in this problem, does have multiple possible outputs for *most* of its inputs, but it only has one possible output for the inputs of *-1.7* and *1.7*. These responses were almost there in demonstrating conceptual understanding of *functions*, but they missed those finer details that were featured in the responses that received a score of *Level 4 – Nailed It*.

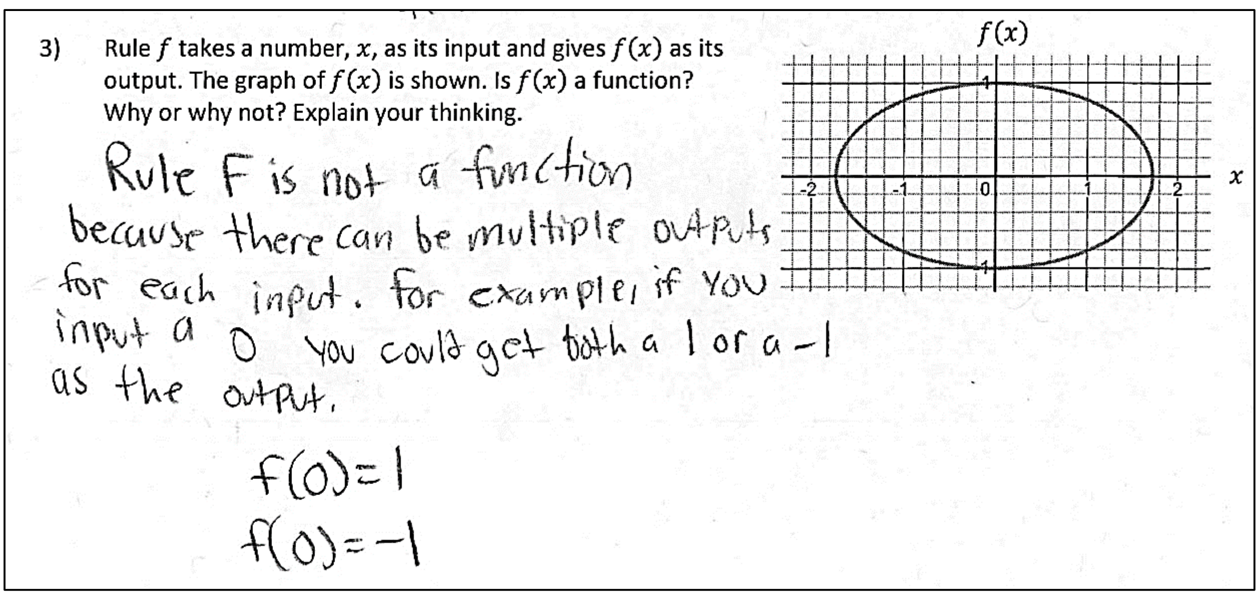
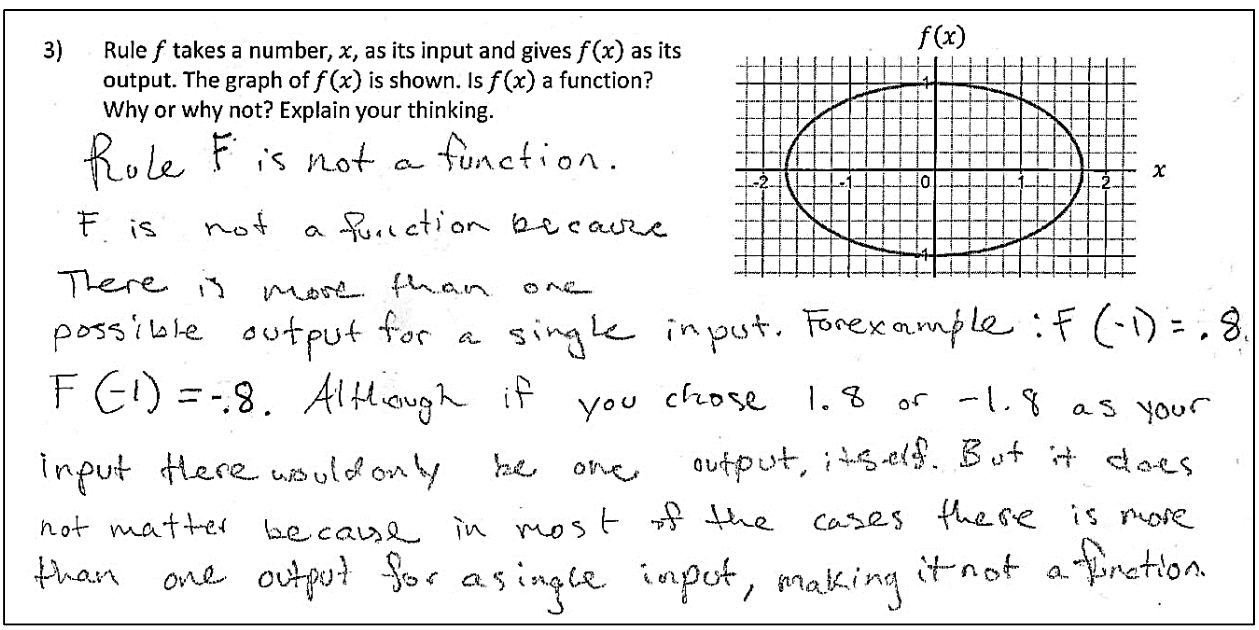


Figure 4. Jyn's Level 3 written response from final assessment in first iteration.

The 13 responses that were coded as *Level 4 – Nailed It* featured explanations that demonstrated understanding of a concept's definition and how to correctly apply that definition to a given problem. They provided a clear and accurate description and example of how the concept was being applied. What elevated these responses from Level 3 to Level 4 was their careful use of phrases like *some* inputs, *many* inputs, or *most* inputs rather than phrases like *all* inputs or *each* input that appeared in many of the Level 3 responses. They did not state that no matter what input was chosen, there *would* be more than one output, which is what many Level 3 responses did. Rather, they emphasized that *some* inputs (or *at least one* input) had more than one possible output. These students paid close attention to what outputs were possible for different inputs, and they avoided making false absolute statements like students whose responses were coded at Level 3. Luke's response (Figure 5) illustrates the care shown by many students whose responses were coded at Level 4. Luke acknowledged there were two points on the graph that had only one output possible, but he also said "it does not matter because in most of the cases there is more than one output for a single input, making it not a function." These Level 4 responses provided ample evidence they understood the concept of a *function rule*, which is why they were considered to have nailed it in their explanation.


3) Rule f takes a number, x, as its input and gives $f(x)$ as its output. The graph of $f(x)$ is shown. Is $f(x)$ a function? Why or why not? Explain your thinking.

Rule F is not a function. F is not a function because There is more than one possible output for a single input. For example: F(-1) = .8, F(-1) = -.8. Although if you chose 1.8 or -1.8 as your input there would only be one output, itself. But it does not matter because in most of the cases there is more than one output for a single input, making it not a function

Figure 5. Luke's Level 4 written response from final assessment in first iteration.

Written Responses Coded after Second Iteration. The final assessment of the second iteration of instructional scaffolding was taken by 31 students. Students were given a one-question quiz on the concept of *solutions to equations* (Appendix G). The given prompt was representative of the prompts that students answered during the *collaborative practice with debriefing* step and *score sample responses* step of instructional scaffolding. Of the 31 responses that were coded, 0 (0%) were coded as *Level 1 – Needs Help*, 3 (9.68%) were coded as *Level 2 – Not Quite*, 10 (32.26%) were coded as *Level 3 – Almost There*, and 18 (58.06%) were coded as *Level 4 – Nailed It*. In all, 28 (90.32%) of the 31 total responses were coded at Level 3 or Level 4, indicating a greater degree of epistemic complexity was featured in these responses.

The 3 responses that were coded as *Level 2 – Not Quite* were able to demonstrate they had some awareness of the definition of *solutions to equations*, but they were unable to provide evidence they fully understood the definition and how to apply it. Instead of making a conclusion about which graph correctly represented all the *solutions* to the given equation by appropriately referencing the definition and using examples to support their reasoning, these responses justified their explanations by only giving three examples of ordered pairs that worked in the equation. Consider Lando's response (Figure 6) in which he made the statement, "The right graph is true to the equation because after plugging 3 points in from the line into the equation they all came out to be true." This explanation did not provide evidence Lando fully understood the definition of *solutions to equations* and knew how to connect the graph to the equation. If anything, Lando's response implied that for a graph to represent all the solutions to an equation, the graph only needs to feature three points that produce true statements when substituted into the equation. His response should have used these examples to support his answer, but he used them as his answer and did not connect them to the definition.

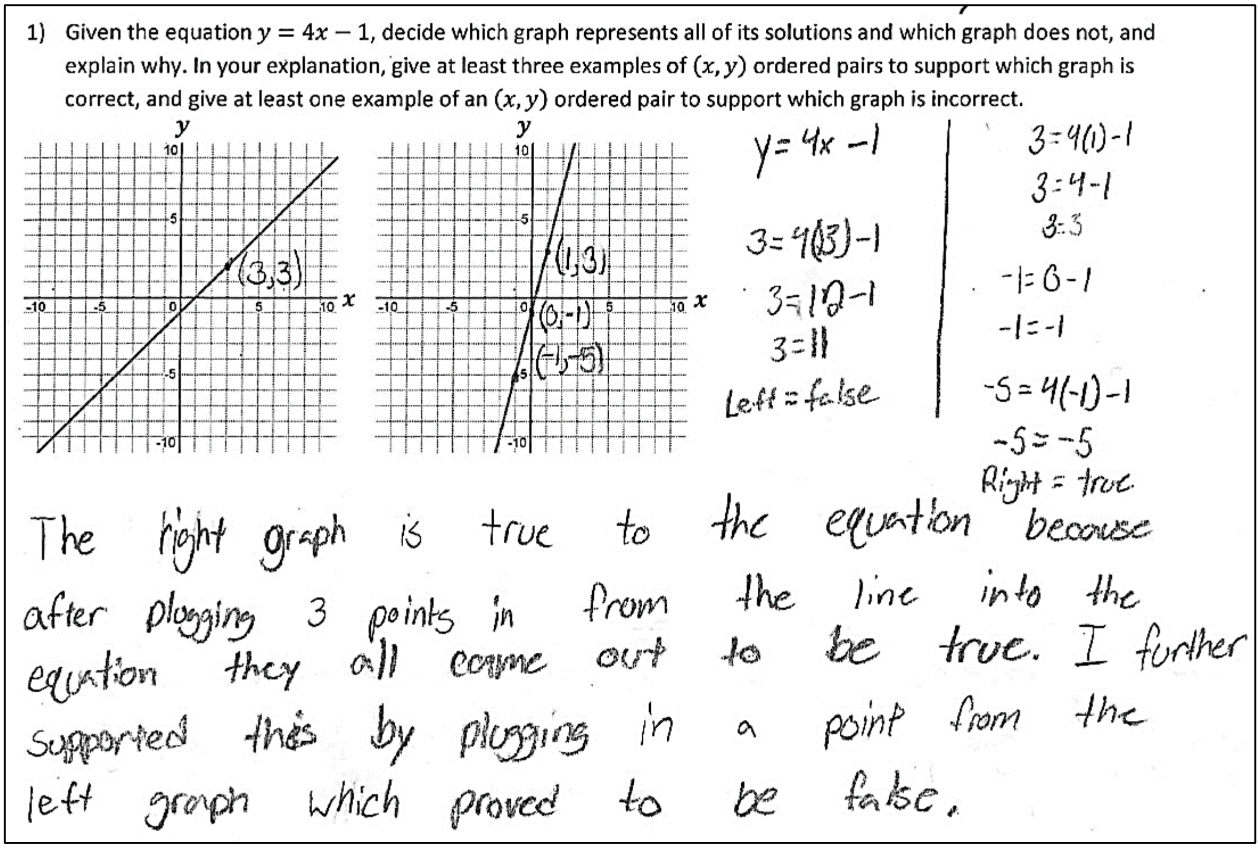


Figure 6. Lando's Level 2 written response from final assessment in second iteration.

The 10 responses that were coded as *Level 3 – Almost There* featured explanations that showed they knew a concept's definition. They attempted to apply that definition and connect it to the given prompt, but they fell short by making an incomplete or slightly incorrect statement. The mistakes featured in these responses tended to be the result of a false absolute statement, which is similar to what prevented many responses from being coded as *Level 4 – Nailed It* in the first iteration. For example, in Jyn's response (Figure 7), she made the statement that "The graph on the left is incorrect because when plugged into the equation, the ordered pairs on that graph do not make a true statement." Multiple responses at this level made a similar statement, but the statement was untrue because the point *(0, -1)* was a solution to the equation, and it was represented on the left graph. These students did not make it clear that a graph needs only one ordered pair to produce a false statement in a given equation for that graph to not represent all the solutions to the equation. By lacking that precision in their answers, these responses were unable to be coded as *Level 4 – Nailed It*.

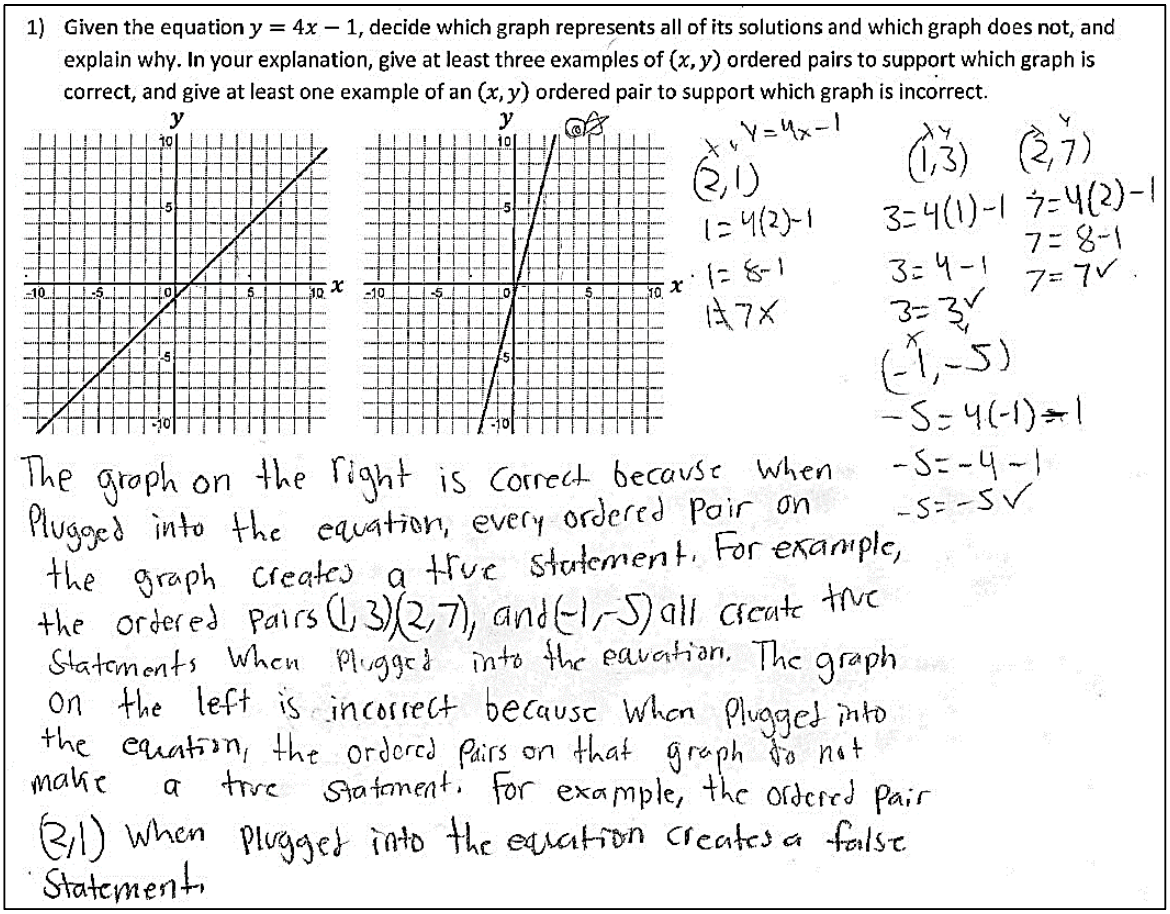


Figure 7. Jyn's Level 3 written response from final assessment in second iteration.

The 18 responses that were coded as *Level 4 – Nailed It* featured explanations that demonstrated understanding of a concept's definition and how to correctly apply that definition to a given problem. They provided a clear and accurate description and example of how the concept was being applied. What elevated these responses from Level 3 to Level 4 was their inclusion of statements about how *every* ordered pair on the correct graph would work in the equation and how *at least one* ordered pair on the incorrect graph would not work in the equation. They avoided making incorrect absolute statements like students whose responses were coded at Level 3. Cassian's response (Figure 8) illustrates the precision of the responses that were coded at Level 4. Cassian directly and correctly answered the question, he connected the fact that any ordered pair on the correct graph would work in the equation, he supported that statement with three accurate examples, and he stated and showed there was *at least one* ordered pair on the left graph that did not work in the equation. Cassian's response and the other Level 4 responses provided ample evidence they understood the concept of *solutions to equations*, which is why they were considered to have nailed it in their explanations.

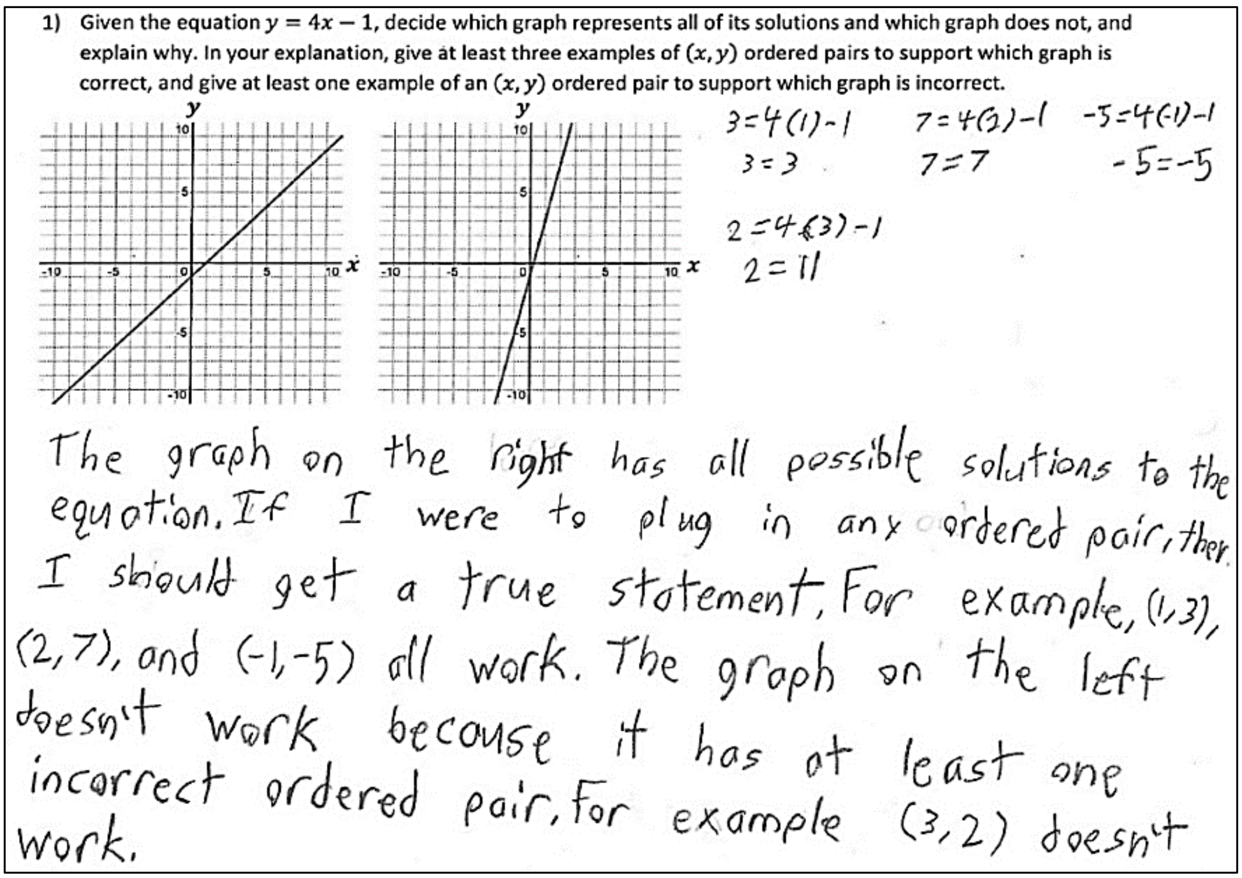
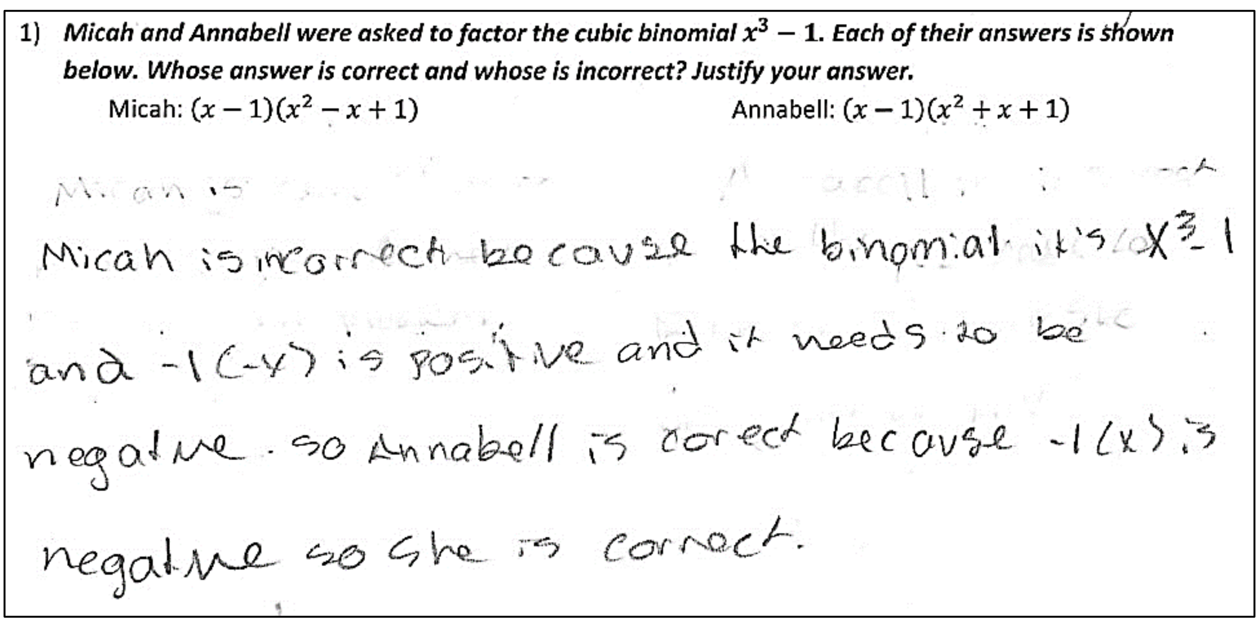


Figure 8. Cassian's Level 4 written response from final assessment in second iteration.

Written Responses Coded after Third Iteration. The final assessment of the third iteration of instructional scaffolding was taken by 32 students. Students were given a one-question quiz on the concept of *factoring polynomials* (Appendix H). The given prompt was representative of the prompts that students answered during the *collaborative practice with* *debriefing* step and *score sample responses* step of instructional scaffolding. Of the 32 responses that were coded, 2 (6.25%) were coded as *Level 1 – Needs Help*, 6 (18.75%) were coded as *Level 2 – Not Quite*, 13 (40.63%) were coded as *Level 3 – Almost There*, and 11 (34.38%) were coded as *Level 4 – Nailed It*. In all, 24 (75%) of the 32 total responses were coded at Level 3 or Level 4, indicating a greater degree of epistemic complexity was featured in these responses.

The 2 responses that were coded as *Level 1 – Needs Help* provided no evidence they understood the concept of *factoring polynomials*. The responses as a whole were entirely incorrect, and their explanations made no connections to the concept in question. Consider Mace's response (Figure 9). While he did get the answer right that Annabell's factorization was correct, his response did nothing to justify that conclusion using the concept of *factoring polynomials*. Instead, his explanation focused on the sign of the product of *-1(-x)* versus the sign of the product of *-1(x)*. It was unclear why Mace focused his response on the signs of those products, and that reasoning alone did not demonstrate he had a conceptual understanding of


1) **Micah and Annabell were asked to factor the cubic binomial $x^3 - 1$. Each of their answers is shown below. Whose answer is correct and whose is incorrect? Justify your answer.**

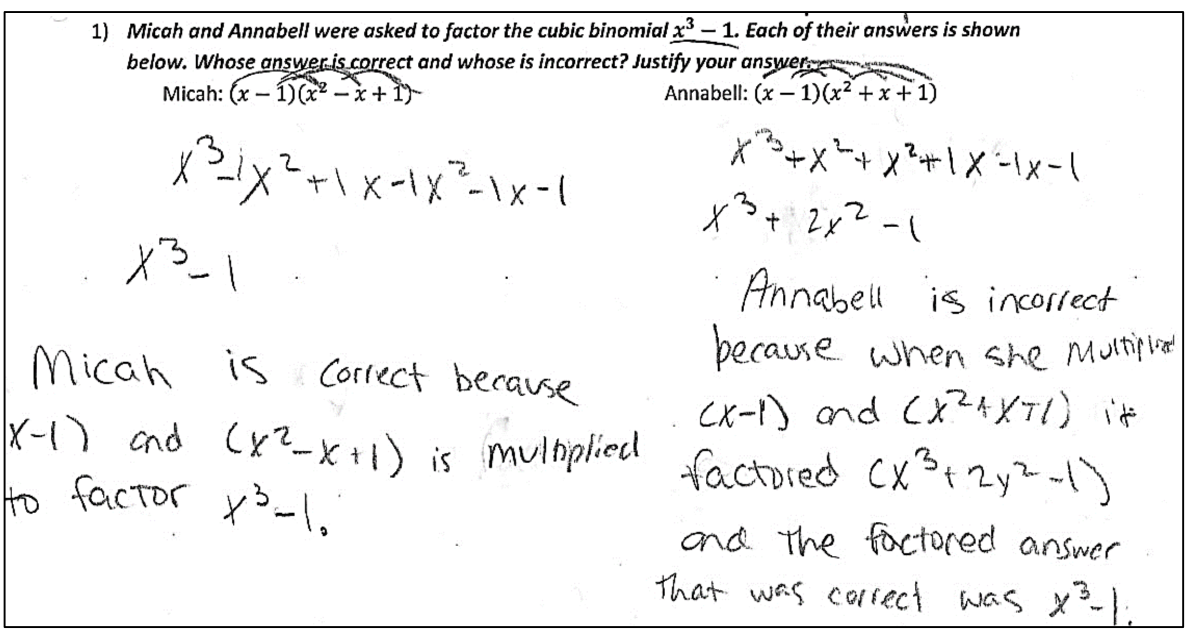
Micah: $(x-1)(x^2 - x + 1)$ Annabell: $(x-1)(x^2 + x + 1)$

Micah is incorrect because the binomial it's $x^3 - 1$ and $-1(-x)$ is positive and it needs to be negative. so Annabell is corect because $-1(x)$ is negative so she is correct.

Figure 9. Mace's Level 1 written response from final assessment in third iteration.

factoring polynomials. This response and the other coded at Level 1 demonstrated these students needed help with learning and understanding the concept.

The 6 responses that were coded as *Level 2 – Not Quite* demonstrated they had some awareness of the definition of *factoring polynomials*, but they were unable to provide evidence they fully understood the definition and how to apply it. The language used by these students was the biggest issue aside from computational errors. Consider Kylo's response (Figure 10) in which he made the statement, "Micah is correct because $(x - 1)$ and $(x^2 - x + 1)$ is multiplied to factor $x^3 - 1$." Kylo also made a similar statement about Annabell's given answer. Each of these statements improperly used the word *factor* in place of the correct term *produce*. A more accurate explanation would state the *factors* multiply to *produce* the original polynomial. In addition to this misuse of terminology, Kylo's multiplication work above his explanations was incorrect, which led him to conclude Micah was correct when Annabell's answer was actually correct. The lack of precision in the terminology Kylo employed and the lack of accuracy in his computational work did not illustrate that he could properly connect the definition of *factoring*


1) ***Micah and Annabell were asked to factor the cubic binomial $x^3 - 1$. Each of their answers is shown below. Whose answer is correct and whose is incorrect? Justify your answer.***

Micah: $(x - 1)(x^2 - x + 1)$

$x^3 - 1x^2 + 1x - 1x^2 - 1x - 1$

$x^3 - 1$

Micah is correct because (x-1) and $(x^2 - x + 1)$ is multiplied to factor $x^3 - 1$.

Annabell: $(x - 1)(x^2 + x + 1)$

$x^3 + x^2 + x^2 + 1x - 1x - 1$

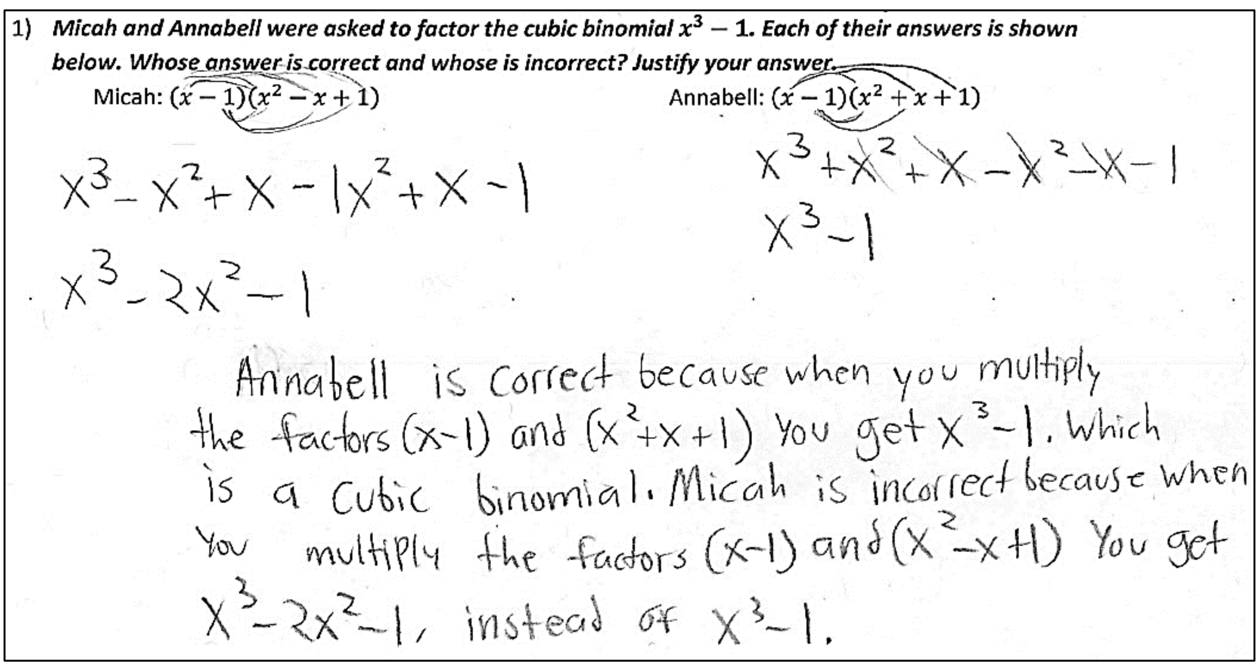
$x^3 + 2x^2 - 1$

Annabell is incorrect because when she multiplied $(x-1)$ and $(x^2 + x + 1)$ it factored $(x^3 + 2y^2 - 1)$ and the factored answer that was correct was $x^3 - 1$.

Figure 10. Kylo's Level 2 written response from final assessment in third iteration.

polynomials to the given problem. This caused his response and the 5 others like it to be considered not quite there in terms of their conceptual understanding.

The 13 responses that were coded as *Level 3 – Almost There* featured explanations that showed they knew a concept's definition. They attempted to apply that definition and connect it to the given prompt, but they fell short by making an incomplete or slightly incorrect statement. In fact, all 13 of these Level 3 responses fell short of earning a score of *Level 4 – Nailed It* due to incorrectly multiplying Micah's given factors to justify that his answer was wrong. Jyn's response (Figure 11) illustrates this common mistake. She accurately verified Annabell's factorization was correct, and her explanation used proper and precise terminology. It was only her incorrect work with Micah's factors that featured an inaccuracy, as she should have combined the like terms of *x* and *x* into *2x*. It could be argued this type of mistake reflects more of a lack of procedural skill than a conceptual misunderstanding. Jyn demonstrated she knew she needed to multiply the given factors to verify which factorization was correct. She used precise


1) ***Micah and Annabell were asked to factor the cubic binomial $x^3 - 1$. Each of their answers is shown below. Whose answer is correct and whose is incorrect? Justify your answer.***

Micah: $(x-1)(x^2-x+1)$

Annabell: $(x-1)(x^2+x+1)$

$x^3 - x^2 + x - 1x^2 + x - 1$

$x^3 - 2x^2 - 1$

$x^3 + x^2 + x - x^2 - x - 1$

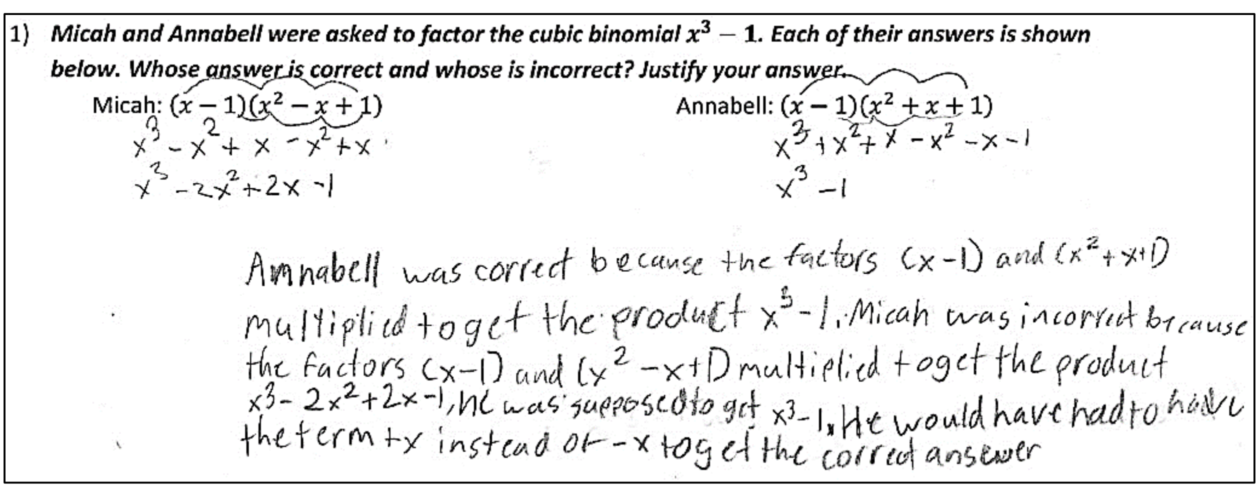
$x^3 - 1$

Annabell is correct because when you multiply the factors $(x-1)$ and (x^2+x+1) You get x^3-1. Which is a Cubic binomial. Micah is incorrect because when You multiply the factors $(x-1)$ and (x^2-x+1) You get x^3-2x^2-1, instead of x^3-1.

Figure 11. Jyn's Level 3 written response from final assessment in third iteration.

and accurate terminology in her explanation. She even correctly multiplied Annabell's factors together and then simplified to show the result was the original polynomial, proving it was the correct factorization. Her only downfall was an error that likely had more to do with a lack of procedural skill of how to multiply and simplify polynomials than a misunderstanding of the concept of factoring and its application to polynomials. With that said, Jyn's task was to justify which factorization was correct *and* which factorization was incorrect. The imprecision in multiplying Micah's incorrect factors prevented her justification from being perfect, which withheld her response from earning a score of *Level 4 – Nailed It*.

The 11 responses that were coded as *Level 4 – Nailed It* featured explanations that demonstrated understanding of the concept's definition and how to correctly apply that definition to a given problem. They accurately verified which of the given factorizations was correct *and* which one was incorrect, and their explanations used proper and precise terminology. What elevated these responses from Level 3 to Level 4 was their lack of any inaccuracies in their multiplication. They avoided making mistakes when multiplying the given factorizations, which was necessary to *justify* which one was correct and which one was incorrect. Rex's response (Figure 12) illustrates the precision of the responses that were coded at Level 4. Rex accurately


1) ***Micah and Annabell were asked to factor the cubic binomial $x^3 - 1$. Each of their answers is shown below. Whose answer is correct and whose is incorrect? Justify your answer.***

Micah: $(x-1)(x^2-x+1)$

$x^3 - x^2 + x - x^2 + x$

$x^3 - 2x^2 + 2x - 1$

Annabell: $(x-1)(x^2+x+1)$

$x^3 + x^2 + x - x^2 - x - 1$

$x^3 - 1$

Annabell was correct because the factors (x-1) and (x²+x+1) multiplied to get the product x³-1. Micah was incorrect because the factors (x-1) and (x²-x+1) multiplied to get the product x³-2x²+2x-1, he was supposed to get x³-1. He would have had to have the term +x instead of -x to get the correct answer

Figure 12. Rex's Level 4 written response from final assessment in third iteration.

multiplied and simplified both of the given factorizations, and he interpreted his work and used proper terminology in his explanation. Rex even pointed out what Micah should have changed in his answer for it to have been correct. Rex's response and the other Level 4 responses provided ample evidence they understood the concept of factoring and how it applied to polynomials, which is why they were considered to have nailed it in their explanations.

Research Question 2: Challenges Encountered during Implementation

Several challenges were encountered throughout the implementation of instructional scaffolding. These challenges were documented in a free-response journal before, during, and after various steps of the instructional scaffolding process. Classes featuring instructional scaffolding also were video recorded throughout the study, and these video recordings were used to reflect and identify further evidence of any challenges that were encountered. Identified challenges were categorized into two types: a) challenges related to the instructional scaffolding process; and b) challenges related to coding responses. Evidence of these challenges is presented in this chapter; how the researcher responded to these challenges is described in Chapter V.

Challenges Related to the Instructional Scaffolding Process. Three primary challenges related to the instructional scaffolding process were encountered during this study: 1) choosing concepts to scaffold; 2) setting expectations for epistemically complex explanations; and 3) responding to student difficulties.

<u>Process Challenge 1: Choosing Concepts to Scaffold</u>. Two free-response journal entries were written describing the challenge of choosing concepts to scaffold. The first entry was written before the first iteration of the instructional scaffolding process, and it featured the following excerpt:

> I am finding it to be a challenge just to think through what concept I am going to focus on teaching my students. There are a lot of factors I have to consider before I start this first round of scaffolding. What is the concept I want them to learn? Why do I want them to learn it through instructional scaffolding?

Before the first iteration had even begun, it was a challenge to decide what mathematical concept should be taught through the instructional scaffolding process. This challenge was encountered again in the second iteration, which was explained in the second journal entry:

> I tried to force my second concept a bit. The other Algebra 2 teacher is so far ahead of me that I was feeling pressure to try to catch up. She was about to test on solving systems of linear equations, and I hadn't even started that unit yet. So, I decided to try to scaffold the concept of a solution to a system of equations with my students. The problem, though, was my students needed to understand a prerequisite concept before moving on to solutions to systems. It became very apparent in my first day on solutions to systems that students didn't have a solid conceptual understanding of how graphs of equations represent all the solutions to those equations. After I modeled what a response should look like to score a 4, I had students practice with partners on a couple problems. They overwhelmingly just tried to mimic exactly what I had written in my example, almost as if they cut and pasted my response and then replaced a few words or points or equations with those featured in the practice problems. It felt very procedural, and their explanations, both written and verbal, revealed they didn't understand the concept that graphs represent all possible solutions to equations. Understanding that concept is required before one should discuss the concept of a solution to a system, so I have decided to change direction. Instead, I am going to focus on the concept of solutions to equations, both algebraically and graphically. I want students to connect their understanding of substituting values into equations and getting true statements to how those solutions are represented graphically. Students should be able to convince me they know that all the solutions to a given equation should be represented as ordered pairs on a graph, and any ordered pair on a graph should be a solution to the given equation.

Thus, the challenge of choosing a concept to scaffold was so great that it actually caused the researcher to abandon the first attempt of the second iteration of instructional scaffolding. Instead, the researcher chose to reevaluate what concept should be taught through instructional scaffolding, and he then restarted the second iteration of the process.

<u>Process Challenge 2: Setting Expectations for Epistemically Complex Explanations</u>. The challenge of setting expectations for epistemically complex explanations of mathematics was discussed in two free-response journal entries. This challenge was first mentioned before the first

iteration of instructional scaffolding. After the researcher wrote about the difficulty of choosing what concept to scaffold, he made the following remarks:

> What language do I want [students] to be able to use to prove to me they have a conceptual understanding? Once I figure out what language I want them to use, I have to think through how I am going to model that use of language. I also have to create examples and prompts I believe will be specific and clear enough to evoke the type of language I want my students to use in their writing.

It was challenging to decide what language students should have been expected to use in their written responses for their explanations to be considered epistemically complex. It also was a challenge to clearly communicate those expectations to students. It was challenging to develop prompts and model responses that would help students know exactly what was expected of their explanations. It also was challenging to create prompts for student practice to help them develop the ability to write epistemically complex explanations.

The researcher even remarked that the challenge of setting expectations seemed to be one cause of the next challenge encountered (responding to student difficulties). This was observed in a free-response journal entry written after the *collaborative practice with debriefing* step of the first iteration:

> I am finding that students are having a much easier time understanding and explaining the concept of a function when it is put into [real-world] context – such as a rule's having an input of an item in a store and an output of the price of that item – than if the rule is left more abstract and mathematical [featuring an equation or a graph]. I suppose this was to be expected, but it presented more of a challenge than I anticipated. When [students] have been responding to prompts giving an equation relating x and y as the rule's definition or the graph of the rule, they have had much more difficulty recognizing whether or not the rule is a function. I felt that during class, too, as I had conversation after conversation with students about connecting the idea of inputs and outputs to the equations and graphs. None of the examples I modeled used an equation or a graph, so that is most likely why they struggled.

This reflection highlights the challenge of setting expectations for epistemically complex explanations. Responses to prompts of one nature – such as a real-world context of items in a store – were modeled. Responses to prompts of another nature – such as a mathematical context of inputs and outputs of equations or graphs – were not modeled. Then, responses to prompts of both of these natures were given to students, and students were unsure of what was expected of them when responding to the latter prompts.

Process Challenge 3: Responding to Student Difficulties. Throughout all three iterations, students had difficulty with writing epistemically complex responses during the *collaborative practice with debriefing* step. In one debriefing session between the researcher and a pair of students during the *collaborative practice with debriefing* step of the first iteration, the pair of students received feedback on their response to the prompt: "Rule *I* takes a price, in dollars, as its input and gives an item from the store that costs that price as its output. Is *I* a function? Why or why not? Explain your thinking." For their response, the students had written "Rule *I* isn't a function because it can have many outputs. For example, if you get a bag of chocolate donuts and you buy a bag of powder donuts, they're [going to] cost the same." After the researcher read that response, the following conversation took place:

Teacher: "For an input, how many outputs should you get if [the rule] is a function?"

Student A: "One."

Teacher: "Just one. So, actually, I think we need to look at your number three … 'It's not a function because it can have many outputs. For example, if you get a bag of chocolate donuts and you buy a bag of powdered donuts, they're [going to] cost the same.' So, in that, I'm not clear what you're saying is the input or the output.

Student A: "Should've [given] the price."

Teacher: "So, are you saying that … the different types of donuts are the input, or are you saying the different types of donuts are the output? I'm not clear on how you worded that."

Student B: "Well, because, it says a price in dollars, into a product. So, a price is your input, a product is your output, and if you input like a certain price, you can get multiple outputs. For example, those two bags of donuts, [because] they'd be the same price."

Student A: **Adds the function notation "*I(2.00) = CD*" and "*I(2.00) = PD*" to the written response.**

Student A: "Like that?"

Teacher: "That's better. Now, I see what you're saying. You input a price of $2.00 and could get the chocolate donuts, could get the powdered donuts. That right there cleared it up for me big time. … This [example] ties into the [written description] much better. But, the way the words were [written], it made me think that you were [saying] *I(powdered donuts) = 3* and *I(chocolate donuts) = 3*, which would be fine."

Student A: "But it's not addressing the question."

Teacher: "That would be taking an item as the input and the price as the output, which we talked about on another example. That would be okay, right?"

Student B: "Yeah."

Student A: "But … the input is the cost."

Teacher: "Right. If the input is the price and the output is a possible item with that price, that's when it's bad because of this type of thinking right here. If you input a 2, you can get more than one item."

After reading the pair's initial explanation, which was incorrect, the teacher spent 15 seconds thinking about how to respond. He then responded by asking the pair about the definition of a function. As the conversation continued, the teacher displayed hesitancy in directly telling the pair what needed to be improved. Instead, he attempted to make statements or ask questions to guide the students in determining what needed to be more clear in their explanation. The challenge was that the two students were not understanding at the same pace. Student A realized almost immediately what needed to be changed about the written explanation, but Student B displayed less certainty. Student B's verbal explanation appeared to contain conceptual understanding, but his written response lacked epistemic complexity. It was a challenge to decide what kind of questions to ask or hints to offer to provide feedback that would help Student B understand what needed to be changed in the written explanation, especially since Student A was eager to move on from the discussion.

Whether the teacher was providing feedback in small-group discussions or whole-class discussion, the challenge of deciding what types of questions to ask or hints to offer remained present. This was evident when comparing video recordings of the two separate Algebra 2 classes during the *collaborative practice with debriefing* step in the first iteration. Students in both classes had difficulty when responding to the following prompts:

- Prompt 1 – Rule y takes a number, x, as its input and gives $y = 4x + 2$ as its output. Is y a function? Why or why not? Explain your thinking.
- Prompt 2 – Rule f takes a number, x, as its input and gives $f(x) = x^2$ as its output. Is f a function? Why or why not? Explain your thinking.

When students in each class asked the teacher for assistance with these prompts, their difficulties were primarily related to the notation. They were not connecting that they needed to input values for x and then simplify to determine what the outputs would be for y (in Prompt 1) or f (in Prompt 2). One conversation between the teacher and a pair of students from the first class is detailed below:

Student A: "So, this is right, because … Rule *y* is a function."

Teacher: "Why?"

Student A: "I [want to] make sure. So, as it says, like, 'takes a number, *x*.' … Would it be like *x* equals something?"

Teacher: "Yeah, you could plug in something for *x*, input a value for *x*. And that means it goes in there for *x*."

Student A: "Okay. Well then, yeah, then it is a function because –"

Teacher: "So, what are these two doing to each other when you input that 2?"

Teacher: **Points to the factors of *4* and *x* that are being multiplied.**

Student A: "[They] complement each other …"

Student B: "I was thinking they like … multiplied. I don't know."

Teacher: "Why would you think that they're multiplying?"

Student B: "Because there's … no space in between it, and so that means that it's multiplying."

Teacher: "Yeah, that's the notation that we use for multiplication. … Once we plug in that 2, you have 4 times 2 plus 2. So, simplify that. What is it [going to] end up being? What's 4 times 2?"

Students: "8."

Teacher: "And then you [must] do what next?"

Students: "Plus 2."

Teacher: "So, you [have] 8 plus 2. What do you get?"

Students: "10."

Teacher: "So, you input a 2. What was the output?"

Students: "10."

Teacher: "A 10. Would it ever be anything besides 10?"

Students: "No."

Teacher: "So?"

Students: "It is a function."

The teacher had similar conversations with other pairs of students in the first class, which could be why he did not give as much wait time after he asked these students some questions. He interrupted Student A when she started to explain why she thought the rule in the prompt was a function, and he began asking questions based on what was said in conversations with other pairs of students. The teacher realized he should have waited to let Student A finish her thought, so he focused on being more patient in the rest of the discussion. He gave more wait time after Student A said the factors of *4* and *x* "complemented" each other, and this wait time provided Student B the opportunity to enter the conversation and explain that he thought those factors should be multiplied. From there, both students quickly caught onto why the rule represented a function, and the rest of the conversation went quickly. Throughout this conversation, the teacher again was challenged to decide when to be patient and ask purposeful questions or to give more direct instruction.

When the second class reached the *collaborative practice with debriefing* step of the first iteration, pairs of students began asking the same types of questions about the same two prompts. Rather than having the same conversation repeatedly with different pairs of students like he did in the first class, the teacher provided feedback to the entire class at once. Responses from two volunteer pairs were displayed on the projection screen, and the following whole-class discussion took place:

Teacher: "Here is what some of your classmates said. They said, 'rule *y* is a function because no matter the [number] for *x*, it will be … [multiply] by 4. So, if you input a [number], it will only come out with one answer. For example, if you input 1, you will get 6 … $y(1) = 4(1) + 2 = 6$ … $y(2) = 4(2) + 2 = 10$.'

Okay, and then this other response … They said, 'Rule *y* is a function. If I input a number, I will only get one output. For example, if I were to input a 3, I would get a 14 … $y = 4(3) + 2 = 12 + 2 = 14$.'

I think both of these are pretty similar … There were a couple things that I saw walking around that I want to make sure I address with you. This [prompt] seems to be giving you guys some trouble recognizing that *x*s are the inputs and *y*s are the outputs. I saw a lot of kind of funky notation being used. That's why I'm wanting to talk about this right now.

You see how, on this [response], they said, 'if you input a 3, you get a 14.' And then … they input that 3 where the *x* was, right? Why did they put that 3 where the *x* was? What about the problem tells them to do that?"

Student A: "Rule *y* takes a number –"

Teacher: "Takes a number, *x*, as its input. So, *x*s are inputs here. … If you input the 3, you're inputting it to this expression right here – 4 times 3 plus 2. … That's only ever [going to] give you a 14. That is why, on this problem, we can say it is a function. … Did they have to pick a 3?

Students: "No."

Teacher: "What else could they have picked for an input, guys?"

Students: **Call out several possible inputs.**

Teacher: "They could have picked any input in the world, because who chooses what the inputs are?"

Students: "Us."

Teacher: "You do. You choose the inputs, and then you go figure out what the output is based on that input. That's what I saw some confusion on when I was walking around. You are choosing what you want *x* to be. That's how inputs work."

Just like the small-group discussions about student difficulties that took place in the first class, the teacher visibly struggled to: a) think of how to word explanations; b) decide what questions to ask; and c) be patient while waiting for students to respond.

Students also had difficulty with accurately coding responses during the *score sample responses* step in each iteration, and it again was challenging to decide how to respond to those difficulties. One instance of this happened in the first iteration during a whole-class discussion on coding two written responses to the same prompt. In the prompt, the graph of a circle was given, and it read: "Rule *y* has been represented by the graph shown at the right. Is *y* a function? Why or why not? Explain your thinking." The class discussed the responses written by Han (Figure 13) and Rey (Figure 14). All students appropriately coded Han's response as *Level 4 – Nailed It*, but many students incorrectly coded Rey's response as Level 4 when it should have been coded as *Level 3 – Almost There*.

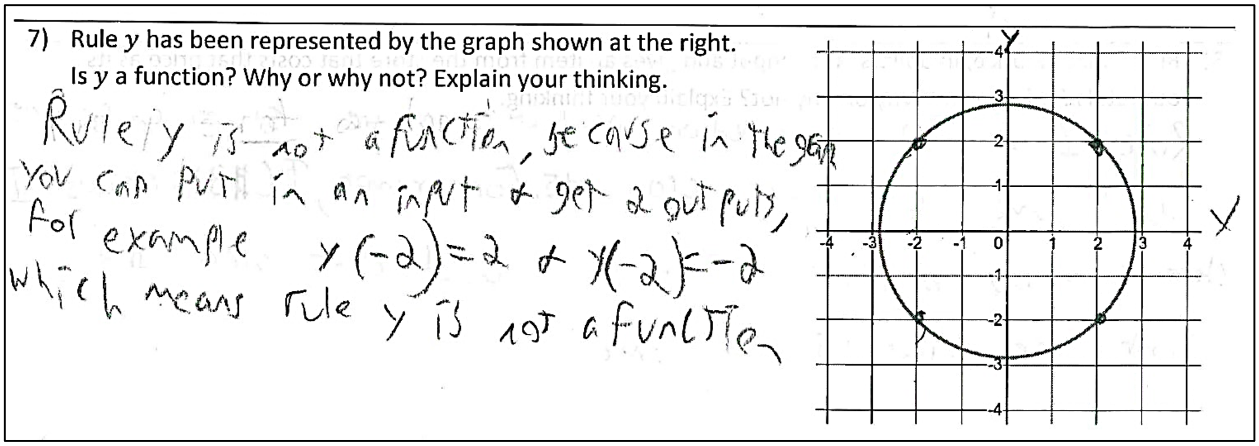


Figure 13. Han’s Level 4 written response from collaborative practice in first iteration.

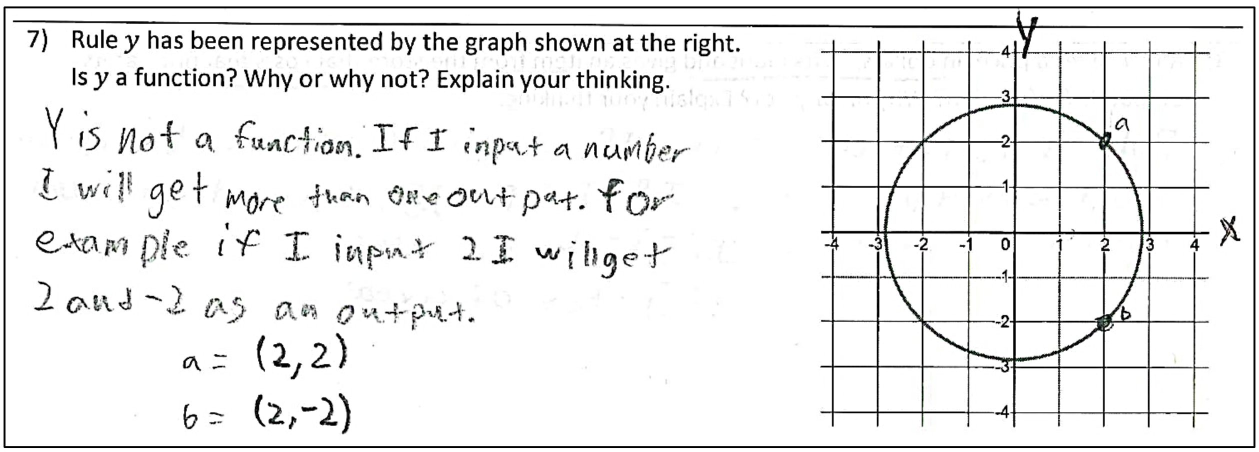


Figure 14. Rey’s Level 3 written response from collaborative practice in first iteration.

The difference that elevated Han’s response to Level 4 but kept Rey’s response as Level 3 was that Rey made an incorrect absolute statement. Rey said “If I input a number I will get more than one output,” whereas Han used the wording “you can put in an input [and] get 2 outputs.” Students had difficulty discriminating the difference between these two statements, and it was a challenge to help them realize why the responses should not have been coded the same. This was evidenced in the following discussion:

Teacher: “Why is [Rey’s] not a 4 when [Han’s] was a 4? It’s subtle, and I almost missed it.”

Student A: “Did you not do it because it didn’t say rule?”

Teacher: "No, that's okay."

Student A: "Darn, that's why I didn't."

Student B: **Asks unintelligible question about how Rey used *a* and *b* in her response.**

Teacher: "I actually don't have a problem with how [Rey] said *a* and *b* because … they called this point *a* and this point *b*. So, they were just labeling. … I actually liked that. … I know exactly what they were referencing on the graph. These [responses] are very similar; aren't they?"

Students: **Ask unintelligible questions about Rey's listed input-output pairs.**

Teacher: "No, [if you] input 2, you [get] a 2; [if you] input 2, you also [get] a -2."

Student C: "Is it because they say 'more than one output' instead of 'two outputs'?"

Teacher: "Nope –"

Student A: "Is it about the example?"

Teacher: "Nope, the example's fine."

Student D: "Does it have something to do with the way they said 'input' and 'output'?"

Teacher: "Are you talking about [Rey's] second sentence?"

Student D: "Yeah."

Teacher: "Yes! Listen to how [Rey] said it: '*If* I put in a number, I *will* get more than one output.' [Han] said: 'You *can* put in *an* input and get two outputs.'"

Student A: "So, [Han] worded it better."

Teacher: "Let's think about this. [Rey's] says, '*If* I put in a number, I *will* get more than one output.'"

Student E: "Oh, that's false!"

Teacher: "Why is it false?"

Students: **Unintelligible chatter that did not correctly explain why Rey's statement was false.**

Teacher: “So, this is very subtle. [Rey’s second sentence] is almost true, but it is not true at two spots on that graph.”

Student E: “Oh! I get it!”

Student F: “At the 3 and -3!”

Teacher: “[At this *x*-intercept], how many outputs are there?”

Students: “One.”

Teacher: “[At the other *x*-intercept], how many outputs are on the graph?”

Students: “One.”

Teacher: “So, is the statement true? … If I put in any number, [am I] guaranteed to get more than one output?”

Students: “No.”

Teacher: “Everywhere in between, that’s a true sentence.”

Student E: “I get that.”

Student F: “So, it can be a function and cannot be a function at the same time?”

Teacher: “No, no, no. [The graph] is still not a function because … I can find an example of an input with more than one output. This is just not a true statement that if I input anything, I’m going to get [multiple outputs]. So, [Rey’s] description was pretty dang good. It was just that [Rey] made a false statement that made this a [Level 3] for me. … I want everything you say to explain it, but I also want it to be true. I don’t want you to explain something that’s actually false.”

This discussion lasted almost four minutes, and it was challenging to refrain from directly explaining the issue with Rey’s statement for as long as the teacher did. Students asked many questions throughout the discussion, but they were not focused on the correct detail of what prevented Rey’s response from being coded as Level 4. Even after Student D asked if the error was related to Rey’s second sentence – “*If* I put in a number, I *will* get more than one output” – Student D and the rest of the class needed further guidance in recognizing how that statement was different compared to Han's better statement – "You *can* put in *an* input and get two outputs." Then, just when it seemed that the class was ready to move on, Student F revealed he did not fully understand that for a rule to not be considered a function, there only needs to be one example of an input with more than one possible output. The teacher had to determine how to tie together everything that had been discussed and explain that while both Han and Rey were correct in their conclusions, Han's response demonstrated greater epistemic complexity.

Challenges Related to Coding Responses. Two primary challenges related to applying the modified coding framework for epistemic complexity (Appendix B) were encountered during this study: 1) assessing only students' writing; and 2) verifying examples supported statements.

Coding Challenge 1: Assessing Only Students' Writing. The goal of coding students' written responses was to assess their abilities to write epistemically complex explanations. As such, assumptions could not be made about what students meant to say; only what was actually written in their explanations could be assessed. This was challenging because it was sometimes easy to infer what a student was *trying* to explain, but the focus had to remain on what they actually wrote. For example, in a response about whether a given graph of a circle (Appendix F) represented a function, a student made the statement that "there can be 1 output but there's also multiple outputs depending on the spot of the graph." It could have been inferred that they were trying to explain that some inputs had only one output but many other inputs had multiple outputs. However, when they explained their thinking in writing, they were unable to write a response that demonstrated full understanding of the concept of a function.

When another student was explaining which of two graphs (Appendix G) represented the solutions to a given equation, they wrote "The graph on the *right* [emphasis added] shows all possible solutions to the equation" and provided three examples to support this statement. Later in that same response, they wrote "The graph on the *right* [emphasis added] is incorrect because it does not have all possible solutions." It could have been inferred this was a simple mistake and that the student meant to write the graph on the *left* was incorrect in their second statement. In fact, had they named an accurate ordered pair from the left graph and demonstrated that it did not produce a true statement in the given equation, it would have been plausible to overlook the misuse of the word *right* when they meant *left*. However, this was not the case. Instead, they listed the point *(1, 2)* to show that it did not work in the equation. The issue, though, was that *(1, 2)* was not included in either graph. As such, it was unclear what statement they were trying to support with that example. This lack of precision caused the response to be coded as *Level 2 – Not Quite*, even though it was tempting to assume the student's conceptual understanding may have been closer to that of a *Level 3 – Almost There*.

There also were several students who made incorrect absolute statements in their responses, and it was challenging not to overlook this lack of precision and infer what they actually meant. For example, one student made the following statement about whether the graph of a circle (Appendix F) represented a function: "there can be multiple outputs for *each* [emphasis added] input." However, the graph in question had two inputs that only had one possible output. Did this student understand that for a rule not to be considered a function, there only needs to be *at least* one input that has multiple outputs? Or, did they think that every input needed to have multiple outputs for a rule not to be considered a function? Without making an assumption about their understanding, those questions could not be answered based on the written explanation.

Similarly, a student made the following incorrect absolute statement when explaining why one of two graphs (Appendix G) did not represent all the solutions to a given equation: "The graph on the left is incorrect because when plugged into the equation, the ordered pairs on that graph do not make a true statement." This statement was untrue because the graph on the left actually had one ordered pair, *(0, -1)*, that would have produced a true statement when substituted into the given equation, $y = 4x - 1$. Based on what the student wrote, it was unclear if they understood that a graph could have *some* ordered pairs that were solutions to a given equation but still not represent *all* the solutions to that equation. To demonstrate high epistemic complexity, students needed to explain that they only needed to show *one* example of an ordered pair that did not work in the given equation to conclude that a graph did not represent all the solutions to the equation. Failure to notice incorrect absolute statements like these could have resulted in coding a response at a higher level than it deserved. It was a challenge to remain objective and only take what students wrote literally while measuring the epistemic complexity of their responses.

Coding Challenge 2: Verifying Examples Supported Statements. The other challenge that presented itself while coding responses was making sure examples that were provided supported the statements being made. There were many students who made accurate conclusion statements, but then they provided examples that did not support those statements. One student whose response was coded as *Level 3 – Almost There* wrote that "certain inputs can give more than 1 output." That was a correct statement for the given prompt (Appendix F), and the example they cited to support this statement was that $f(-1) = 1$ and $f(-1) = -1$. However, this was not supported graphically, as the input-output pairs for an input of -1 would have actually been the ordered pairs *(-1, 0.8)* and *(-1, -0.8)*. Another student whose response was coded as *Level 3 – Almost There* wrote that "if you input all but two of the points you will get more than one output." Again, this sounded like it was on track to be coded as *Level 4 – Nailed It*, but the example they provided was *F(-1) = (-1, -.8), (1, -.8)*. These examples were insufficient because the ordered pairs they provided showed the same *output*, *-.8*, for two different *inputs*, *-1* and *1*, which is perfectly acceptable in relation to the definition of a function.

On the second iteration's final assessment (Appendix G), one student wrote that "The graph on the left is incorrect because any ordered pair will not work in the equation. For example, the point *(3, 1)*." They then illustrated that substituting *(3, 1)* into the given equation would produce a false statement. The issue was that *(3, 1)* was not a point on the graph they referenced; the point was actually *(3, 2)*. It was a challenge to pay close attention to little details like the use of *(3, 1)* instead of *(3, 2)* in a response that was otherwise correct. Part of writing an epistemically complex response that is aligned with the intentions of the Standards for Mathematical Practice is attending to precision. What these students wrote lacked precision. As a result, their examples did not support their explanations. Ultimately, this withheld their responses from being coded as *Level 4 – Nailed It*. While coding responses, it was challenging to be careful to verify that provided examples supported statements that were made. If the examples did not truly support the statements, the response could not be scored as highly on the coding framework for epistemic complexity.

Summary

This study examined the implementation of instructional scaffolding in the mathematics classroom to assist students in learning to write epistemically complex explanations of mathematics. Data analysis focused on addressing two research questions:

1. Does engaging secondary mathematics students in an instructional scaffolding teaching strategy support their abilities to write explanations with high degrees of epistemic complexity?

2. What challenges were encountered while implementing instructional scaffolding to support students' abilities to write explanations with high degrees of epistemic complexity?

Research question 1 was addressed through the analysis of 94 written responses collected during the study. A modified coding framework (Appendix B) was used to measure the epistemic complexity of these written responses. The distribution of how responses were coded throughout the three iterations of implementation is given in Figure 15. A total of 77 (81.91%) of the 94 responses were coded at Level 3 or Level 4, indicating a greater degree of epistemic complexity was featured in these responses. In each iteration of implementing instructional scaffolding, at least 75% of the responses collected were coded as *Level 3 – Almost There* or *Level 4 – Nailed It*.

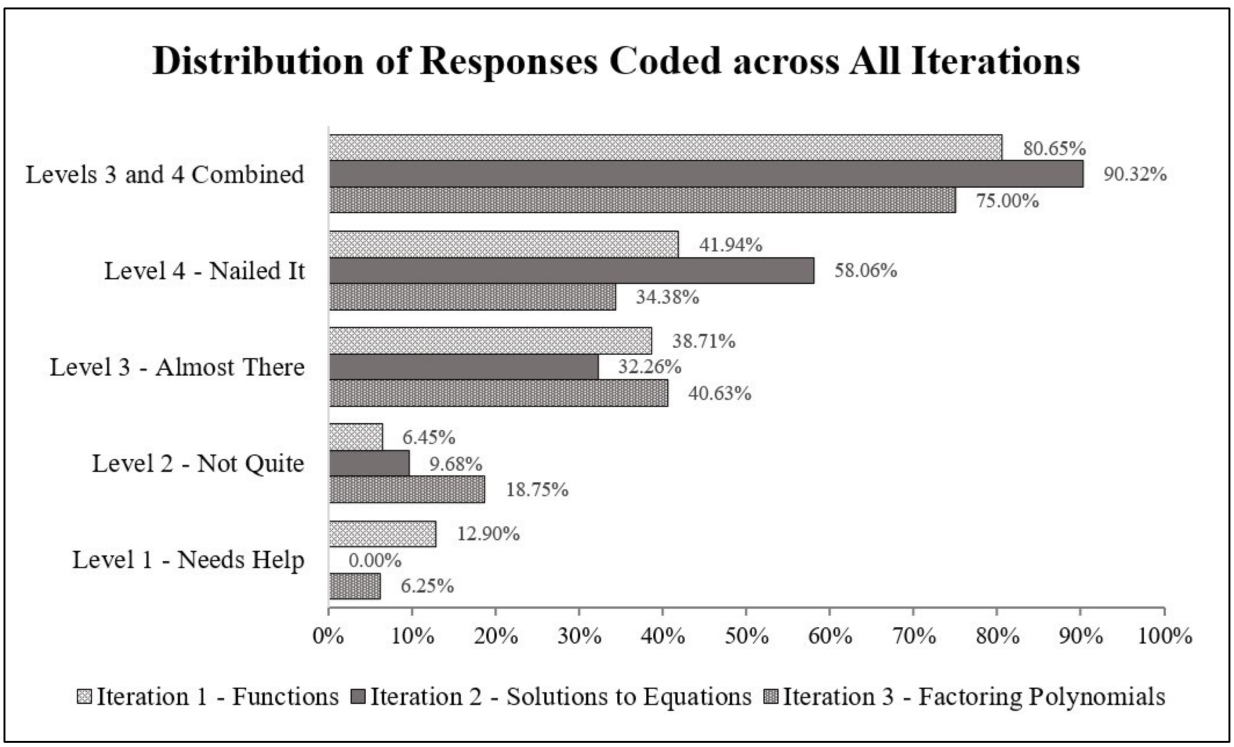


Figure 15. Distribution of responses coded across all iterations of implementation.

Research question 2 was addressed through the analysis of the following sources of data: a) 94 written responses; b) 7 free-response journal entries made by the researcher, and c) 18 video recordings of classes during which instructional scaffolding was implemented. These data were analyzed to look for common challenges that were encountered throughout the three iterations of implementation. The analysis of the written responses, free-response journal entries, and video recordings led to the identification of two themes regarding challenges that were encountered during the implementation of instructional scaffolding. The first theme included challenges related to the instructional scaffolding process, such as: 1) choosing concepts to scaffold; 2) setting expectations for epistemically complex explanations; and 3) responding to student difficulties. The second theme included challenges related to coding responses, such as: 1) assessing only students' writing; and 2) verifying examples supported statements. The analysis of these data sources provided evidence that although there are challenges associated with implementing instructional scaffolding in the secondary mathematics classroom, it is a viable and effective teaching strategy that helps students develop the ability to write epistemically complex explanations of mathematics.

CHAPTER V: DISCUSSION AND CONCLUSION

This study examined the implementation of instructional scaffolding in the secondary mathematics classroom to assist students in learning to write epistemically complex explanations of mathematics. Students' responses to prompts asking them to explain their thinking were analyzed qualitatively using a coding framework to measure epistemic complexity (Appendix B). In addition, challenges experienced during the implementation of instructional scaffolding were identified. These challenges fell into two themes: a) challenges related to the instructional scaffolding process; and b) challenges related to coding responses. Adjustments were made to the initial design of instructional scaffolding that was recommended by Nachowitz (personal communication, 2020) based on the challenges that were encountered. The final design of instructional scaffolding provides a lens through which other teachers can support their students in learning to write epistemically complex explanations of mathematics.

Summary of Findings

The methodology and data analysis employed in this study focused on addressing two research questions:

1. Does engaging secondary mathematics students in an instructional scaffolding teaching strategy support their abilities to write explanations with high degrees of epistemic complexity?
2. What challenges were encountered while implementing instructional scaffolding to support students' abilities to write explanations with high degrees of epistemic complexity?

The findings related to research question 1 established that instructional scaffolding is a viable and effective teaching strategy that helps students learn to write epistemically complex explanations of mathematics. The distribution of codes applied to the 94 written responses collected during the study was as follows:

- 6 responses (6.38%) were coded as *Level 1 – Needs Help*.
- 11 responses (11.70%) were coded as *Level 2 – Not Quite*.
- 35 responses (37.23%) were coded as *Level 3 – Almost There*.
- 42 responses (44.68%) were coded as *Level 4 – Nailed It*.

A total of 77 (81.91%) responses were coded at Level 3 or Level 4, indicating a greater degree of epistemic complexity was featured in these responses. The findings related to research question 2 identified two themes regarding challenges that were encountered during the implementation of instructional scaffolding. The first theme consisted of challenges related to the instructional scaffolding process, including: 1) choosing concepts to scaffold; 2) setting expectations for epistemically complex explanations; and 3) responding to student difficulties. The second theme consisted of challenges related to coding responses, including: 1) assessing only students' writing; and 2) verifying examples supported statements.

Discussion

Numerous studies have established several benefits of including writing in the mathematics classroom (Baxter et al., 2005; Bicer et al., 2018; Casler-Failing, 2013; Evans, 2017; Kostos & Shin, 2010; Martin & Polly, 2013, 2016; Martin et al., 2017; McIntosh, 1991; Miller & England, 1989; Miller, 1991, 1992a, 1992b, 1993; Nachowitz, 2018). While some of these studies have stated teachers can use written responses to assess student understanding and adjust instruction (Baxter et al., 2005; Kostos & Shin, 2010; Martin & Polly, 2013; McIntosh, 1991; Miller & England, 1989; Miller, 1991, 1992b), they have not described what those adjustments looked like or provided evidence that the adjustments resulted in the enhancement of students' abilities to demonstrate conceptual understanding of mathematics in writing. Nachowitz (2018) observed that students' written explanations did not demonstrate conceptual understandings of mathematics, and research has been recommended on strategies teachers can implement to teach students how to write about mathematics conceptually (Baxter et al., 2005; Martin & Polly, 2013, 2016; Martin et al., 2017; Nachowitz, 2018). Nachowitz specifically recommended research investigate the viability of instructional scaffolding as an effective teaching practice to help students learn to write epistemically complex explanations of mathematics.

This study provided evidence that instructional scaffolding is a viable teaching strategy in the secondary mathematics classroom. In Nachowitz's (2018) study – which was purely observational and featured no classroom interventions – only 8% of the analyzed responses were coded as *Level 4 – Partial Explanation* using the coding framework for epistemic complexity employed in the study (Appendix A). No responses were coded as *Level 5 – Well-organized Explanation*. In this study, 37.23% of the analyzed responses were coded as *Level 3 – Almost There* using the modified coding framework for epistemic complexity (Appendix B), and 44.68% of the responses were coded as *Level 4 – Nailed It*. The *Level 4 – Partial Explanation* code featured in the framework used by Nachowitz was comparable to the *Level 3 – Almost There* code featured in the modified framework used in this study. Similarly, the codes of *Level 5 – Well-organized Explanation* and *Level 4 – Nailed It* were comparable. The percentages of how these codes were applied in the two studies are compared in Figure 16. Based on the findings of this study, it can be concluded that the implementation of instructional scaffolding helped students learn to write mathematical explanations of high epistemic complexity.

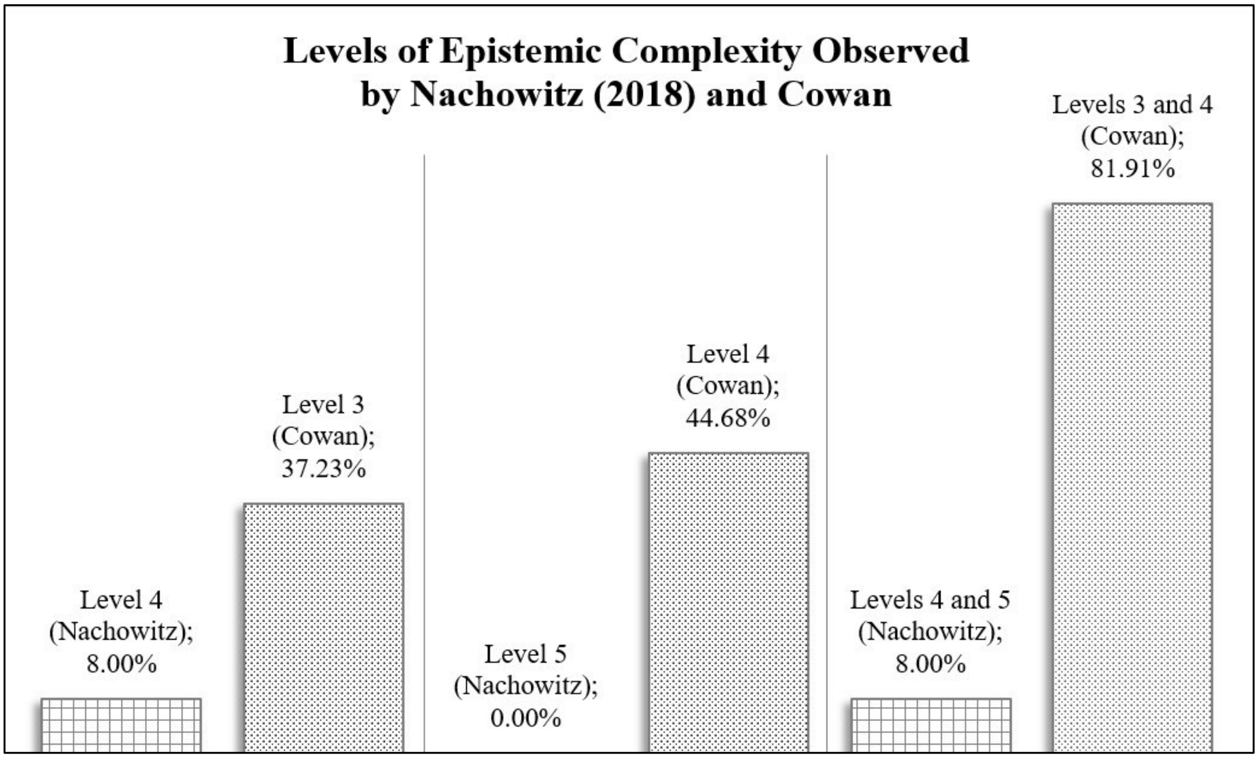


Figure 16. Levels of epistemic complexity observed by Nachowitz (2018) and Cowan.

By analyzing samples of student writing featured in the studies conducted by Baxter et al. (2005), Martin and Polly (2013, 2016), Martin et al. (2017), and Nachowitz (2018), it could be suggested that students simply were not taught how to write about mathematics in a way that demonstrated their conceptual understandings, or perhaps they were just not being held accountable for creating such written responses. Going into this study, it was never assumed that students would be incapable of writing epistemically complex explanations. Rather, the primary assumption was that they had never been taught how to do so. If that assumption was incorrect, then it at least was unknown what actions teachers were taking in an attempt to teach their students how to write conceptually. The findings of this research provided evidence of a specific teaching strategy that can be used to help students learn to communicate their conceptual understandings of mathematics in writing. Secondary mathematics teachers can employ instructional scaffolding in their classrooms and be confident it will help their students learn to write epistemically complex explanations of mathematics.

It has been noted that the design of instructional scaffolding that was implemented in this study was adjusted from the design Nachowitz recommended (personal communication, 2020). Three main adjustments were made to the design of the instructional scaffolding process:

1. *Debriefing* through small-group discussions during *collaborative practice*.
2. Replacing *independent practice* with the *score sample responses* step.
3. Providing students with a rubric during the *scores sample responses* step.

Nachowitz recommended that *debriefing* take place through whole-class discussion after students participate in *collaborative practice*. In the first iteration of implementation in this study, *debriefing* was carried out in this way. However, *debriefing* with the entire class seemed less effective for helping students than having focused, small-group discussions with pairs while they worked together. One issue was the flow of the class was disrupted because pairs were progressing at different paces. Some pairs chose to wait until the rest of the class was finished to make progress, while others chose to practice writing responses to the subsequent prompts on the assignment. When the class did come together to have a whole-class discussion about a particular volunteer's written response, the majority of students did not seem engaged in the discussion, especially those who had already progressed past the prompt being discussed. Students were inattentive, and they seemed less willing to offer their feedback for a response in front of the entire class versus in a more private, small-group discussion.

With these perceived drawbacks in mind, the method of *debriefing* was adjusted when students returned the next day to work on finishing their *collaborative practice*. Pairs were told to work at their own pace, and *debriefing* took place in small-group discussions with the pairs intermittently as they were working. Students seemed to respond more productively to this targeted feedback. The pairs appeared more engaged when receiving direct feedback on their own written responses than when there was whole-class discussion on someone else's response. It was easier to clearly point out specific details that were correct or needed improvement, and it was easier to answer students' clarification questions. Giving pairs the opportunity to work on the assignment at their own pace also seemed to keep them more focused and on task; there was no struggle to regain their attention like there was when trying to bring everyone together for a whole-class discussion. The second day of *collaborative practice* felt like it went more smoothly as a result of this adjustment. As such, it was decided to keep this adjustment and focus more on *debriefing* in small-group discussions during *collaborative practice* in subsequent iterations.

The second adjustment was the replacement of the *independent practice* step recommended by Nachowitz (personal communication, 2020) with the *score sample responses* step featured in the implemented design. After students finished *collaborative practice* in the first iteration, some certainly needed more individual practice at learning how to write an epistemically complex response. However, it did not seem like *independent practice* would have been the most effective way to help those students realize what improvements needed to be made, especially because giving individualized, targeted feedback was time-consuming. Rather, it seemed students would benefit from comparing examples of responses that would have been coded at different levels. They needed to see the difference between a *Level 3 – Almost There* response and a *Level 4 – Nailed It* response to help them make that leap in their own writing. The belief was that replacing *independent practice* with the *score sample responses* step would help students discriminate between different details that resulted in different codes for responses.

Students needed to determine what a given response was lacking or did well so they could learn what they needed to avoid or include in their own responses.

The third adjustment – providing students with a rubric (Appendix I) during the *score sample responses* step – was not made until after the first iteration was complete. When students worked on the *score sample responses* step of the first iteration, they were instructed to code responses as *Level 1 – Needs Help*, *Level 2 – Not Quite*, *Level 3 – Almost There*, or *Level 4 – Nailed It*. No elaboration was provided on the details of what constituted each level. This seemed to result in students' awarding scores inaccurately compared to how the teacher would have coded the same responses. When pressed to explain why they coded different responses at certain levels, students gave superficial reasons that were not focused on the epistemic complexity of the responses. In fact, many pairs placed more importance on the length of a response or its inclusion of key words instead of the accuracy and precision of mathematical vocabulary that was featured.

For example, how students coded responses to one prompt illustrates their inaccuracies. The prompt was: "Rule *I* takes a price, in dollars, as its input and gives an item from the store that costs that price as its output. Is *I* a function? Why or why not? Explain your thinking." The responses students coded inaccurately were:

- Response 1: "Rule *I* is not a function because there are multiple outputs possible for *I*. *I(2.16) = milk, yogurt*. For example, a gallon of milk or a pack of yogurt would have the same price."
- Response 2: "It wouldn't be a function b/c two items in a store could have the same price. For example, a bag of chips could be $3 and a box of crackers could also be $3. *I(chips) = $3 I(crackers) = $3*"

Many pairs of students coded Response 1 as *Level 1 – Needs Help* or *Level 2 – Not Quite*, and many coded Response 2 as *Level 3 – Almost There* or even *Level 4 – Nailed It*. The main reason students cited for coding Response 1 lower was its short length. Students who coded Response 2 higher cited how the explanation involved the word *function* and included the use of function notation in the provided example. The inconsistency of this reasoning was surprising, considering Response 1 also mentioned *function* and used function notation in the response, and Response 2 was only a few words longer.

The expectation during the *score sample responses* step was for students to check if statements made were correct and supported by valid examples. In this case, a correct response should have made a statement that rule *I* was not a function because it had at least one input with more than one possible output, and they should have given an example to support that statement, such as how Response 1 gave the example *I(2.16) = milk, yogurt*. Had students looked for those features, they should have scored Response 1 as *Level 3 – Almost There* and Response 2 as *Level 1 – Needs Help*. This was the exact opposite trend of how students scored the responses.

Response 1 was only lacking because it said there were "multiple outputs possible for *I*" when it should have specified there were multiple outputs possible for any given *input* of *I*. Many *functions* have multiple possible outputs; there just should not be any single *input* that has more than one possible output for a rule to be considered a *function*. Response 1 attempted to explain that but fell slightly short due to its incomplete statement. Response 2, on the other hand, should have been coded as *Level 1 – Needs Help* because its examples used inputs and outputs that were switched, listing items as inputs and prices as outputs. The response was completely incorrect because it did not properly apply the given inputs and outputs. In fact, if the examples of input-output pairs provided in Response 2 had been accurate, then rule *I* would have been a *function*. Thus, Response 2's examples did not support its conclusions; they actually would have supported the exact opposite conclusion.

If students had been given a rubric to apply when coding these responses, some of these difficulties may have been avoided. The language of the rubric for the descriptions of each level may have given students more direction for what to seek while they were coding. It may have even affected the language they used when defending how they coded a particular response. At the very least, a rubric could have been a tool the teacher could have referenced during the whole-class discussions that took place after students finished coding responses. With these considerations in mind, it was decided to provide students with a rubric to use during the *score sample responses* step of the second and third iterations. In the subsequent iterations, it seemed like having a rubric for reference helped students code responses more accurately. There appeared to be fewer instances of students' codes being significantly different from how the responses were coded by the teacher. Students also seemed to cite specific language from the rubric when defending why they believed a response should earn a particular score instead of only focusing on superficial features like the length of a response or its inclusion of key words. Unfortunately, though, no formal evidence was collected to support these claims.

The adjustments made to the design of instructional scaffolding resulted in observations that support other studies' findings regarding the benefits of including writing in the mathematics classroom. Writing has been shown to help both teachers and students assess students' conceptual understandings (Baxter et al., 2005; Bicer et al., 2018; Casler-Failing, 2013; Evans, 2017; Kostos & Shin, 2010; Martin & Polly, 2013, 2016; Martin et al., 2017; McIntosh, 1991; Miller & England, 1989; Miller, 1991, 1992a, 1992b; Nachowitz, 2018). *Debriefing* with students in small-group discussions and giving feedback to students during the *score sample responses* step revealed the level of students' understandings of concepts. Reading students' explanations provided valuable insight into the level of their conceptual understandings instead of just assessing their procedural skills, which are so often the focus of assessment in typical mathematics classrooms in the United States (Hiebert et al., 2005; NCTM, 2018). Students' written responses provided a window into students' thinking about concepts. The value of that window should not be taken for granted. It provides teachers with tangible evidence for how to proceed in their courses, whether that entails reteaching concepts, refining students' use of written language, or moving forward to the next topic of study.

These adjustments enabled students to learn not only about the nature of their own understandings, but they also were able to learn about the understandings of their peers. Within the *score sample responses* step, students were required to think through definitions and examples to decide whether or not the responses demonstrated conceptual understanding. Students applied a rubric to score these responses, and they were required to justify the scores they awarded during small-group and whole-class discussions. These discussions evoked specific questions about the wording or structure of responses or what details were lacking from a response that was not coded as highly as another. Students debated and convinced each other why they believed a response should earn a particular score. They sought clarity on their own understandings and what they needed to be able to explain in their own writings to achieve a score of *Level 4 – Nailed It.* Thus, the *score sample responses* step helped students assess their own conceptual understandings and those of their classmates.

Writing has also been shown to engage learners in mathematical thinking (Baxter et al., 2005; Evans, 2017; McIntosh, 1991; Miller, 1992a, 1992b). In this study, both the *score sample responses* step and the adjusted way *debriefing* took place in small-group discussions were effective in engaging students in mathematical thinking. During the *score sample responses* step, students checked to see if responses appropriately explained mathematical concepts based on the statements that were made and the examples that were provided. They asked specific questions to each other and to the teacher about how responses should be structured or what proper terminology should be included. Students were also engaged in mathematical thinking while working in their pairs during the *collaborative practice with debriefing* step. At the very least, half of the students were engaged in mathematical thinking during this step as one student in each pair was required to answer a given prompt with a think-aloud protocol. Then, by *debriefing* in small-group discussions and giving feedback directly to the pairs, students were engaged in thinking about the mathematical questions posed to them.

Writing has also been shown to encourage precision and the proper use of mathematics vocabulary in both students and teachers (Kostos & Shin, 2010; Miller, 1992b, 1993). In this study, students' responses had to be precise and use proper vocabulary to be coded as *Level 4 – Nailed It*. Students asked questions or made observations about this precision of writing and proper use of terminology while *debriefing*, both in small-group discussions during *collaborative practice* and in whole-class discussions during the *score sample responses* step. One example of this happened in the third iteration during the *score sample responses* step. In a whole-class discussion, a student justified why she thought a response should have been coded at Level 3 instead of Level 4 by citing an improper use of vocabulary. The response justified that a given factorization was correct because "when you *factor* [emphasis added] $(x - 4)$ and $(x^2 + 4x + 16)$ the product is $x^3 - 64$." The student pointed out the response should have used the word *multiply* instead of *factor*. The student believed this misuse of language should have prevented the response from being coded as *Level 4 – Nailed It*, and she was absolutely correct. Students' success relied on precision and accuracy, and they learned how to be precise and accurate as a result of the feedback they received during *debriefing* and while coding responses.

Recommendations for Future Research

The results of this study provided evidence that instructional scaffolding is a viable and effective practice in the secondary mathematics classroom. Teachers can employ this instructional strategy to develop students' abilities to write epistemically complex explanations of mathematics. Further research should be conducted on the viability of this practice in more mathematics classes at all levels of education. Is the design of instructional scaffolding that was implemented in this study transferable to other mathematics subjects, or would it need to be adjusted further in different courses? While the *modeling with a think-aloud protocol* step was an integral feature of the implemented design, the *collaborative practice with debriefing* and *score sample responses* steps seemed to be more impactful in students' development. Would this be the case in elementary mathematics classrooms or classrooms with younger secondary students? Would students in higher-level courses, such as dual credit or Advanced Placement classes, need to spend as much time on each step as the Algebra 2 student participants needed in this study? These questions are worth further exploration to determine the breadth of viability of instructional scaffolding in mathematics classrooms.

There was also discussion about how providing students with a rubric to reference while scoring sample responses seemed to help them code the responses with greater accuracy. However, no evidence was collected during this study to support or refute that claim. Students should be asked to score sample responses without a rubric and with a rubric, and the codes they apply should be compared to the actual scores of those responses. It also would be valuable to investigate how students would code their own responses compared to how they would code others' responses. Would they tend to be stricter or more lenient when coding their own responses, or would they be consistent regardless of whose responses they were scoring? Perhaps there would even be no observable pattern. Future research should be conducted to investigate these questions.

Another question worth investigating is: "What are the residual effects, if any, of implementing instructional scaffolding to improve the quality of mathematical explanations?" This study investigated if implementing instructional scaffolding would support students' abilities to write epistemically complex explanations of mathematics. In each iteration, the teacher modeled how to respond to certain types of prompts, students practiced responding to similar prompts, and students scored sample responses. There was no point in time when students were asked to respond to prompts without going through all the steps of that process. After employing the instructional scaffolding process to teach some concepts, it would be interesting to examine the quality of students' written responses in subsequent units that were not taught through instructional scaffolding. Would there be residual improvements to their writing because of their past work within the instructional scaffolding process? In other words, would students be able to generalize what they directly learned from instructional scaffolding and then apply it to write epistemically complex explanations about concepts they did not learn through instructional scaffolding? There could be residual benefits of students' engaging in instructional scaffolding, and perhaps they would require less scaffolding to be able to demonstrate conceptual understanding in writing as they continued to mature as writers of mathematics. Future research is necessary to determine if that conjecture would prove to be true.

Recommendations for Future Practice

The development of students' procedural skills has typically been the focus of mathematics education in the United States (Hiebert et al., 2005; NCTM, 2018) even though the need to balance students' procedural skills and conceptual understandings has been recommended by many experts in mathematics education (CCSSI, 2020a; Hiebert et al., 2005). This study provides secondary mathematics teachers with details of an instructional scaffolding teaching strategy that can be used to shift focus back to balancing students' procedural skills and conceptual understandings. That balance cannot be emphasized enough. Students certainly need to know how to employ mathematical procedures, but this should not come at the expense of their understandings of the concepts underpinning those procedures. Mathematics teachers should incorporate instructional scaffolding into their classrooms to shift instruction toward focusing on teaching and assessing concepts behind the mathematics students learn. This task may seem daunting to some, especially considering all of the mandated state and national standards and objectives that already place a heavy burden on teachers. Teachers will encounter challenges when implementing instructional scaffolding in their classrooms, but these challenges can be overcome.

The first process challenge encountered in this study was determining what concepts to scaffold so students could learn how to write epistemically complex explanations about those concepts. This challenge was resolved by choosing concepts that were the foundation of many topics that would be covered in the course. Teachers can strategically choose concepts to teach through the instructional scaffolding process to maximize its effect on students' learning. The concepts of *functions*, *solutions to equations*, and *factoring polynomials* were taught through instructional scaffolding in this study because those concepts would be featured in the Algebra 2 course repeatedly. Even though it took multiple days to get through the instructional scaffolding process for each concept, the effort was incredibly worthwhile for the understandings students developed. These understandings made for less frustration in subsequent lessons involving those three concepts and the related vocabulary because students had developed the required prior knowledge at a deeper level than if instructional scaffolding had not been utilized. That benefit was realized in the units of study taught after the implementation of instructional scaffolding, and other teachers likely would enjoy experiencing similar results. As such, teachers should think about what standards, objectives, and concepts are the most important for their students to learn; they should determine what key conceptual understandings they want to help their students develop. When those questions are answered, teachers will know exactly what is worth teaching through instructional scaffolding.

The second process challenge encountered in this study was setting the expectations for epistemically complex explanations. The *modeling with a think-aloud protocol* step of the process was crucial for addressing this challenge. First, prompts needed to be carefully written in a way that would elicit epistemically complex responses. Nachowitz (2018) discussed how most writing prompts analyzed in his study manifested as *show your work* rather than *show your thinking*. Students were not asked *why* they took certain steps or *why* they understood how to answer a given prompt, so explanations typically only involved summaries of the steps taken. With that in mind, the prompts in this study were specific in asking students to *justify* their answer or to *explain their thinking*. To clarify what was expected by the instructions to *explain their thinking*, it was then pivotal to develop a model response by focusing on what details should be included for an explanation to be considered epistemically complex.

Consider, for example, the unit on *functions*. Students first needed to make a statement like "*F(x)* is a function because each input has only one possible output" or "*F(x)* is not a function because at least one input has more than one possible output." Statements like these would demonstrate students understood the definition of the concept in question. Beyond just referencing the definition, students then needed to apply it in the context of a given problem. To achieve this, they needed to provide a clear and accurate description or example of how the concept was being applied. For example, if the graph of a rule included ordered pairs where there was at least one input with multiple outputs, students needed to reference that when justifying why the graph did not represent a function. Model responses were developed by thinking through details like these, and that development helped to resolve the challenge of setting expectations for epistemically complex explanations.

The third process challenge encountered in this study was responding to student difficulties. The adjustments to the design of the instructional scaffolding teaching strategy were made primarily in response to this challenge. By *debriefing* with students in small-group discussions, it was easier to be patient and pose intentional questions for students to consider instead of just directly telling them what needed to be fixed in their responses. At times, students just needed to be allowed to work through their struggles with their partners. While remaining patient or determining the right question to ask was difficult at times, direct assistance was offered to students only when they seemed stuck and unable to make progress toward the learning goals. Because of this restraint, many students were able to help each other better understand concepts and how their written responses should be revised.

It also seemed like students were able to better understand the nature of responses that would be coded at different levels because of their practice scoring responses during the *score sample responses* step. Then, it appeared that students coded responses more accurately in the second and third iterations because they had a rubric to apply. The rubric, at a minimum, was a tool that could be referenced when explaining how certain responses should have been coded or when asking students to consider which code should have been applied to a certain response based on the descriptions of each level featured in the rubric. While responding to student difficulties was certainly a challenge in implementing instructional scaffolding, it is also a challenge that teachers typically face. Teachers are always required to react to their students' struggles; the fact that students will have difficulty in learning to write epistemically complex explanations of mathematics should not discourage teachers from implementing instructional scaffolding in their classes.

Two challenges related to coding students' responses were encountered in this study, and teachers may encounter the same challenges when implementing instructional scaffolding in their classes. When coding responses, it was a challenge to: 1) assess only students' writing; and 2) verify examples supported statements. Having the coding framework for epistemic complexity (Appendix B) helped to resolve both of these challenges. The codes' descriptions were helpful for focusing on what details a response should have featured to receive a particular code. It also was helpful not to rush through coding responses and to be consistent in how codes were awarded. Specific details were noticed of responses that were coded at one level, which was often helpful in realizing when to apply the same code to other responses. For example, some students made incorrect absolute statements like "none of the ordered pairs work" to justify why a graph did not represent all the solutions to a given equation. These statements were incorrect when made about graphs that did have *some* (but not all) ordered pairs that worked for the given equation. A statement like this would have caused the response to be coded, at most, as *Level 3 – Almost There*. It also can be helpful to establish interrater reliability when coding responses. In practice, a teacher may only need to train a coworker – perhaps even one with a common prep – how to code a set of responses and then compare their results with their coworker's. Then, they can discuss the similarities and differences between the codes that were applied. This can help teachers gain confidence that the codes they are applying are legitimate and consistent.

Conclusion

Students enrolled in mathematics courses need to understand more than just how to memorize facts and step-by-step procedures. In addition, students need to develop understandings of the concepts that underlie and motivate the mathematics they learn. Just as important is that teachers need to be able to effectively assess their students' degree of understanding of mathematics concepts. Instructional scaffolding is a teaching strategy that can be used both to develop students' conceptual understandings and to teach students how to write epistemically complex explanations of mathematics. A teacher who includes instructional scaffolding as part of their pedagogy will encounter challenges, but those challenges are worth the benefits. Teachers can implement this practice strategically to develop students' understandings of the most important concepts of a course, and they can assess those understandings based on the epistemic complexity of their students' written responses. In the end, the most meaningful and impactful lesson learned from conducting this research may sound elementary, but that does not diminish its significance – students are capable of demonstrating conceptual understandings of mathematics in writing; they simply must be taught how.

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APPENDICES

Appendix A: Nachowitz's (2018) Coding Framework for Epistemic Complexity

Table 1. Coding framework for epistemic complexity.

Level and definition	Example
1. *Separated pieces of facts*. A statement consisting of a list or table of facts with hardly any integration or connections/or explanation.	"My estimate was correct"
2. *Partially organized facts*. A statement consisting of facts that were loosely organized together. The facts were stated without relating them to each other by means of causal or some other connections. Only a minimal amount of inference seemed to be involved.	"1) You have to read all the problem carefully so you won't miss something important. 2) You get both of the numbers 8 foot and 10 foot. 3) Then you times 8 foot times 2. 4) After you do that you times 10 times 2. 5) Then you add both of these numbers and you should get 36 feet. 6) OK you got the answer for the fencing one, so now its time to do the netting one. 7) You times 8 foot times 2. 8) You times 10 foot times h. 9) Then you should get 36 feet and your done."
3. *Well-organized facts*. A statement consisting of rather well organized factual or descriptive information. Although the ideas did not explicitly provide an explanation, it was meaningfully organized and had a potential for facilitating understanding of the issue in question.	"I started 4 subtraction problems with the country to compare which one has the highest temperature and the lowest temperature. And I found out that Washington has the range of 166° and Kentucky has the range of 151°. As you move west the temperature gets lower and lower as you move east the temperature is colder."
4. *Partial explanation*. A statement represents an explicit attempt to construct an explanation and to provide new information, but the explanation was only partially articulated. It was only an explanatory sketch that was not further elaborated.	"Did you know that baking a cake is a lot like… fractions… ."
5. *Well-organized explanation*. A statement containing postulations of common causes, reasons and other explanatory relations, or theoretical entities.	*No examples from the study met the criteria.*

Appendix B: Cowan's Modified Coding Framework for Epistemic Complexity

Coding Framework for Epistemic Complexity	
Level	Definition
1. *Needs help.*	An explanation that demonstrates little, if any, awareness of the concept being assessed. This could be a response that is entirely incorrect, or it could be a response that is incorrect due to a misunderstanding of the foundational ideas behind a concept.
2. *Not quite.*	An explanation that demonstrates awareness of a concept's definition but does not illustrate how a particular problem or situation is connected to that definition.
3. *Almost there.*	An explanation that demonstrates knowledge of a concept's definition and attempts to make a connection between that definition and a given problem or prompt. It may describe an example that almost connects to the definition but falls short by making an incomplete or slightly incorrect statement.
4. *Nailed it.*	An explanation that demonstrates understanding of a concept's definition and its application. It makes a direct connection between that definition and a given problem or prompt by providing a clear and accurate description, justification, or example of how the concept is being applied.

Appendix C: Informed Consent Agreement

INFORMED CONSENT AGREEMENT

Consent to Participate in a Research Study
Missouri State University
College of Natural and Applied Sciences

WRITING FOR CONCEPTUAL UNDERSTANDING IN THE MATHEMATICS CLASSROOM

Introduction
Students are being asked to participate in a research study. Before a student agrees to participate in this study, it is important that each student reads and understands the following explanation of the study and the procedures involved. The investigators will also explain the project to students in detail. If a student has any questions about the study or his or her role in it, the student should ask the investigators, Dr. Patrick Sullivan and Camry Cowan. If you agree to participate in this study and have questions later, the investigators will answer them for you. Their contact information is listed below.

Voluntary Participation
Participation in this study is completely voluntary and will have no effect on a student's Algebra 2 course grade as participants will learn the same course content and complete the same activities and assignments as non-participants. Students and their parents will need to sign this form giving us their permission to be involved in this study.

It is each student's decision to partake in this study. After providing consent to participate in this study, a student may choose to withdraw at any time without giving a reason. Withdrawing from this study will not affect a student's relationship with the researcher in any way. If a student withdraws from the study before its conclusion, any personal data that was collected will not be included in the study's analyses and conclusions.

Purpose of the Study
This study aims to examine the implementation of a teaching strategy, instructional scaffolding, in the mathematics classroom to assist students in improving their abilities to write at a level that demonstrates conceptual understanding of mathematics.

Description of Procedures
Participation in this study will require students to engage in nothing other than regularly planned classroom activities. I will be implementing a teaching practice called instructional scaffolding with the goal of teaching students how to write about mathematics in a way that demonstrates to the reader they understand the concepts behind the mathematics they are learning. Students' written responses to questions asking them to justify their answers and/or explain their thinking will be collected as evidence and then analyzed by investigators. This study will take place in all of my Algebra 2 courses for the entirety of the fall semester. On occasion, I will have other teachers or my supervisors from MSU observe the class and take notes on their observations. I also will video record some lessons so that I can review and reflect deeply on the instructional scaffolding teaching practice. These video recordings will only be accessible to me and my three supervisors from MSU. I also will

keep a journal to reflect on my own experiences, making note of the challenges and successes I believe I am experiencing while implementing the instructional scaffolding teaching practice.

Potential Risks

The instructional scaffolding process will feature students' working in pairs to explain their thinking to each other, and students may experience some discomfort during these exchanges due to fear of being embarrassed. The instructional scaffolding process will also feature some students' sharing their written explanations in front of the class. These students may experience some discomfort in this circumstance, but sharing in front of the class will be a voluntary decision made by willing students. Students may also experience some discomfort during a lesson that is being video recorded or observed by an outsider.

Benefits

One potential benefit that participants in this study can experience is the development of conceptual understanding of mathematics topics instead of just procedural understanding of mathematics topics. Students will learn mathematics concepts at a deeper level than students are used to learning them.

Another potential benefit is students will be able to communicate their conceptual understandings of mathematics to others. Instead of just knowing what steps to follow to solve a problem, students will understand the reasoning behind that problem-solving process and be able to explain it.

This study could also benefit the field of mathematics education by providing mathematics teachers with a picture of how they can help students improve their abilities to write about mathematics in a way that demonstrates conceptual understanding.

Confidentiality

The name of the school district, teacher, and students will not appear on any information that we share with others. None of these identifiers will be identified by name in any publications that result from this research. We will use pseudonyms to identify any written work or video that may involve the image of a student. The information gathered will be accessible only by the investigators and it will be in a locked facility on campus. Any data saved electronically will be secured on a password-protected computer and cloud storage system. All information not used in publications will be destroyed 10 years after the study ends.

All participants' information will be confidential except in the case of the researcher's being legally obligated to disclose that information. Such a case could include, but is not limited to, the risk of student abuse, neglect, or suicide.

Contact Information

Camry Cowan
[redacted]

Patrick Sullivan, Ph.D.
Missouri State University
901 S. National Avenue, Springfield, MO 65897
(417) 836-5496
PatrickSullivan@MissouriState.edu

Consent to Participate

If you agree to participate in this study, Writing for Conceptual Understanding in the Mathematics Classroom, please provide a signature below. A parent/guardian must also provide their signature giving their consent for your participation in this study.

I, ________________________________ (print student name), have read and understand the information in this form. I have been encouraged to ask questions, and all of my questions have been answered to my satisfaction. By signing this form, I agree voluntarily to participate in this study. I know that I can withdraw from the study at any time. I have received a copy of this form for my own records.

I, ________________________________ (print parent/guardian name), have read and understand the information in this form. I have been encouraged to ask questions, and all of my questions have been answered to my satisfaction. By signing this form, I agree voluntarily that my child can participate in this study. I know that my child can withdraw from the study at any time. I have received a copy of this form for my own records.

______________________________________ ____________________

Student signature Date

______________________________________ ____________________

Parent/guardian signature Date

Videotaping of sessions: It is possible that audio and video excerpts could be used in conference presentations, articles submitted to professional journals, and teacher training. Audio and video excerpts of students at work will be used in professional capacities only if I have explicit parental and student consent from each participant in the group. I will keep these recordings in a password-protected file. The only people who will have access to these recordings will be the principal investigators, Dr. Patrick Sullivan and Camry Cowan.

May the researcher use your child's video or voice records for future research?
<u>Please check two</u> options:

________ I *do not* give permission for my child's image and recorded voice to be **archived** for **future research**, reports, and publications. The records will be destroyed by December 31, 2030.

________ I *do not* give permission for my child's image and recorded voice to be **archived** for **educational** and **training** purposes. The records will be destroyed by December 31, 2030.

________ I give permission for my child's image and recorded voice to be **archived** for use in **future research** reports and publications.

________ I give permission for my child's image and recorded voice to be **archived** for **educational** and **training** purposes

____

Parent Signature Date

I, the undersigned, verify that the above informed consent procedures will be followed.

7/22/2020

____

Person Obtaining Consent – Researcher Date
Dr. Patrick Sullivan

7/22/2020

____

Person Obtaining Consent – Researcher Date
Camry Cowan

Appendix D: IRB Approval

Date: 9-22-2020

IRB #: IRB-FY2020-665
Title: Writing for Conceptual Understanding in the Mathematics Classroom
Creation Date: 4-25-2020
End Date:
Status: Approved
Principal Investigator: Patrick Sullivan
Review Board: MSU
Sponsor:

Study History

Submission Type Initial	Review Type Expedited	Decision Approved

Key Study Contacts

Member Gay Ragan	Role Co-Principal Investigator	Contact gayragan@missouristate.edu
Member Kurt Killion	Role Co-Principal Investigator	Contact kurtkillion@missouristate.edu
Member Patrick Sullivan	Role Principal Investigator	Contact patricksullivan@missouristate.edu
Member Camry Cowan	Role Primary Contact	Contact cowan2@live.missouristate.edu
Member Sarah Nixon	Role Investigator	Contact sarahnixon@missouristate.edu

Appendix E: School District Approval

[redacted]
Assistant Superintendent
[redacted]
Professional Learning and Instructional Support

[redacted]

[redacted] Ed.D Superintendent of Schools

[redacted]
Executive Director of Operations
[redacted]
Director of Communications
[redacted]
Director of Elementary Learning

October 5, 2020

Re: Request to Conduct Research in the [redacted] School District

Title of Research: Writing for Conceptual Understanding in the Mathematics Classroom

Dear Camry Cowan,

Your request to conduct research described in your application submitted on 8/30/2020 is:

__x__ approved with the understanding that no in person observations will be made by the university staff. Upon completion of your study, please provide the office of Curriculum, Instruction, and Assessment a summary of your findings.

_____ pending approval upon completion of the following components

_____ denied at this time. Please contact [redacted] for further details.

Thank you for your interest in conducting research to benefit our instructional practices.

Sincerely,

[redacted]
Asst. Supt of Academic Services

[redacted]
Director of Curriculum, Instruction & Assessment

[redacted]

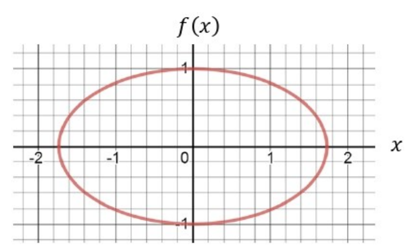
Appendix F: Final Assessment from First Iteration

Algebra 2 | T.1 Function Rules | QUIZ

1) Rule L takes a word as its input at gives the first letter of that word as its output. Is L a function? Why or why not? Explain your thinking.

2) Rule f takes a number, x, as its input and gives $f(x) = x + 5$ as its output. Is f a function? Why or why not? Explain your thinking.

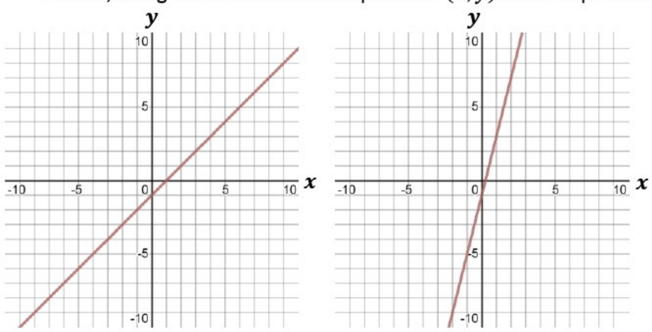
3) Rule f takes a number, x, as its input and gives $f(x)$ as its output. The graph of $f(x)$ is shown. Is $f(x)$ a function? Why or why not? Explain your thinking.



Appendix G: Final Assessment from Second Iteration

Algebra 2 | T.2 Solutions to Equations | QUIZ Name: ______________________

1) Given the equation $y = 4x - 1$, decide which graph represents all of its solutions and which graph does not, and explain why. In your explanation, give at least three examples of (x, y) ordered pairs to support which graph is correct, and give at least one example of an (x, y) ordered pair to support which graph is incorrect.



Appendix H: Final Assessment from Third Iteration

Algebra 2 | T.3 Factoring | Quiz Name: ___________________________

1) ***Micah and Annabell were asked to factor the cubic binomial $x^3 - 1$. Each of their answers is shown below. Whose answer is correct and whose is incorrect? Justify your answer.***

   Micah: $(x-1)(x^2-x+1)$ Annabell: $(x-1)(x^2+x+1)$

Appendix I: Student Rubric for Score Sample Responses Step

1. ***Needs help.***	2. ***Not quite.***	3. ***Almost there.***	4. ***Nailed it.***
An explanation that demonstrates little, if any, awareness of the concept being assessed. This could be a response that is entirely incorrect, or it could be a response that is incorrect due to a misunderstanding of the foundational ideas behind a concept.	An explanation that demonstrates awareness of a concept's definition but does not illustrate how a particular problem or situation is connected to that definition.	An explanation that demonstrates knowledge of a concept's definition and attempts to make a connection between that definition and a given problem or prompt. It may describe an example that almost connects to the definition but falls short by making an incomplete or slightly incorrect statement.	An explanation that demonstrates understanding of a concept's definition and its application. It makes a direct connection between that definition and a given problem or prompt by providing a clear and accurate description, justification, or example of how the concept is being applied.